Exam 3 Review

1. Find the derivative:
   (a) $f(x) = 3^{2x-1}$ (b) $f(x) = \log_4(x^2 - x)$
2. Find the slope of the tangent line to
   $f(x) = \ln|2 - \ln x|$ at $x = e$.
3. Find the slope of the tangent line to the curve given by $\sqrt{3x-y} - e^{x+y} = 1 + \ln x$ at $(1, -1)$.
4. Find the first three derivatives of
   $f(x) = \ln(x^2 + 1)$.
5. Find each value of x at which $f(x) = \ln\left(\dfrac{2x-6}{\sqrt{x^2+3}}\right)$ has a horizontal tangent line.
6. Find $f'(x)$ if $f(x) = \ln \dfrac{e^{x-3}\sqrt[3]{6+3x}}{(3x+1)^2}$.
7. Use Logarithmic Differentiation to find the slope of the tangent line to $f(x) = x^{\sqrt{x}}$ at $x = 4$.
8. Find each value at which $f(x) = \dfrac{x^4}{4} - \dfrac{x^3}{3} - x^2$ has a relative maximum or minimum.
   Find the absolute extrema of $f(x)$ on $[-2, 1]$.
9. Find all critical numbers and relative extrema
   of $g(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{x^2}$.
10. Let $f(x) = \dfrac{\sqrt[3]{3x-2}}{x}$.
    (a) Find $f'(x)$ and write as a single fraction.
    (b) Find the equation of each horizontal and vertical tangent line of $f(x)$.
    (c) Find each x-value at which $f(x)$ has a critical number.
    (d) Find the relative extreme values of $f(x)$.
11. The cost function for a product is
    $C(x) = 1.25x^2 + 25x + 8000$.
    (a) Suppose the company decides to increase production by 4 units per day when the current daily production level is 50 units. Find the rate of change of cost with respect to time.
    (b) If $C(x) = 1.25x^2 + 25x + 8000$, find each interval on which **average cost** is increasing and decreasing. For what production level x is average cost minimized?

12. The demand function for a certain product is given by
$p(x) = -0.02x + 400$, $0 \le x \le 20,000$, where p is the unit price when x items are sold. The cost function for the product is
$C(x) = 100x + 300,000$.
    (a) Find the marginal profit of the product when $x = 2000$.
    (b) Find the actual profit from the sale of the 2001st item. Compare to your answer in (a).
    (c) Find each interval on which the profit function $P(x) = -0.02x^2 + 300x - 300,000$ is increasing and decreasing. Remember that $0 \le x \le 20,000$. How many items should be sold to maximize profit? At what price?

13. Find all relative extrema of
$f(x) = 2x^{5/3} - 5x^{2/3}$. Then find the absolute extrema of $f(x)$ on $[-8, 0]$. Compare the two methods.

14. Find the absolute maximum and minimum
values of $f(x) = e^{x^3-12x}$ on $[0, 3]$.

15. Find the maximum and minimum values of $f(x) = x^2 - 8\ln x$ on $[1, e]$.

16. The position (in centimeters) of a particle moving in a straight line at time t (in seconds) is given by $s(t) = t^3 - 6t^2 + 9t$ for $0 \le t \le 6$.
    (a) Find the velocity function $v(t)$.
    (b) At what time(s) is the particle at rest?
    (c) For what time interval(s) over the first six seconds is the particle traveling in a positive direction?
    (d) Find the average velocity from $t = 0$ to $t = 4$ seconds.
    (e) What is the acceleration of the particle after 3/2 second? Include units in your answer.
    (f) Find each interval on which the particle is (1) speeding up and (2) slowing down

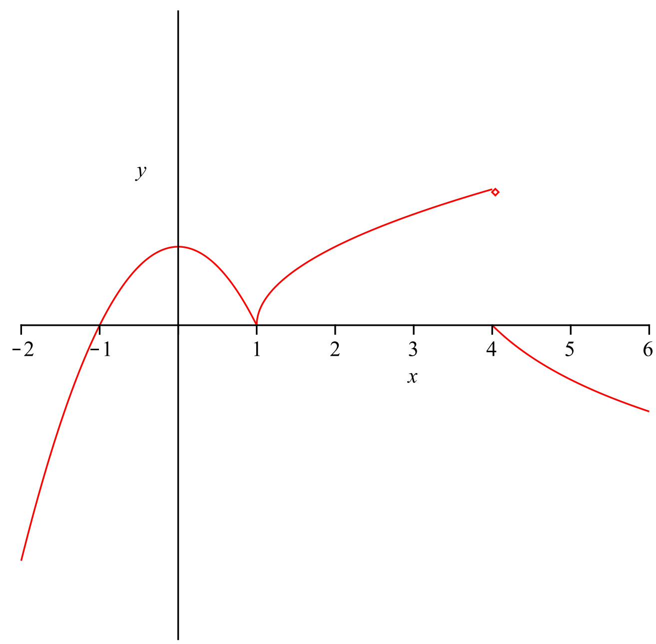
17. Find each interval on which $f(x) = e^{1-x^2}$ is concave up and down, and find each inflection point of the graph of f.

