The Final exam covers Lectures 1 – 34

MAC 2233: Exam 1 Review
Unit 1 Exam Review covers Lectures 1 – 14

1. Solve for x: $2(x+1)^{-1/3}x^{4/3} - (x+1)^{2/3}x^{-2/3} = 0$

2. Perform the operation and simplify the expression: $\dfrac{\dfrac{3x}{\sqrt{x^2+4}} - \sqrt{x^2+4}}{2\sqrt{x^2+4}}$

3. Solve the inequality: $\dfrac{x+4}{x-1} \leq 2$

4. Find and simplify $\dfrac{f(x+h)-f(x)}{h}$ for

a) $f(x) = 2x^2 - x - 3$ and b) $f(x) = \dfrac{x}{x+4}$.

5. Let $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{2}{x} + 1$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$. Include domains.

6. Let $f(x) = \sqrt{x-1}$ and $g(x) = \dfrac{x}{\sqrt{x-1}}$. Find $\dfrac{f}{g}(x)$ and its domain.

7. Sketch the graph of $f(x) = 3 - 2x - x^2$ by using a formula to find the vertex. Show all intercepts. Confirm your work by writing your function in standard form $f(x) = a(x-h)^2 + k$ by completing the square, and using translations to graph.

8. Sketch the graph of $f(x) = 2 - \sqrt{1-x}$. Starting with $y = \sqrt{x}$, list each translation used to graph $f(x)$.

9. Use the definition of absolute value to write the function $g(x) = x|x|$ as a piecewise defined function. Then sketch its graph.

10. Find the inverse of $f(x) = \sqrt{4-x}$. Be sure to include domain.

11. Find the inverse of one-to-one function $f(x) = \dfrac{x+2}{x-3}$. Use that inverse function to find the range of $f(x)$. Then find the horizontal asymptote of $f(x)$ if possible.

12. Find the domain of the following functions:

    (a) $f(x) = \sqrt{x^3 - x^2 - 6x}$ (b) $f(x) = \ln\left(\dfrac{8}{x} - 2\right)$

13. Find the solution set of each of the following equations:

    (a) $\log_3(2x^2 - 5) - \log_3 x = 1$ (b) $4^{3-x^2} = \left(\dfrac{1}{8}\right)^{x+1}$

    (c) $\ln(x+8) + \ln(x-2) = \ln(3x+2)$

14. Find the inverse of $f(x) = e^{x+3} - 4$. Sketch the graph of f and f^{-1} on the same axes. Include at least one point and any asymptotes of each function.

15. Let $f(x) = \begin{cases} x+4 & x < -2 \\ 2 - |x| & -2 \leq x < 2 \\ \ln(x-1) & x > 2 \end{cases}$.

(a) Find if possible: $f(-4)$, $f(-2)$, $f(0)$, $f(2)$, $f(e+1)$.

(b) Sketch the graph of $y = f(x)$. (c) Use your graph to evaluate the following

limits if they exist:

1) $\lim_{x \to -2} f(x)$ 2) $\lim_{x \to 0} f(x)$ 3) $\lim_{x \to 2} f(x)$

16. Let $f(x) = \dfrac{x^2 - 4}{x^2 - 2x - 8}$. Find:

(a) domain of f

(b) all intercepts (express as ordered pairs)

(c) all vertical and horizontal asymptotes

(d) Sketch the graph of $y = f(x)$. Include the coordinates of any holes in the function.

(e) Use your graph to find $\lim_{x \to -2} f(x)$.

17. There is a linear relationship between temperature in degrees Celsius C and degrees Fahrenheit F. Water freezes at $0°C\,(32°F)$ and boils at $100°C\,(212°F)$. Write the model expressing C as function of F. What is the temperature in degrees Fahrenheit if the temperature is $30°C$? What does the slope of the line tell you?

18. The demand and supply functions for a given product are given by

    $p = D(q) = 60 - 2q^2$ and $p = S(q) = q^2 + 9q + 30$ where q is quantity in thousands and p is the unit price. Find the equilibrium quantity and price.

    How many items will the supplier provide if the unit price of the product is \$40? What will be the demand for the product when the unit price is \$40? What should happen to the price of the product?

