Practice Exam #3 Solutions

#1 Determine # of Solutions To This System.

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 3 & -1 & -2 & 1 \\ 2 & 3 & -5 & -3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -7 & 7 & 7 \\ 0 & -1 & 1 & 1 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -3 & -2 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Infinitely Many Solutions:

$z = t$, $y = -1 + t$, $x = t$

#2 Solve The System + Determine The Interval In Which x, y, z Lie.

$$\left[\begin{array}{ccc|c} 0 & 2 & 3 & 7 \\ 3 & 6 & -12 & -3 \\ 5 & -2 & 2 & -7 \end{array}\right] \to \left[\begin{array}{ccc|c} 3 & 6 & -12 & -3 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 5 & -2 & 2 & -7 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & -12 & 22 & -2 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 2 & -4 & -1 \\ 0 & 2 & 3 & 7 \\ 0 & 0 & 40 & 40 \end{array}\right] \Rightarrow \begin{array}{l} z = 1 \\ y = 2 \\ x = -1 \end{array}$$

These Lie In $[-7, 2.5]$

So The Answer Is (1)

#3 Let x = "Acceptable", y = "Defective But Repairable", z = "Irreparable"

$$\begin{array}{l} x + y + z = 500 \\ y + z = 50 \\ x - z = 430 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 0 & 1 & 1 & 50 \\ 1 & 0 & -1 & 430 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 0 & 1 & 1 & 50 \\ 0 & -1 & -2 & -70 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & 1 & 500 \\ 0 & 1 & 1 & 50 \\ 0 & 0 & -1 & -20 \end{array}\right]$$

$\Rightarrow z = 20$, $y = 30$, $x = 450$.

#4

$$ABC = \begin{bmatrix} 12 & 14 & 20 & 10 \end{bmatrix} \begin{bmatrix} 20 & 30 & 40 \\ 20 & 20 & 55 \\ 15 & 35 & 45 \\ 25 & 30 & 40 \end{bmatrix} \begin{bmatrix} 800 \\ 10000 \\ 7000 \end{bmatrix} = \begin{bmatrix} 1070 & 1640 & 2550 \end{bmatrix} \begin{bmatrix} 800 \\ 10000 \\ 7000 \end{bmatrix}$$

$= 35,106,000$

= Total of Fares Collected

#5

$$\left[\begin{array}{ccc|ccc} 1 & 4 & -1 & 1 & 0 & 0 \\ 2 & 3 & -2 & 0 & 1 & 0 \\ -1 & 2 & 3 & 0 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 4 & -1 & 1 & 0 & 0 \\ 0 & -5 & 0 & -2 & 1 & 0 \\ 0 & 6 & 2 & 1 & 0 & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 2/5 & -1/5 & 0 \\ 0 & 6 & 2 & 1 & 0 & 1 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 4 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 2 & -\frac{7}{5} & \frac{6}{5} & 1 \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 4 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{7}{10} & \frac{3}{5} & \frac{1}{2} \end{array}\right] \to \left[\begin{array}{ccc|ccc} 1 & 4 & 0 & 3/10 & 3/5 & \frac{1}{2} \\ 0 & 1 & 0 & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -\frac{7}{10} & \frac{3}{5} & \frac{1}{2} \end{array}\right]$$

$$\to \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{13}{10} & \frac{7}{5} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{2}{5} & -\frac{1}{5} & 0 \\ 0 & 0 & 1 & -7/10 & \frac{3}{5} & \frac{1}{2} \end{array}\right]$$

$(1,3)$ Entry Is $\frac{1}{2}$

#6 $A = \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}$ $D = \begin{bmatrix} 50 \\ 10 \end{bmatrix}$

$I - A = \begin{bmatrix} 0.6 & -0.2 \\ -0.1 & 0.8 \end{bmatrix}$ $(I-A)^{-1} = \frac{1}{(0.6)(0.8) - (0.1)(0.2)} \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.6 \end{bmatrix} = \frac{1}{0.46} \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.6 \end{bmatrix}$

$\Rightarrow X = (I-A)^{-1}D = \frac{1}{0.46} \begin{bmatrix} 0.8 & 0.2 \\ 0.1 & 0.6 \end{bmatrix} \begin{bmatrix} 50 \\ 10 \end{bmatrix} = \frac{1}{0.46} \begin{bmatrix} 42 \\ 11 \end{bmatrix} = \begin{bmatrix} 91.3 \\ 23.9 \end{bmatrix}$

(4) $AX = X - D = \begin{bmatrix} 91.3 \\ 23.9 \end{bmatrix} - \begin{bmatrix} 50 \\ 10 \end{bmatrix} = \begin{bmatrix} 41.3 \\ 13.9 \end{bmatrix}$