Exam 3 Review

ANSWERS

1. (a) $f'(x) = 2 \cdot 3^{2x-1}(\ln 3)$ (b) $f'(x) = \dfrac{2x-1}{(x^2-x)(\ln 4)}$

2. $m = -\dfrac{1}{e}$

3. $m = -1$

4. $f'(x) = \dfrac{2x}{x^2+1}$; $f''(x) = \dfrac{2-2x^2}{(x^2+1)^2}$; $f'''(x) = \dfrac{4x(x^2-3)}{(x^2+1)^3}$

5. $f'(x) = \dfrac{1}{x-3} - \dfrac{x}{x^2+3} = 0$ when $x = -1$ but -1 is not in the domain of $f(x)$ so the graph has no horizontal tangent lines.

6. $1 + \dfrac{1}{6+3x} - \dfrac{6}{3x+1}$

7. $8 + 4\ln 4$

8. horizontal tangent lines at $x = -1$, $x = 0$ and $x = 2$
   relative maximum at $x = 0$; relative minima at $x = -1$ and $x = 2$
   absolute maximum on $[-2, 1]$: $\dfrac{8}{3} = f(-2)$ and
   absolute minimum on $[-2, 1]$: $-\dfrac{13}{12} = f(1)$

9. critical number: $x = 4$ only
   relative maximum value is $f(4) = -\dfrac{3}{16}$, no relative minima

10. (a) $f'(x) = \dfrac{2-2x}{x^2(3x-2)^{2/3}}$
    (b) HTL: $y = 1$, VTL: $x = \dfrac{2}{3}$
    (c) $x = 1$ and $x = \dfrac{2}{3}$ ($f(x)$ has a vertical asymptote at $x = 0$ so not a critical number)
    (d) local maximum: $f(1) = 1$, no local minima

11. (a) $\dfrac{dC}{dt} = 600$ so cost is increasing by \$600 per day

(b) Average cost $\overline{C}(x) = \dfrac{C(x)}{x}$ is decreasing on interval (0, 80) and increasing for $x > 80$ so average cost is minimized when 80 items are produced.

12. $P(x) = -0.02x^2 + 300x - 300,000$

    (a) When $x = 2000$, $MP = 220$ so the profit from the 2001st item is approximately \$220.

    (b) $\Delta P = P(2001) - P(2000) = 219.98$

    (c) increasing: $(0, 7500)$ and decreasing: $(7500, 20,000)$ Profit is maximized when 7500 items are sold at a unit price of \$250.

13. $f'(x) = \dfrac{10x - 10}{3x^{1/3}}$

relative maximum is $f(0) = 0$; relative minimum is $f(1) = -3$

on $[-8, 0]$: absolute maximum is $f(0) = 0$ and absolute minimum is $f(-8) = -84$

14. maximum: $1 = f(0)$, minimum: $\dfrac{1}{e^{16}} = f(2)$

15. maximum: $1 = f(1)$, minimum: $4 - 8\ln 2 = f(2)$

16. (a) $v(t) = 3t^2 - 12t + 9$

    (b) $t = 1$ and $t = 3$ seconds

    (c) $(0, 1)$ and $(3, 6)$

    (d) 1 cm/sec

    (e) $a(t) = 6t - 12$; $a(3/2) = -3$ cm/sec^2

    (f) (1): (1, 2) and (3, 6)   (2): (0, 1) and (2, 3)

17. concave up: $\left(-\infty, -\dfrac{1}{\sqrt{2}}\right) \cup \left(\dfrac{1}{\sqrt{2}}, \infty\right)$, concave down: $\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

inflection points: $\left(-\dfrac{1}{\sqrt{2}}, \sqrt{e}\right)$ and $\left(\dfrac{1}{\sqrt{2}}, \sqrt{e}\right)$

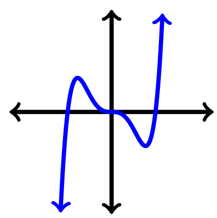
18. $(-\infty, -3)$ and $(-1, 0)$

19. $(2, \infty)$; inflection point is $\left(2, \ln 2 + \dfrac{1}{2}\right)$

20. maximum at $x = 1$, minimum at $x = -2$ and $x = 5$

21. $f(x)$ is increasing on interval $(-1, 4)$ and decreasing on interval $(\infty, -1)$ and $(4, \infty)$
relative maximum at $x = 4$ and relative minimum at $x = -1$
concave up: $(-\infty, 0)$ and $(1, 4)$, concave down: $(0, 1)$ and $(4, \infty)$
inflection points at $x = 0$, $x = 1$ and $x = 4$

22. relative maxima: $x = -1$, relative minimum: $x = 1$
inflection points at $x = \pm\frac{1}{\sqrt{2}}$, $x = 0$



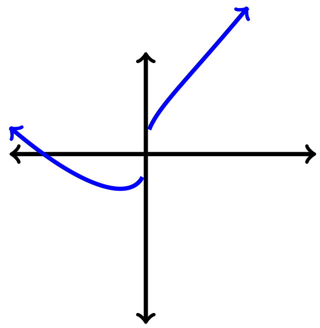
23. (a) after 20 minutes; population is $P(20) = 4200$ viruses
(b) $t = 10$ minutes

24. Dimensions: $x = 550$ ft, $y = \frac{2200}{3}$ ft

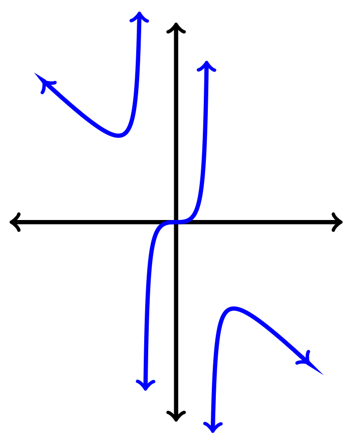
25. 45 items at a price of \$42 per unit

26. $R(x)$ is increasing on $(0, 400)$; maximum revenue is $R(400) = \$3200$.
Point of diminishing returns: $(200, 1600)$ is an inflection point of the graph of $R(x)$.

27.



28.



29. graph has a relative minimum at $x = -1$ and a relative maximum at $x = 3$; inflection points at $x = 0$, $x = 1$ and $x = 2$