19. A financial manager at Target has made the following observations about a certain product in one of its districts: an average of 250 units will sell in a month when the price is \$15, but an average of 50 more will sell if the price is reduced by \$1. Assuming the demand function is linear,

(a) Express p as a function of x.

(b) Find the revenue function $R(x)$. Find the production level x that will maximize revenue. What is the maximum revenue?

(c) If fixed costs are \$800 and the marginal cost is \$10 per item, find each value of x at which the company will break even. What is the profit for those values?

(d) Find the profit function $P(x)$. What price should the manager charge to maximize profit on this item?

20. A farmer plans to spend \$6000 to enclose a rectangular field with two kinds of fencing. Two opposite sides will require heavy-duty fencing that costs \$3 per linear foot, while the other two sides can be constructed with standard fencing that costs \$2 per foot. Express the area of the field, A, as a function of x, the length of a side that requires the more expensive fence. Find the value of x that will maximize the area of the field, and the length of a side that uses standard fencing.

21. Rewrite the expression as the sum, difference, or multiple of logarithms:

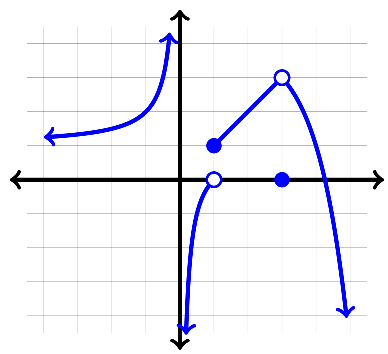
(a) $\log \dfrac{x^2}{1000}$ (b) $\ln \sqrt[3]{\dfrac{e^{x+1}(x-2)^4}{x^6}}$

22. Mr. Jones invested \$2500 at 5.5% compounded continuously. How long will it take his account to grow to \$4000 if he adds no new funds to the account?

23. How much money must be invested now at 3 1/4% compounded quarterly in order to have \$6000 in three years?

24. Iodine - 131 has a half-life of 8 days. Suppose some hay was contaminated with ten times the allowable amount of I-131. How long must the hay be stored before it can be fed to cattle? Hint: the hay must have one-tenth of its current amount of I-131.

25. Use the following graph of a function $f(x)$ to evaluate the limits and function value if possible. If the limit does not exist, write "dne".



a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$

d) $\lim_{x\to 1+} f(x)$ e) $\lim_{x\to 1^-} f(x)$ f) $\lim_{x\to 1} f(x)$

g) $\lim_{x\to 3} f(x)$ h) $f(3)$ i) $\lim_{x\to -1} f(x)$

26. Use the properties of limits to evaluate $\lim_{x\to a} \frac{(fg)(x)}{\sqrt[3]{g(x)} - 1}$ if $\lim_{x\to a} f(x) = -\frac{1}{3}$ and $\lim_{x\to a} g(x) = 9$.

27. Evaluate (a) $\lim_{x \to -1} \frac{x + \sqrt{x+2}}{x+1}$ and (b) $\lim_{x \to 2} \frac{\frac{2}{x} - 1}{x-2}$.

28. If $f(x) = \begin{cases} \frac{x^2 - 16}{x^2 + 3x - 4} & x \neq -4 \\ 0 & x = -4 \end{cases}$

find $p = \lim_{x \to -4} f(x)$ and $q = \lim_{x \to 1^-} f(x)$.

29. Sketch the graph of $f(x) = \dfrac{|6-2x|}{x-3}$. Hint: rewrite as a piecewise function without absolute value bars.

Use the graph to find: (a) $\lim_{x\to 3^-} f(x)$, (b) $\lim_{x\to 3^+} f(x)$, and (c) $\lim_{x\to 3} f(x)$.

Now find those limits algebraically without using the graph.

30. If $f(x) = \dfrac{x^3+3x^2+2x}{x-x^3}$, find a) $\lim_{x\to 0^+} f(x)$ b) $\lim_{x\to -1^+} f(x)$, c) $\lim_{x\to 1^-} f(x)$ and d) $\lim_{x\to -\infty} f(x)$. Find each vertical and horizontal asymptote of $f(x)$.

