Unit 3 Exam Review – Lectures 26 - 31

1. Find each value at which $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$ has a relative maximum or minimum.

   Find the absolute extrema of $f(x)$ on $[-2, 1]$.

2. Find all critical numbers and relative extrema of $g(x) = \frac{1}{2\sqrt{x}} - \frac{1}{x^2}$.

3. Let $f(x) = \frac{\sqrt[3]{3x-2}}{x}$.

   (a) Find $f'(x)$ and write as a single fraction.

   (b) Find the equation of each horizontal and vertical tangent line of $f(x)$.

   (c) Find each x-value at which $f(x)$ has a critical number.

   (d) Find the relative extreme values of $f(x)$.

4. The cost function for a product is $C(x) = 1.25x^2 + 25x + 8000$.

   (a) Suppose the company decides to increase production by 4 units per day when the current daily production level is 50 units. Find the rate of change of cost with respect to time.

   (b) If $C(x) = 1.25x^2 + 25x + 8000$, find each interval on which **average cost** is increasing and decreasing. For what production level x is average cost minimized?

5. The demand function for a certain product is given by

   $p(x) = -0.02x + 400$, $0 \le x \le 20,000$, where p is the unit price when x items are sold. The cost function for the product is

   $C(x) = 100x + 300,000$.

   (a) Find the marginal profit of the product when $x = 2000$.

   (b) Find the actual profit from the sale of the 2001st item. Compare to your answer in (a).

   (c) Find each interval on which the profit function $P(x) = -0.02x^2 + 300x - 300,000$ is increasing and decreasing. Remember that $0 \le x \le 20,000$. How many items should be sold to maximize profit? At what price?

6. Find all relative extrema of

   $f(x) = 2x^{5/3} - 5x^{2/3}$. Then find the absolute extrema of $f(x)$ on $[-8, 0]$. Compare the two methods.

7. Find the absolute maximum and minimum values of $f(x) = e^{x^3 - 12x}$ on $[0, 3]$.

8. Find the maximum and minimum values of $f(x) = x^2 - 8\ln x$ on $[1, e]$.

9. The position (in centimeters) of a particle moving in a straight line at time t (in seconds) is given by $s(t) = t^3 - 6t^2 + 9t$ for $0 \le t \le 6$.

   (a) Find the velocity function $v(t)$.

   (b) At what time(s) is the particle at rest?

   (c) For what time interval(s) over the first six seconds is the particle traveling in a positive direction?

   (d) Find the average velocity from $t = 0$ to $t = 4$ seconds.

   (e) What is the acceleration of the particle after 3/2 second? Include units in your answer.

   (f) Find each interval on which the particle is (1) speeding up and (2) slowing down

10. Find each interval on which $f(x) = e^{1-x^2}$ is concave up and down, and find each inflection point of the graph of f.

11. Find all intervals on which the graph of

    $f(x) = \dfrac{x^4}{4} + 2x^3 + \dfrac{9}{2}x^2 + 8$ is both decreasing

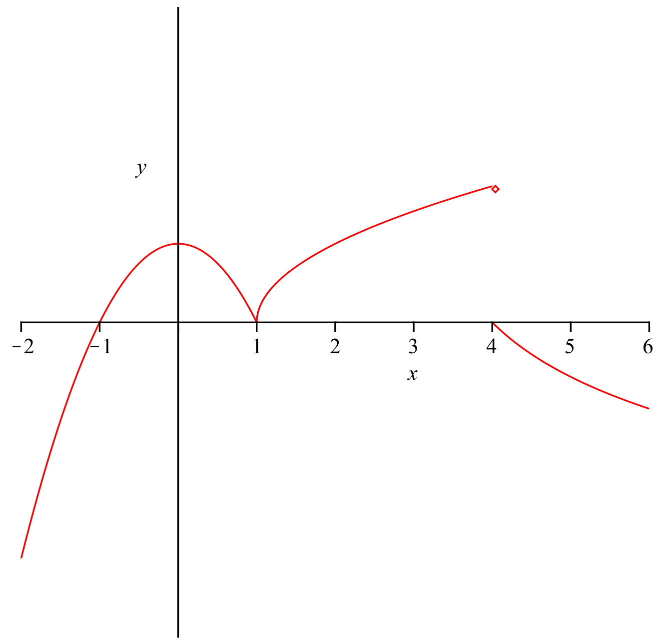
    and concave up.

