PRACTICE EXAM #1 SOLUTIONS

#1 $\int (4x^2 - x^{-3} + e^{2x})\,dx = \frac{4}{3}x^3 + \frac{1}{2x^2} + \frac{1}{2}e^{2x} + C$

#2 $\int \frac{3(\ln x)^2}{x}dx$ LET $u = \ln x$. THEN $du = \frac{1}{x}dx$. SO

$$\int \frac{3(\ln x)^2}{x}dx = 3\int u^2\,du = u^3 + C = (\ln x)^3 + C$$

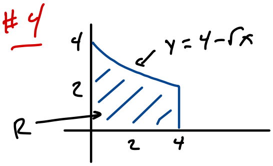
#3 FIND THE LEFT ENDPOINT RIEMANN SUM L_4 FOR APPROXIMATING $\int_0^2 (8 - x^3)\,dx$

HERE $\Delta x = \frac{2-0}{4} = \frac{1}{2}$ THE LEFT ENDPOINTS ARE $0, \frac{1}{2}, 1, \frac{3}{2}$

$$0 \quad \tfrac{1}{2} \quad 1 \quad \tfrac{3}{2} \quad 2$$

$$\begin{aligned}
\text{So, } L_4 &= f(0)(\tfrac{1}{2}) + f(\tfrac{1}{2})(\tfrac{1}{2}) + f(1)(\tfrac{1}{2}) + f(\tfrac{3}{2})(\tfrac{1}{2}) \\
&= \tfrac{1}{2}\left\{8 + (8 - \tfrac{1}{8}) + (8-1) + (8 - \tfrac{27}{8})\right\} \\
&= \tfrac{1}{2}\left\{32 - \tfrac{1+8+27}{8}\right\} = \tfrac{1}{2}\left[32 - \tfrac{36}{8}\right] = \tfrac{1}{2}\left[\tfrac{256-36}{8}\right] = \tfrac{1}{2}\left(\tfrac{220}{8}\right) = \tfrac{55}{4} \\
&= \underline{13.75}
\end{aligned}$$

#4



$$\begin{aligned}
A = \int_0^4 (4 - \sqrt{x})\,dx &= 4x - \tfrac{2}{3}x^{3/2}\Big|_0^4 \\
&= \left(4(4) - \tfrac{2}{3}4^{3/2}\right) - \left(4(0) - \tfrac{2}{3}(0)\right) \\
&= 16 - \tfrac{2}{3}(8) \\
&= \tfrac{32}{3}
\end{aligned}$$

#5 $\int x^2 \ln x\,dx$ $\quad u = \ln x \quad v = \frac{1}{3}x^3$

$du = \frac{1}{x}dx \quad dv = x^2\,dx$

$$\begin{aligned}
\int x^2 \ln x\,dx &= \tfrac{1}{3}x^3 \ln x - \int \tfrac{1}{3}x^3(\tfrac{1}{x})\,dx \\
&= \tfrac{1}{3}x^3 \ln x - \tfrac{1}{3}\int x^2\,dx \\
&= \tfrac{1}{3}x^3 \ln x - \tfrac{1}{9}x^3 + C
\end{aligned}$$

#6 $\int_1^3 x e^{-x}\,dx$ First Integrate By Parts To Find The Antiderivative:

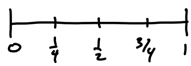
$$u = x \qquad v = -e^{-x}$$
$$du = dx \qquad dv = e^{-x}dx$$

$$\int x e^{-x}\,dx = -xe^{-x} + \int e^{-x}\,dx = -xe^{-x} - e^{-x} + C$$

So, $\int_1^3 x e^{-x}\,dx = -xe^{-x} - e^{-x}\Big|_1^3$

$$= \left(-3/e^3 - \frac{1}{e^3}\right) - \left(-\frac{1}{e} - \frac{1}{e}\right)$$
$$= \frac{2}{e} - \frac{4}{e^3} \approx 0.53661$$

#7 $\int_0^1 \sqrt{1+x^4}\,dx$ $\quad n=4 \quad \frac{b-a}{n} = \frac{1-0}{4} = \frac{1}{4}$



Trapezoidal: $\frac{\frac{1}{4}}{2}\left(f(0) + 2f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 2f(\tfrac{3}{4}) + f(1)\right)$

$$= \frac{1}{8}\left(1 + 2\sqrt{1+(\tfrac{1}{4})^4} + 2\sqrt{1+(\tfrac{1}{2})^4} + 2\sqrt{1+(\tfrac{3}{4})^4} + \sqrt{2}\right)$$