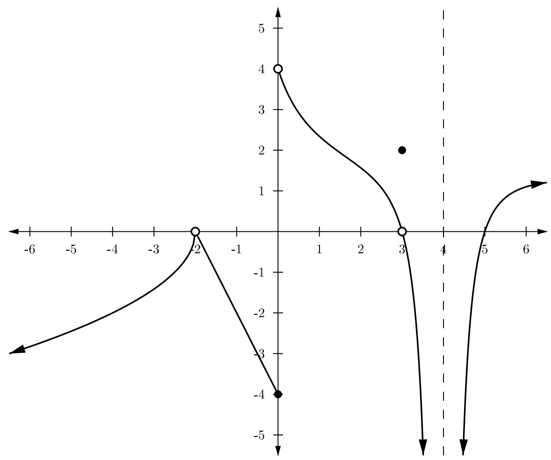
31. If $f(x) = \dfrac{2}{e^{-x} - 3}$, find if possible:

1) $\lim\limits_{x \to -\infty} f(x)$ 2) $\lim\limits_{x \to +\infty} f(x)$ 3) Each asymptote of the graph of $f(x)$.

32. The Intermediate Value Theorem guarantees that the function $f(x) = x^3 - \dfrac{1}{x} - 5x + 3$ has a zero on which of the following intervals?

a) $[-1, 1]$ b) $[1, 3]$ c) $[3, 5]$ d) $[-3, -2]$

33. Consider a function $f(x)$ which has the following graph.



(a) On which interval(s) is $f(x)$ continuous?

(b) $f(x)$ has a jump discontinuity at $x =$________.

(c) $f(x)$ has an infinite discontinuity at $x =$________.

(d) $f(x)$ has a removable discontinuity at $x =$________.

(e) How would you define or redefine $f(x)$ at the point(s) in part (d) in order to make $f(x)$ continuous?

MAC 2233: Unit 2 Exam Review
Lectures 15 – 24

1. Use the definition of derivative to evaluate $f'(x)$ if $f(x) = \sqrt{2x-1}$. Check your answer using a derivative rule.

2. (a) Use the **definition of derivative** to find $f'(x)$ if $f(x) = \dfrac{x}{2x-1}$. Check your answer using the Quotient Rule.

   (b) Find each interval over which $f(x)$ is differentiable.

   (c) Write the equation of the tangent line to $f(x) = \dfrac{x}{2x-1}$ at $x = -1$.

3. Indicate whether each of the following statements is true or false.

   (a) If f is continuous at $x = a$, then f is differentiable at $x = a$.

   (b) If f is not continuous at $x = a$, then f is not differentiable at $x = a$.

   (c) If f has a vertical tangent line at $x = a$, then the graph of $f'(x)$ has a vertical asymptote at $x = a$.

4. If an object is projected upward from the roof of a 200 foot building at 64 ft/sec, its height h in feet above the ground after t seconds is given by

   $h(t) = 200 + 64t - 16t^2$. Find the following:

   (a) The average velocity of the object from time $t = 0$ until it reaches its maximum height (hint: consider the graph of the function)

   (b) The instantaneous velocity of the object at time $t = 1$ second using the limit definition.

5. Find each value at which $f(x) = \dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x$ is parallel to the line $2y - 8x + 9 = 0$.

6. Find the value of a so that the tangent line to $y = x^2 - 2\sqrt{x} + 1$ is perpendicular to the line $ay + 2x = 2$ when $x = 4$.

7. If $f(x) = (x^3 - 2x)(2\sqrt{x} + 1)$, find $f'(x)$ two ways: rewriting $f(x)$ and differentiating, and using the Product Rule.

8. Find each value of x at which $f(x) = (1 - x)^5(5x + 2)^4$ has a horizontal tangent line.

9. Let $f(x) = \dfrac{(\sqrt{x} - 1)^2}{x}$. Find $f'(x)$ and write as a single fraction. Write the equation of the tangent line to $f(x)$ at $x = 4$.