12. Find each interval on which $f(x) = \ln x + \dfrac{1}{x}$

    is both increasing and concave down. Find each inflection point.

13. Suppose that $f(x)$ has horizontal tangent lines at $x = -2$, $x = 1$ and $x = 5$. If $f''(x) > 0$ on intervals $(-\infty, 0)$ and $(2, \infty)$ and $f''(x) < 0$ on the interval $(0, 2)$, find the x- values at which $f(x)$ has relative extrema. Assume that f and f' are continuous on $(-\infty, \infty)$ and use the Second Derivative test.

14. Given the graph of **the derivative** $f'(x)$, find each interval on which the function $f(x)$ is increasing and decreasing, and find the x-coordinate of each point at which $f(x)$ has a local maximum or minimum value. Find each interval on which $f(x)$ is concave up and down, and the x-coordinate of each inflection point. Assume that the domain of $f(x)$ is $(-\infty, \infty)$.



15. Suppose that $f'(x) = 15x^4 - 15x^2$. Find each x-value at which the function $f(x)$ has relative extrema. Find the x-coordinate of each inflection point. Sketch a possible graph of $f(x)$ if $f(x)$ passes through the origin.

16. A drug that stimulates reproduction is introduced into a population of viruses. That population can be modeled by the function $P(t) = 30t^2 - t^3 + 200$, $0 \le t \le 30$, where $P(t)$ is the population after t minutes.

    (a) At what time does the population reach its maximum? What is the maximum population?

    (b) At what time is the rate of growth of the population maximized?

17. A farmer wishes to fence an area next to his barn. He needs a wire fence that costs \$1 per linear foot in front of the barn and and wooden fencing that costs \$2 per foot on the other sides. Find the lengths x (sides perpendicular to the barn) and y (side across from the barn) so that he can enclose the maximum area if his budget for materials is \$4400.

18. Frye's Electronics has started selling a new video game. In one of its Dallas stores, an average of 50 games sell per month at the regular price of \$40. The manager of the department has observed that when the video game is put on sale, an average of 5 more games will sell for each \$2 price decrease. If each video game costs the store \$24 and there are fixed costs of \$5600, how many should be sold in a given month to maximize profit? What price should they charge? Assume that the demand function $p(x)$ is linear.

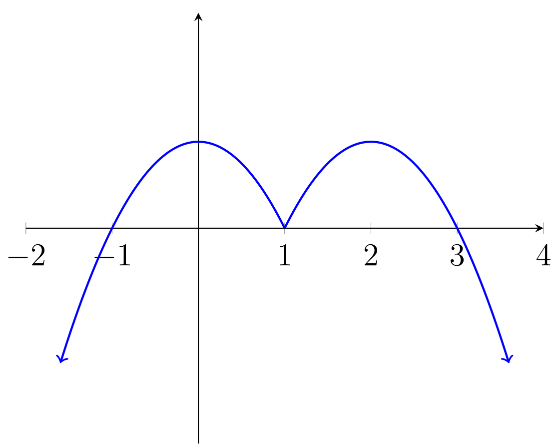
19. The revenue $R(x)$ generated from sales of a certain product is related to the amount of money spent on advertising according to the model $R(x) = \dfrac{1}{10,000}(600x^2 -$

x^3), $0 \le x \le 600$, where x and $R(x)$ are measured in thousands of dollars. Find each interval over which $R(x)$ is increasing. For the interval on which $R(x)$ is increasing, find the point of diminishing returns. Why is it significant?

