Exam 3 Review

Disclaimer: Exam 3 covers Chapters 9.1-9.4, 10.1-10.3, and 11.1-11.3. This review may not cover all the material that will be on the exam.
Exam 3 can cover any material from the lectures, homework, quizzes, etc.
Final answers to these problems will only be released after the exam review. This is to focus your studying on understanding the process of solving them, rather than the final answer. Feel free to use the discussion boards to discuss these problems.

1. Solve the differential equation.

   (a) $y' = xy^2$

   (b) $y' = \dfrac{y \ln x}{x}$

   (c) $y' = 2 - y$ with initial condition $y(0) = 3$.

   (d) $y' = 3xy - 2x$ with initial condition $y(0) = 1$.

2. Use Euler's method to approximate the indicated function value to 3 decimal places, using $h = 0.1$.

   (a) $\dfrac{dy}{dx} = x^2 + y^2$, $y(0) = 2$; find $y(0.5)$.

   (b) $\dfrac{dy}{dx} = 1 + \dfrac{y}{x}$, $y(1) = 0$; find $y(1.4)$.

3. Find the value of k that will make f a probability density function on the indicated domain.

   (a) $f(x) = k\sqrt{x}$; $1 \leq x \leq 4$

   (b) $f(x, y) = ky\sqrt{x}$; $D = \{0 \leq x \leq 2;\ 1 \leq y \leq 2\}$

4. f is a probability density function for a random variable X defined on the given interval. Find the indicated probabilities.

   (a) $f(x) = \dfrac{1}{4\sqrt{x}}$; $\quad [1, 9]$

      (i) $P(X \geq 4)$

      (ii) $P(X \leq 4)$

      (iii) $P(1 \leq X < 8)$

      (iv) $P(X = 3)$

   (b) $f(x) = 2xe^{-x^2}$; $\quad [0, \infty)$

      (i) $P(X \leq 4)$

      (ii) $P(X \geq 2)$

      (iii) $P(1 < X < 2)$

5. f is a joint probability density function for the random variables X and Y on D. Find the indicated probabilities.

   (a) $f(x,y) = xy; \quad D = \{0 \le x \le 1;\ 0 \le y \le 2\}$

      (i) $P(0 \le X \le 1;\ 0 \le Y \le 1)$

      (ii) $P(X + 2Y \le 1)$

   (b) $f(x,y) = \dfrac{1}{12}(x+y); \quad D = \{0 \le x \le 2;\ 1 \le y \le 3\}$

      (i) $P(1 \le X \le 2;\ 0 \le Y \le 1)$ (Not a typo)

      (ii) $P(Y \ge 1;\ X + Y \le 3)$

6. Find the mean, variance, and standard deviation of the random variable X associated with the probability density function over the indicated interval.

   (a) $f(x) = \dfrac{3}{8}x^2 \quad [0,2]$

   (b) $f(x) = \dfrac{3}{x^4} \quad [1,\infty)$

   (c) $f(x) = \dfrac{5}{2}x^{3/2} \quad [0,1]$

7. Let Z be the standard normal random variable. Find the value of the given probability, then make a sketch of the area under the standard normal curve associated with this probability.

   (a) $P(Z < 1.38)$

   (b) $P(Z > 2.27)$

   (c) $P(-1.48 < Z < 1.54)$

8. The medical records of infants delivered at Shands Hospital show that the infants' lengths at birth (in inches) are normally distributed with a mean of 20 in and a standard deviation of 2.6 in. Find the probability that an infant selected at random from among those at the hospital measures:

   (a) More than 22 in.

   (b) Less than 18 in.

   (c) Between 19 and 21 in.

9. Let Z be the standard normal random variable. Find the values of z such that:

   (a) $P(Z < z) = 0.8907$

   (b) $P(Z > z) = 0.9678$

10. Find the third degree Taylor polynomial of the function $f(x)$ centered at $x = a$.

    (a) $f(x) = \ln(x+1); \quad a = 0$

    (b) $f(x) = \sqrt{x}; \quad a = 4$

    (c) $f(x) = \dfrac{1}{1-x}; \quad a = 2$

11. Using your answer from 10(a), estimate

$$\int_0^{1/2} \ln(x+1)dx.$$

    Compare your result with the exact value of the integral using integration by parts. What is the exact error associated with the approximation?

12. Approximate $\sqrt{1.1}$ using the second degree Taylor polynomial of $f(x) = \sqrt{x+1}$ centered at $x = 0$. Without calculating $\sqrt{1.1}$, find a bound for the error in the approximation.

13. Write down the first three terms of the sequence $\{a_n\}$. Then determine the convergence or divergence of the sequence.

    (a) $a_n = 2^{n-1}$

    (b) $a_n = \dfrac{2n^2}{n^2+1}$

14. Find the n^{th} term of the sequence (assuming that the 'obvious' pattern continues). Then determine whether the sequence converges or diverges.

    (a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \ldots$

    (b) $1, 4, 7, 10, \ldots$

15. Given a_n, determine the convergence or divergence of both $\{a_n\}$ and $\sum_{n=0}^{\infty} a_n$. If the sequence/series converges, determine what it converges to.

    (a) $a_n = \dfrac{1}{n+1} - \dfrac{1}{n+2}$

    (b) $a_n = 2e^{-n}$

16. Determine whether the series converges or diverges. If it converges, find its sum.

    (a) $\displaystyle\sum_{n=0}^{\infty} 4\left(-\frac{2}{3}\right)^n$

    (b) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{5}{4}\right)^n$

    (c) $1 - \dfrac{1}{3} + \dfrac{1}{9} - \dfrac{1}{27} + \cdots$