10. Write the equation of the tangent line to $f(x) = \left(x - \frac{6}{x}\right)^3$ at $x = 3$.

11. Find each value of x at which the function $f(x) = \frac{\sqrt[3]{6x+1}}{x}$ has

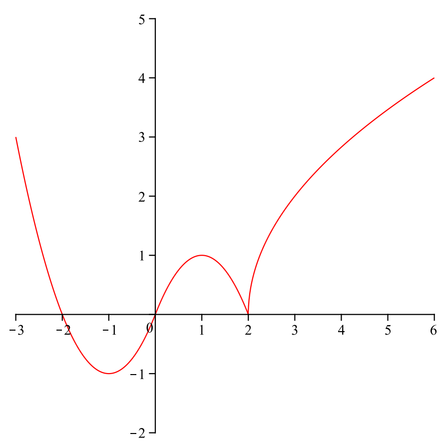
(a) horizontal and (b) vertical tangent lines.

Write the equation of each of those lines.

12. Suppose that $f(4) = -1$, $g(4) = 2$, $f(-4) = 1$, $g(-4) = 3$, $f'(4) = -2$, $g'(4) = 12$, $f'(-4) = 6$, and $g'(-1) = -2$.

Find: (a) $h'(4)$ if $h(x) = g(f(x))$ and (b) $H'(4)$ if $H(x) = \sqrt{xf(x) + \frac{x^2}{2}}$.

13. Sketch a possible graph of the derivative of the function $y = f(x)$ shown below.



14. Find the derivative:

 (a) $f(x) = 3^{2x-1}$ (b) $f(x) = \log_4(x^2 - x)$

15. Find the slope of the tangent line to the curve given by $\sqrt{3x - y} - e^{x+y} = 1 + \ln x$ at $(1, -1)$.

16. Find the first three derivatives of
 $f(x) = \ln(x^2 + 1)$.

17. Find each value of x at which $f(x) = \ln\left(\frac{2x-6}{\sqrt{x^2+3}}\right)$ has a horizontal tangent line.

18. Find $f'(x)$ if $f(x) = \ln \frac{e^{x-3}\sqrt[3]{6+3x}}{(3x+1)^2}$.

19. Use Logarithmic Differentiation to find the slope of the tangent line to $f(x) = x^{\sqrt{x}}$ at $x = 4$.

20. Find each value at which $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$ has a relative maximum or minimum.

Find the absolute extrema of $f(x)$ on $[-2, 1]$.

21. Let $f(x) = \frac{\sqrt[3]{3x-2}}{x}$.

(a) Find $f'(x)$ and write as a single fraction.

(b) Find the equation of each horizontal and vertical tangent line of $f(x)$.

(c) Find each x-value at which $f(x)$ has a critical number.

(d) Find the relative extreme values of $f(x)$.

22. The cost function for a product is
$C(x) = 1.25x^2 + 25x + 8000.$

(a) Suppose the company decides to increase production by 4 units per day when the current daily production level is 50 units. Find the rate of change of cost with respect to time.

(b) If $C(x) = 1.25x^2 + 25x + 8000$, find each interval on which **average cost** is increasing and decreasing. For what production level x is average cost minimized?

23. The demand function for a certain product is given by

$p(x) = -0.02x + 400$, $0 \leq x \leq 20,000$, where p is the unit price when x items are sold. The cost function for the product is

$C(x) = 100x + 300,000$.

(a) Find the marginal profit of the product when $x = 2000$.

(b) Find the actual profit from the sale of the 2001st item. Compare to your answer in (a).

(c) Find each interval on which the profit function $P(x) = -0.02x^2 + 300x - 300,000$ is increasing and decreasing. Remember that $0 \leq x \leq 20,000$. How many items should be sold to maximize profit? At what price?

MAC 2233: Unit 3 Exam Review
Lectures 25 – 31

1. Find each value at which $f(x) = \dfrac{x^4}{4} - \dfrac{x^3}{3} - x^2$ has a relative maximum or minimum.

   Find the absolute extrema of $f(x)$ on $[-2, 1]$.

2. Find all critical numbers and relative extrema of $g(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{x^2}$.

3. Let $f(x) = \dfrac{\sqrt[3]{3x-2}}{x}$.

   (a) Find $f'(x)$ and write as a single fraction.

   (b) Find the equation of each horizontal and vertical tangent line of $f(x)$.