18. Find all intervals on which the graph of
$f(x) = \dfrac{x^4}{4} + 2x^3 + \dfrac{9}{2}x^2 + 8$ is both decreasing
and concave up.

19. Find each interval on which $f(x) = \ln x + \dfrac{1}{x}$
is both increasing and concave down. Find each inflection point.

20. Suppose that $f(x)$ has horizontal tangent lines at $x = -2$, $x = 1$ and $x = 5$. If $f''(x) > 0$ on intervals $(-\infty, 0)$ and $(2, \infty)$ and $f''(x) < 0$ on the interval $(0, 2)$, find the x- values at which $f(x)$ has relative extrema. Assume that f and f' are continuous on $(-\infty, \infty)$ and use the Second Derivative test.

21. Given the graph of **the derivative** $f'(x)$, find each interval on which the function $f(x)$ is increasing and decreasing, and find the x-coordinate of each point at which $f(x)$ has a local maximum or minimum value. Find each interval on which $f(x)$ is concave up and down, and the x-coordinate of each inflection point. Assume that the domain of $f(x)$ is $(-\infty, \infty)$.



22. Suppose that $f'(x) = 15x^4 - 15x^2$. Find each x-value at which the function $f(x)$ has relative extrema. Find the x-coordinate of each inflection point. Sketch a possible graph of $f(x)$ if $f(x)$ passes through the origin.

23. A drug that stimulates reproduction is introduced into a population of viruses. That population can be modeled by the function $P(t) = 30t^2 - t^3 + 200$, $0 \le t \le 30$, where $P(t)$ is the population after t minutes.

    (a) At what time does the population reach its maximum? What is the maximum population?

    (b) At what time is the rate of growth of the population maximized?

24. A farmer wishes to fence an area next to his barn. He needs a wire fence that costs \$1 per linear foot in front of the barn and and wooden fencing that costs \$2 per foot on the other sides. Find the lengths x (sides perpendicular to the barn) and y (side across from the barn) so that he can enclose the maximum area if his budget for materials is \$4400.

25. Frye's Electronics has started selling a new video game. In one of its Dallas stores, an average of 50 games sell per month at the regular price of \$40. The manager of the department has observed that when the video game is put on sale, an average of 5 more games will sell for each \$2 price decrease. If each video game costs the store \$24 and there are fixed costs of \$5600, how many should be sold in a given month to maximize profit? What price should they charge? Assume that the demand function $p(x)$ is linear.

26. The revenue $R(x)$ generated from sales of a certain product is related to the amount of money spent on advertising according to the model $R(x) = \frac{1}{10,000}(600x^2 - x^3)$, $0 \le x \le 600$, where x and $R(x)$ are measured in thousands of dollars. Find each interval over which $R(x)$ is increasing. For the interval on which $R(x)$ is increasing, find the point of diminishing returns. Why is it significant?

27. Consider the function $f(x) = x^{1/3}(x+3)$ and its first two derivatives,
$f'(x) = \frac{4x+3}{3x^{2/3}}$ and $f''(x) = \frac{4x-6}{9x^{5/3}}$.
Find all intercepts, asymptotes, relative extrema and inflection points. Sketch the graph of $f(x)$.

28. Sketch the graph of $f(x) = \frac{x^3}{1-x^2}$ if
$f'(x) = \frac{x^2(3-x^2)}{(1-x^2)^2}$ and $f''(x) = \frac{2x(x^2+3)}{(1-x^2)^3}$.

29. Given the graph of the derivative $f'(x)$, find a possible graph of the function $f(x)$. Assume that $f(-1) = -2$, $f(0) = 0$, $f(1) = 1$, $f(2) = 3$ and $f(3) = 5$, and that $f(x)$ is a continuous function. Be sure to find all extrema and inflection points.