MAC 2233: Unit 2 Exam Review
Lectures 15 – 24

1. Use the definition of derivative to evaluate $f'(x)$ if $f(x) = \sqrt{2x-1}$. Check your answer using a derivative rule.

2. (a) Use the **definition of derivative** to find $f'(x)$ if $f(x) = \dfrac{x}{2x-1}$. Check your answer using the Quotient Rule.

   (b) Find each interval over which $f(x)$ is differentiable.

   (c) Write the equation of the tangent line to $f(x) = \dfrac{x}{2x-1}$ at $x = -1$.

3. Indicate whether each of the following statements is true or false.

   (a) If f is continuous at $x = a$, then f is differentiable at $x = a$.

   (b) If f is not continuous at $x = a$, then f is not differentiable at $x = a$.

   (c) If f has a vertical tangent line at $x = a$, then the graph of $f'(x)$ has a vertical asymptote at $x = a$.

4. If an object is projected upward from the roof of a 200 foot building at 64 ft/sec, its height h in feet above the ground after t seconds is given by

   $h(t) = 200 + 64t - 16t^2$. Find the following:

   (a) The average velocity of the object from time $t = 0$ until it reaches its maximum height (hint: consider the graph of the function)

   (b) The instantaneous velocity of the object at time $t = 1$ second using the limit definition.

5. Find each value at which $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x$ is parallel to the line $2y - 8x + 9 = 0$.

6. Find the value of a so that the tangent line to $y = x^2 - 2\sqrt{x} + 1$ is perpendicular to the line $ay + 2x = 2$ when $x = 4$.

7. If $f(x) = (x^3 - 2x)(2\sqrt{x} + 1)$, find $f'(x)$ two ways: rewriting $f(x)$ and differentiating, and using the Product Rule.

8. Find each value of x at which $f(x) = (1 - x)^5(5x + 2)^4$ has a horizontal tangent line.

9. Let $f(x) = \dfrac{(\sqrt{x} - 1)^2}{x}$. Find $f'(x)$ and write as a single fraction. Write the equation of the tangent line to $f(x)$ at $x = 4$.

10. Write the equation of the tangent line to $f(x) = \left(x - \frac{6}{x}\right)^3$ at $x = 3$.

11. Find each value of x at which the function $f(x) = \frac{\sqrt[3]{6x+1}}{x}$ has

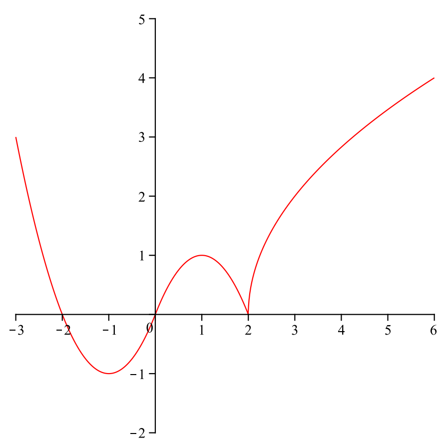
(a) horizontal and (b) vertical tangent lines.

Write the equation of each of those lines.

12. Suppose that $f(4) = -1$, $g(4) = 2$, $f(-4) = 1$, $g(-4) = 3$, $f'(4) = -2$, $g'(4) = 12$, $f'(-4) = 6$, and $g'(-1) = -2$.

Find: (a) $h'(4)$ if $h(x) = g(f(x))$ and (b) $H'(4)$ if $H(x) = \sqrt{xf(x) + \dfrac{x^2}{2}}$.

13. Sketch a possible graph of the derivative of the function $y = f(x)$ shown below.



14. Find the derivative:

(a) $f(x) = 3^{2x-1}$ (b) $f(x) = \log_4 (x^2 - x)$

15. Find the slope of the tangent line to the curve given by $\sqrt{3x - y} - e^{x+y} = 1 + \ln x$ at $(1, -1)$.

16. Find the first three derivatives of $f(x) = \ln(x^2 + 1)$.

17. Find each value of x at which $f(x) = \ln\left(\frac{2x-6}{\sqrt{x^2+3}}\right)$ has a horizontal tangent line.

18. Find $f'(x)$ if $f(x) = \ln \frac{e^{x-3}\sqrt[3]{6+3x}}{(3x+1)^2}$.

19. Use Logarithmic Differentiation to find the slope of the tangent line to $f(x) = x^{\sqrt{x}}$ at $x = 4$.

20. Find each value at which $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$ has a relative maximum or minimum.

Find the absolute extrema of $f(x)$ on $[-2, 1]$.

21. Let $f(x) = \frac{\sqrt[3]{3x - 2}}{x}$.

(a) Find $f'(x)$ and write as a single fraction.

(b) Find the equation of each horizontal and vertical tangent line of $f(x)$.

(c) Find each x-value at which $f(x)$ has a
critical number.

(d) Find the relative extreme values of $f(x)$.

22. The cost function for a product is
$C(x) = 1.25x^2 + 25x + 8000.$

(a) Suppose the company decides to increase production by 4 units per day when the current daily production level is 50 units. Find the rate of change of cost with respect to time.

(b) If $C(x) = 1.25x^2 + 25x + 8000$, find each interval on which **average cost** is increasing and decreasing. For what production level x is average cost minimized?

23. The demand function for a certain product is given by

$p(x) = -0.02x + 400$, $0 \leq x \leq 20,000$, where p is the unit price when x items are sold. The cost function for the product is

$C(x) = 100x + 300,000$.

(a) Find the marginal profit of the product when $x = 2000$.

(b) Find the actual profit from the sale of the 2001st item. Compare to your answer in (a).

(c) Find each interval on which the profit function $P(x) = -0.02x^2 + 300x - 300,000$ is increasing and decreasing. Remember that $0 \leq x \leq 20,000$. How many items should be sold to maximize profit? At what price?