   (c) Find each x-value at which $f(x)$ has a critical number.

   (d) Find the relative extreme values of $f(x)$.

4. The cost function for a product is $C(x) = 1.25x^2 + 25x + 8000$.

   (a) Suppose the company decides to increase production by 4 units per day when the current daily production level is 50 units. Find the rate of change of cost with respect to time.

   (b) If $C(x) = 1.25x^2 + 25x + 8000$, find each interval on which **average cost** is increasing and decreasing. For what production level x is average cost minimized?

5. The demand function for a certain product is given by $p(x) = -0.02x + 400$, $0 \le x \le 20,000$, where p is the unit price when x items are sold. The cost function for the product is $C(x) = 100x + 300,000$.

   (a) Find the marginal profit of the product when $x = 2000$.

   (b) Find the actual profit from the sale of the 2001st item. Compare to your answer in (a).

   (c) Find each interval on which the profit function $P(x) = -0.02x^2 + 300x - 300,000$ is increasing and decreasing. Remember that $0 \le x \le 20,000$. How many items should be sold to maximize profit? At what price?

6. Find all relative extrema of $f(x) = 2x^{5/3} - 5x^{2/3}$. Then find the absolute extrema of $f(x)$ on $[-8, 0]$. Compare the two methods.

7. Find the absolute maximum and minimum

   values of $f(x) = e^{x^3-12x}$ on $[0, 3]$.

8. Find the maximum and minimum values of $f(x) = x^2 - 8\ln x$ on $[1, e]$.

9. The position (in centimeters) of a particle moving in a straight line at time t (in seconds) is given by $s(t) = t^3 - 6t^2 + 9t$ for $0 \le t \le 6$.

   (a) Find the velocity function $v(t)$.

   (b) At what time(s) is the particle at rest?

   (c) For what time interval(s) over the first six seconds is the particle traveling in a positive direction?

   (d) Find the average velocity from $t = 0$ to $t = 4$ seconds.

   (e) What is the acceleration of the particle after 3/2 second? Include units in your answer.

   (f) Find each interval on which the particle is (1) speeding up and (2) slowing down

10. Find each interval on which $f(x) = e^{1-x^2}$ is concave up and down, and find each inflection point of the graph of f.

11. Find all intervals on which the graph of

    $f(x) = \dfrac{x^4}{4} + 2x^3 + \dfrac{9}{2}x^2 + 8$ is both decreasing

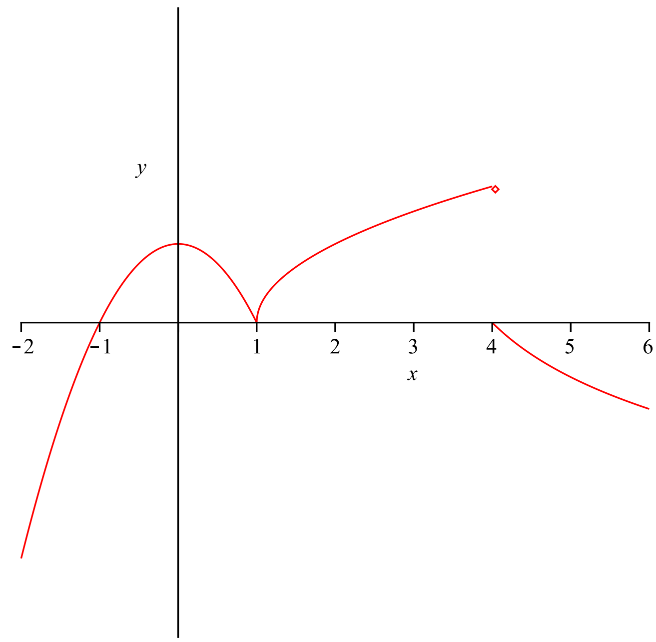
    and concave up.

12. Find each interval on which $f(x) = \ln x + \dfrac{1}{x}$

    is both increasing and concave down. Find each inflection point.

13. Suppose that $f(x)$ has horizontal tangent lines at $x = -2$, $x = 1$ and $x = 5$. If $f''(x) > 0$ on intervals $(-\infty, 0)$ and $(2, \infty)$ and $f''(x) < 0$ on the interval $(0, 2)$, find the x- values at which $f(x)$ has relative extrema. Assume that f and f' are continuous on $(-\infty, \infty)$ and use the Second Derivative test.