20. Consider the function $f(x) = x^{1/3}(x+3)$ and its first two derivatives,
$f'(x) = \dfrac{4x+3}{3x^{2/3}}$ and $f''(x) = \dfrac{4x-6}{9x^{5/3}}$.
Find all intercepts, asymptotes, relative extrema and inflection points. Sketch the graph of $f(x)$.

21. Sketch the graph of $f(x) = \dfrac{x^3}{1-x^2}$ if
$f'(x) = \dfrac{x^2(3-x^2)}{(1-x^2)^2}$ and $f''(x) = \dfrac{2x(x^2+3)}{(1-x^2)^3}$.

22. Given the graph of the derivative $f'(x)$, find a possible graph of the function $f(x)$. Assume that $f(-1) = -2$, $f(0) = 0$, $f(1) = 1$, $f(2) = 3$ and $f(3) = 5$, and that $f(x)$ is a continuous function. Be sure to find all extrema and inflection points.



23. The slope of a curve $y = f(x)$ at any point is given by $f'(x) = \dfrac{6x-1}{\sqrt{x}}$. If the curve passes through the point $(1, 1)$, find $f(4)$.

a. 29 b. $13 - \ln 4$ c. 27 d. $\dfrac{11}{2}$ e. 28

24. If $f'(x) = \dfrac{(\ln x)^2}{x}$ and $f(e) = -1$, find $f(e^4)$.

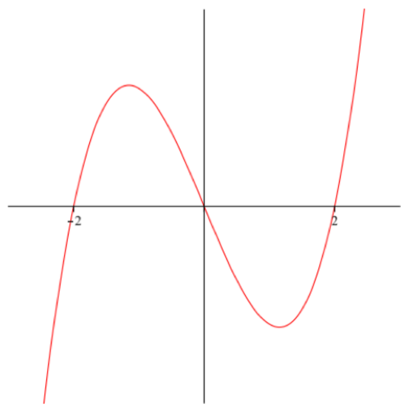
a. $\dfrac{1}{3}$ b. $\ln 4$ c. $\dfrac{16}{e^4}$ d. $\dfrac{60}{3}$ e. 4

25. Let $f(x) = \dfrac{2}{x}$. What is the exact area of the region bounded by $f(x)$

and the x-axis from $x = 1$ to $x = 3$?

a. $\ln 9$ b. $\dfrac{\ln 3}{2} - \dfrac{1}{2}$ c. $\ln 3$ d. $\ln 9 - 2$ e. 9

26. The graph of a function $f(x)$ is sketched below. Which of the following is true of an antiderivative $F(x)$ of f? Hint: consider the number lines for the derivatives of $F(x)$.



P. $F(x)$ has a local maximum at $x = 0$.
Q. The graph of $F(x)$ is concave down on $(-1, 1)$.
R. $F(x)$ has local minima at $x = -2$ and $x = 2$.

a. P and R only b. Q only c. Q and R only

d. P and Q only e. P, Q and R

27. Find the area (in square units) of the region(s) bounded by $f(x) = 4 - 2x$ and the x-axis from $x = 0$ to $x = 3$. Be sure to sketch the area.

a. 3 b. 4 c. 5 d. $\dfrac{7}{2}$ e. 6

28. Evaluate $\displaystyle\int_0^3 (4 - 2x)\,dx$. Compare to the previous problem.

a. 3 b. 4 c. 5 d. $\dfrac{7}{2}$ e. 6

29. Find the maximum and minimum values of $f(x) = xe^{-x}$ on $[0, 3]$.

a. $\dfrac{3}{e^3}$ and $-e$ b. $\dfrac{1}{e}$ and $\dfrac{3}{e^3}$ c. $\dfrac{1}{e}$ and 0

d. $\dfrac{3}{e^3}$ and 0 e. 0 and $-e$

Indicate whether each statements is true or false.