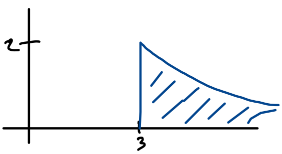
$$\approx 1.09679547\ldots$$

Simpson's: $\frac{\frac{1}{4}}{3}\left(f(0) + 4f(\tfrac{1}{4}) + 2f(\tfrac{1}{2}) + 4f(\tfrac{3}{4}) + f(1)\right)$

$$= \frac{1}{12}\left(1 + 4\sqrt{1+(\tfrac{1}{4})^4} + 2\sqrt{1+(\tfrac{1}{2})^4} + 4\sqrt{1+(\tfrac{3}{4})^4} + 2\right)$$

$$\approx 1.0894134\ldots$$

#8



$$\int_3^\infty \frac{16}{(x+1)^{3/2}}\,dx = \lim_{b\to\infty} \int_3^b \frac{16}{(x+1)^{3/2}}\,dx = \lim_{b\to\infty} \frac{16}{-1/2}(x+1)^{-1/2}\Big|_3^b$$

$$= \lim_{b\to\infty} \frac{-32}{(b+1)^{1/2}} + \frac{32}{(4)^{1/2}}$$

$$= 0 + \frac{32}{2} = \underline{\underline{16}}$$

#9 $f(x) = 0.006x(10-x) \quad 0 \le x \le 10$ $\qquad$ Note: $f(x) \ge 0$ on $[0, 10]$

$$\int_0^{10} 0.006x(10-x)\,dx = \int_0^{10} 0.006(10x - x^2)\,dx$$

$$= 0.006\left(5x^2 - \frac{x^3}{3}\right)\Big|_0^{10}$$

<u>YES</u>

$$= 0.006\left[500 - \frac{1000}{3}\right] = 0.006\left[\frac{500}{3}\right] = 1$$

#10 LET T BE THE RANDOM VARIABLE MEASURING THE LIFE SPAN OF A BULB

(a) $P(T \le 200) = \int_0^{200} 0.001 e^{-0.001t}\,dt = -e^{-0.001t}\Big|_0^{200}$

$= -e^{-0.2} + 1$

$= 0.181269\ldots$

(b) $P(T \geq 800) = \int_{800}^{\infty} 0.001 e^{-0.001t}\,dt = \lim_{b\to\infty} -e^{-.001t}\Big|_{800}^{b}$

$= \lim_{b\to\infty} -e^{-.001b} + e^{-0.8}$

$= e^{-0.8}$

$= 0.44932896\ldots$

#11 $f(x) = 64/x^5 \quad 2 \le x < \infty$

$\mu = \int_2^{\infty} x \cdot \frac{64}{x^5}\,dx = \int_2^{\infty} \frac{64}{x^4}\,dx = \frac{-64}{3x^3}\Big|_2^{\infty} = \frac{64}{3\cdot 8} = \frac{8}{3}$

$V = \int_2^{\infty} x^2\, \frac{64}{x^5}\,dx - \left(\frac{8}{3}\right)^2 = \int_2^{\infty} \frac{64}{x^3}\,dx - \frac{64}{9}$

$= \frac{-32}{x^2}\Big|_2^{\infty} - \frac{64}{9}$

$= \frac{32}{4} - \frac{64}{9} = 8 - \frac{64}{9} = \frac{8}{9}$

$S = \sqrt{V} = \frac{\sqrt{8}}{3}$ So, $\mu + V + S = \frac{8}{3} + \frac{8}{9} + \frac{\sqrt{8}}{3} = \frac{24 + 8 + 3\sqrt{8}}{9}$

$\approx 4.498364\ldots$

#12 $f(x) = \sqrt{1+x} \quad a = 3$

$f(x) = \sqrt{1+x} \qquad f(3) = 2$

$f'(x) = \frac{1}{2\sqrt{1+x}} \qquad f'(3) = \frac{1}{4}$

$f''(x) = -\frac{1}{4(1+x)^{3/2}} \qquad f''(3) = \frac{-1}{4\cdot 8} = \frac{-1}{32}$

$P_2(x) = 2 + \frac{1}{4}(x-3) - \frac{1}{64}(x-3)^2$

$\frac{f''(3)}{2!} = \frac{-1}{32\cdot 2}$