14. Given the graph of **the derivative** $f'(x)$, find each interval on which the function $f(x)$ is increasing and decreasing, and find the x-coordinate of each point at which $f(x)$ has a local maximum or minimum value. Find each interval on which $f(x)$ is concave up and down, and the x-coordinate of each inflection point. Assume that the domain of $f(x)$ is $(-\infty, \infty)$.



15. Suppose that $f'(x) = 15x^4 - 15x^2$. Find each x-value at which the function $f(x)$ has relative extrema. Find the x-coordinate of each inflection point. Sketch a possible graph of $f(x)$ if $f(x)$ passes through the origin.

16. A drug that stimulates reproduction is introduced into a population of viruses. That population can be modeled by the function $P(t) = 30t^2 - t^3 + 200$, $0 \leq t \leq 30$, where $P(t)$ is the population after t minutes.

    (a) At what time does the population reach its maximum? What is the maximum population?

    (b) At what time is the rate of growth of the population maximized?

17. A farmer wishes to fence an area next to his barn. He needs a wire fence that costs \$1 per linear foot in front of the barn and and wooden fencing that costs \$2 per foot on the other sides. Find the lengths x (sides perpendicular to the barn) and y (side across from the barn) so that he can enclose the maximum area if his budget for materials is \$4400.

18. Frye's Electronics has started selling a new video game. In one of its Dallas stores, an average of 50 games sell per month at the regular price of \$40. The manager of the department has observed that when the video game is put on sale, an average of 5 more games will sell for each \$2 price decrease. If each video game costs the store \$24 and there are fixed costs of \$5600, how many should be sold in a given month to maximize profit? What price should they charge? Assume that the demand function $p(x)$ is linear.

19. The revenue $R(x)$ generated from sales of a certain product is related to the amount of money spent on advertising according to the model $R(x) = \dfrac{1}{10,000}(600x^2 -$

x^3), $0 \le x \le 600$, where x and $R(x)$ are measured in thousands of dollars. Find each interval over which $R(x)$ is increasing. For the interval on which $R(x)$ is increasing, find the point of diminishing returns. Why is it significant?

20. Consider the function $f(x) = x^{1/3}(x+3)$ and its first two derivatives,

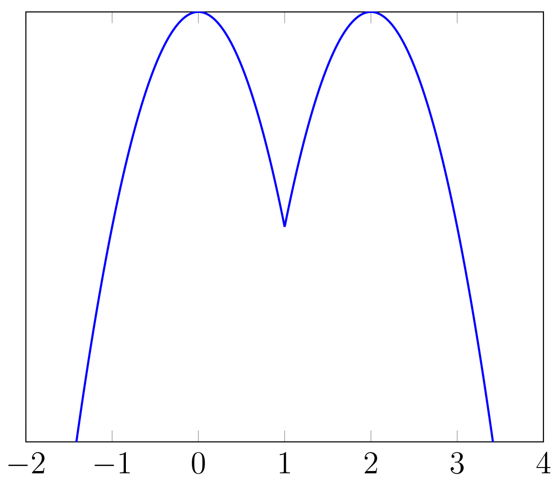
    $f'(x) = \dfrac{4x+3}{3x^{2/3}}$ and $f''(x) = \dfrac{4x-6}{9x^{5/3}}$.

    Find all intercepts, asymptotes, relative extrema and inflection points. Sketch the graph of $f(x)$.

21. Sketch the graph of $f(x) = \dfrac{x^3}{1-x^2}$ if

    $f'(x) = \dfrac{x^2(3-x^2)}{(1-x^2)^2}$ and $f''(x) = \dfrac{2x(x^2+3)}{(1-x^2)^3}$.

22. Given the graph of the derivative $f'(x)$, find a possible graph of the function $f(x)$. Assume that $f(-1) = -2$, $f(0) = 0$, $f(1) = 1$, $f(2) = 3$ and $f(3) = 5$, and that $f(x)$ is a continuous function. Be sure to find all extrema and inflection points.



