MAC 2234: Survey of Calculus II

Practice Exam # 2

The actual exam will be very similar to this practice test. You will have 120 minutes to complete the exam in Canvas. I suggest you attempt this under time restrictions to get the best practice possible.

(1) Compute the first partial derivatives of $f(x, y) = \sqrt{x^3y + y^2}$

(2) The productivity of a country in Africa is given by the equation

$$f(x, y) = 25x^{1/3}y^{2/3}$$

when x units of labor and y units of capital are used. Compute the marginal productivity of labor and the marginal productivity of capital when 27 units of labor and 64 units of capital are used. Should the government encourage increased expenditure on labor rather than capital investiment at this time in order to increase the country's productivity?

(3) The total daily revenue (in dollars) that a publishing company realizes in publishing and selling its dictionaries is given by

$$R(x, y) = -0.005x^2 - 0.003y^2 - 0.002xy + 20x + 15y$$

where x denotes the number of deluxe copies and y denotes the number of standard copies published and sold daily. The total daily cost of publishing these dictionaries is given by

$$C(x, y) = 6x + 3y + 200$$

dollars. Determine how many copies of each style the company should publish each day to maximize its profits. What is the maximum profit realizable?

(4) The monthly profit (in dollars) of a department store depends on the level of inventory x (in thousands of dollars) and floor space y (in thousands of square feet) available for display of the merchandise, as given by

$$P(x, y) = -0.02x^2 - 15y^2 + xy + 39x + 25y - 20,000$$

Currently the level of inventory is \$4,000,000 ($x = 4000$) and the floor space is $150,000\,\text{ft}^2$ ($y = 150$). Find the anticipated change in monthly profit if management increases the level of inventory by \$500,000 and decreases the floor space for display of merchandise by 10,000 ft^2.

(5) Use the method of Lagrange multipliers to find the minimum of $f(x, y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.

(6) Evaluate the double integral of $f(x, y) = y$, where R is the region bounded by $x = 0$, $x = \sqrt{4 - y^2}$, and $y = 0$.

(7) Solve the differential equation

$$y' = \frac{2y+3}{x^2}$$

(use C for the constant of integration).

(8) The personnel manager of a company estimates that the number of insurance claims an experienced clerk can process in a day is 40. Furthermore, the rate at which a clerk can process claims per day during the t-th week of training is proportional to the difference between the maximum number possible (40) and the number he or she can process per day in the t-th week. If the number of claims the average trainee can process after 2 weeks on the job is 10 per day, determine how many claims the average trainee can process after 6 weeks on the job. (Assume the trainee can initially process 0 claims per day.)

(9) Use Euler's method with $n = 5$ to obtain approximations to the solution of the initial value problem over the indicated interval:

$$y' = 2x - y + 1, \quad y(0) = 2, \quad 0 \leq x \leq 2$$