30. $\frac{d}{dx}\int_1^2 f(x)\,dx = 0$. Assume f is continuous on $[1, 2]$.

    a. True b. False

31. If $F(x)$ and $G(x)$ are both antiderivatives of $f(x)$ on an interval, then $F(4)$ must equal $G(4)$.

    a. True b. False

32. $\frac{d}{dx}\int f(x)\,dx = f(x)$.

    a. True b. False

33. If $f(x) = e^{6x}$, then $\int f(x)\,dx = 6f(x) + C$.

    a. True b. False

34. $\int 4^{2x+1}\,dx = \frac{(\ln 4)4^{2x+1}}{2}$.

    a. True b. False

35. If $f(3) = -4$, $f'(x)$ is continuous, and $\int_3^6 f'(x)\,dx = 8$, then $f(6) = 4$.

    a. True b. False

36. (a) If $f(x) = x^2 + 2x - 3$ find the actual change in f when x changes from 0 to 0.15.

    (b) Find the approximate change in $f(x)$ using differentials.

37. The demand function for a product is $p(x) = 45 - \frac{\sqrt{x}}{2}$ where p is the price at which x items will sell.

    (a) Use differentials to approximate the change in revenue when the number of units sold decreases from 1600 to 1590. What is the exact change in revenue?

    (b) Now suppose that the number of units sold is increasing by 20 per week. At what rate is revenue changing with respect to time at a production level of 900 units?

    (c) What price should be charged for the product in order to maximize revenue?

38. Find $f(x)$ so that $\int f(x)\,dx = (1+\ln x)^2 + C$

39. Evaluate each integral:

a) $\displaystyle\int_3^8 \frac{3x}{x^2-4}\,dx$ b) $\displaystyle\int \frac{(\sqrt{x}+1)^2}{x}\,dx$ c) $\displaystyle\int_1^e \frac{(1+2\ln x)^2}{x}\,dx$

d) $\displaystyle\int \frac{e^{\frac{1}{2x-1}}}{(2x-1)^2}\,dx$ e) $\displaystyle\int_0^4 \sqrt{3x+4}\,dx$ f) $\displaystyle\int_0^2 xe^{3x^2}\,dx$

g) $\displaystyle\int_2^5 \frac{x}{\sqrt{x-1}}\,dx$

40. It is estimated that humans are consuming zinc at the rate $R'(t) = 15e^{0.06t}$ million metric tons per year, with $t = 0$ in 2012. If 10 million metric tons were consumed in 2012, find a formula $R(t)$, the amount of zinc consumed in year t.

41. The marginal revenue for a product is $100 + 0.4x - 0.3x^2$. If the revenue from the sale of 20 items is \$1280, find the revenue and demand functions for the product.

42. The acceleration of an object is given by $a(t) = \dfrac{2}{(t+1)^2}$ cm/sec^2. If the velocity after 3 seconds is $\dfrac{3}{2}$ cm/sec, find the displacement of the object from its starting point after the first two seconds.

43. The price of a particular model of a Toyota is increasing at the rate of $\dfrac{3t}{\sqrt{3t^2+4}}$ thousand dollars t years after its introduction. If the retail price of the car when it was first introduced was \$24,000, find the retail price of that same model two years later.

44. A car moving along a straight track has acceleration function $a(t) = e^{2t}$ m/sec^2. The initial velocity is 3 m/sec and the initial distance from an observer is 2 meters. Find its position function $s(t)$ which gives the position of the car from the observer after t seconds .

45. A rectangular shipping crate is to be constructed with a square base. The material for the two square ends costs \$3 per square foot and the material for the sides costs \$2 per square foot. What dimensions will minimize the cost of constructing the crate if it must have a volume of 12 cubic feet? What is the minimum cost? Let x be the length of the side of a square end, and y be the height of the crate. Be sure to check your answer.

46. A sporting goods store has started selling a new fitness tracker. In one of its local districts, an average of 50 trackers sell per month at the regular price of \$40. The financial manager has observed that when the tracker is put on sale, an average of 5 more will sell for each \$2 price decrease. If each unit costs the store \$24 and there are fixed costs of \$5600, how many should be sold in a given month to maximize profit? What price should they charge? Assume that the demand function $p(x)$ is linear.