MAC 2233: Review: Lectures 30 to 34

Part I: Multiple Choice

1. The slope of a curve $y = f(x)$ at any point is given by $f'(x) = \dfrac{6x-1}{\sqrt{x}}$. If the curve passes through the point $(1,1)$, find $f(4)$.

   a. 29 b. $13 - \ln 4$ c. 27 d. $\dfrac{11}{2}$ e. 28

2. If $f'(x) = \dfrac{(\ln x)^2}{x}$ and $f(e) = -1$, find $f(e^4)$.

   a. $\dfrac{1}{3}$ b. $\ln 4$ c. $\dfrac{16}{e^4}$ d. $\dfrac{60}{3}$ e. 4

3. Let $f(x) = \frac{2}{x}$. What is the exact area of the region bounded by $f(x)$ and the x-axis from $x = 1$ to $x = 3$?

a. $\ln 9$ b. $\frac{\ln 3}{2} - \frac{1}{2}$ c. $\ln 3$ d. $\ln 9 - 2$ e. 9

4. The marginal revenue and cost functions for a new product are $R'(x) = 72 - 0.2x$ and $C'(x) = 0.4x$ respectively where x is the number of items sold. Find the profit function $P(x)$ if the developers of the product will lose their initial investment of \$1200 if there are no sales. **Hint:** Profit = Revenue − Cost. What is $P(0)$?

a. $P(x) = 72 - 0.6x - 1200$

b. $P(x) = 72x + 0.1x^2 - 1200$

c. $P(x) = 72x - 0.3x^2 - 1200$

d. $P(x) = 72x + 0.1x^2 + 1200$

5. Find the area (in square units) of the region(s) bounded by $f(x) = 4-2x$ and the x-axis from $x = 0$ to $x = 3$. Be sure to sketch the area.

a. 3 b. 4 c. 5 d. $\frac{7}{2}$ e. 6

6. Evaluate $\int_0^3 (4-2x)\,dx$. Compare to the previous problem.

a. 3 b. 4 c. 5 d. $\frac{7}{2}$ e. 6

7. Find the maximum and minimum values of $f(x) = xe^{-x}$ on $[0, 3]$.

a. $\frac{3}{e^3}$ and $-e$

b. $\frac{1}{e}$ and $\frac{3}{e^3}$

c. $\frac{1}{e}$ and 0

d. $\frac{3}{e^3}$ and 0

e. 0 and $-e$

8. Evaluate $\int \left(x - \frac{1}{x}\right)^2 dx$.

a. $\frac{x^3}{3} - \frac{1}{x} + C$

b. $\frac{x^3}{3} + \ln(2x) + C$

c. $\frac{1}{3}\left(x - \frac{1}{x}\right)^3 + C$

d. $\frac{x^3}{3} - 2x - \frac{1}{x} + C$

e. $\frac{x^3}{3} - 2x + \ln(x^2) + C$

Indicate whether each statements is true or false.

9. If $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$ on an interval, then $F(4)$ must equal $G(4)$.

   a. True b. False

10. If $f(x) = e^{6x}$, then $\int f(x)\,dx = 6f(x) + C$.

   a. True b. False

11. $\int 4^{2x+1}\,dx = \dfrac{(\ln 4)4^{2x+1}}{2}$.

   a. True b. False

12. If $f(3) = -4$, $f'(x)$ is continuous, and $\int_3^6 f'(x)\,dx = 8$, then $f(6) = 4$.

   a. True b. False

Part II: Work each problem

1. (a) If $f(x) = x^2 + 2x - 3$ find the actual change in f when x changes from 0 to 0.15.

   (b) Find the approximate change in $f(x)$ using differentials.

2. The demand function for a product is $p(x) = 45 - \frac{\sqrt{x}}{2}$ where p is the price at which x items will sell.

   (a) Use differentials to approximate the change in revenue when the number of units sold decreases from 1600 to 1590. What is the exact change in revenue?

(b) Now suppose that the number of units sold is increasing by 20 per week. At what rate is revenue changing with respect to time at a production level of 900 units?

(c) What price should be charged for the product in order to maximize revenue?

3. Find $f(x)$ so that $\int f(x)\,dx = (1+\ln x)^2 + C$

4. Evaluate each integral:

a) $\displaystyle\int_3^8 \frac{3x}{x^2-4}\,dx$ b) $\displaystyle\int \frac{(\sqrt{x}+1)^2}{x}\,dx$ c) $\displaystyle\int_1^e \frac{(1+2\ln x)^2}{x}\,dx$

d) $\displaystyle\int \frac{e^{\frac{1}{2x-1}}}{(2x-1)^2}\,dx$ e) $\displaystyle\int_0^4 \sqrt{3x+4}\,dx$ f) $\displaystyle\int_0^2 xe^{3x^2}\,dx$

g) $\displaystyle\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$

5. It is estimated that humans are consuming zinc at the rate $R'(t) = 15e^{0.06t}$ million metric tons per year, with $t = 0$ in 2012. If 10 million metric tons were consumed in 2012, find a formula $R(t)$, the amount of zinc consumed in year t.

6. The marginal revenue for a product is $100 + 0.4x - 0.3x^2$. If the revenue from the sale of 20 items is \$1280, find the revenue and demand functions for the product.

7. The acceleration of an object is given by $a(t) = \dfrac{2}{(t+1)^2}$ cm/sec^2. If the velocity after 3 seconds is $\dfrac{3}{2}$ cm/sec, find the displacement of the object from its starting point after the first two seconds.

8. The price of a particular model of a Toyota is increasing at the rate of $\dfrac{3t}{\sqrt{3t^2+4}}$ thousand dollars t years after its introduction. If the retail price of the car when it was first introduced was \$24,000, find the retail price of that same model two years later.

9. A car moving along a straight track has acceleration function $a(t) = e^{2t}$ m/sec^2. The initial velocity is 3 m/sec and the initial distance from an observer is 2 meters. Find its position function $s(t)$ which gives the position of the car from the observer after t seconds .

10. A rectangular shipping crate is to be constructed with a square base. The material for the two square ends costs \$3 per square foot and the material for the sides costs \$2 per square foot. What dimensions will minimize the cost of constructing the crate if it must have a volume of 12 cubic feet? What is the minimum cost? Let x be the length of the side of a square end, and y be the height of the crate. Be sure to check your answer.

11. A sporting goods store has started selling a new fitness tracker. In one of its local districts, an average of 50 trackers sell per month at the regular price of \$40. The financial manager has observed that when the tracker is put on sale, an average of 5 more will sell for each \$2 price decrease. If each unit costs the store \$24 and there are fixed costs of \$5600, how many should be sold in a given month to maximize profit? What price should they charge? Assume that the demand function $p(x)$ is linear.

12. Consider the area given by $\displaystyle\int_0^4 \frac{x}{x^2+1}\,dx$.

(a) Approximate the area using a Riemann sum with $n = 4$ and using the left endpoint of each subinterval to find the height of each rectangle.

(b) Approximate the same area with $n = 4$ using midpoints to approximate the integral.

(c) Find the exact value of the integral.

13. Find the following definite integral by finding the area of an appropriate geometric figure. Be sure to sketch the regions involved.

$$\int_{-2}^{3} f(x)\,dx \text{ with } f(x) = \begin{cases} 3 & x \leq 0 \\ \sqrt{9-x^2} & 0 < x \leq 3 \end{cases}$$

14. Find the area of the region bounded by $f(x) = |x-1| + x$ and the x-axis on $[-1, 3]$. Be sure to rewrite the function without absolute value bars, then sketch.

15. (a) Sketch the graph of the region bounded by the function

$$f(x) = \begin{cases} x+1 & x \leq 0 \\ e^{x/2} & x > 0 \end{cases}$$ and the x-axis from $x = -1$ to $x = 2$.

(b) Find the area of the region described in part (a) by evaluating two separate integrals.

16. Find the values of a and b so that $f(x)$ will be continuous for all x if $f(x) = \begin{cases} x - a & x < 3 \\ 2 & x = 3 \\ \dfrac{x^2}{3} + b & x > 3 \end{cases}$. Then find the area of the region bounded by $f(x)$ and the x-axis on the interval $[1, 4]$.