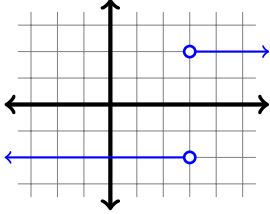


**Solutions:**

$$1. f(x) = \begin{cases} -2 & x < 3 \\ 2 & x > 3 \end{cases}$$



- (a)  $-2$       (b)  $2$       (c) DNE

2. (a)  $2$     (b)  $\frac{1}{2}$     (c)  $+\infty$     (d)  $-1$

$x = -1$ : removable,  $x = 0$ : removable,  $x = 1$ : infinite

vertical asymptote:  $x = 1$ , horizontal asymptote:  $y = -1$

3. 1)  $0$       2)  $-\frac{2}{3}$       3)  $x = \ln \frac{1}{3}$ ;  $y = 0$  and  $y = -\frac{2}{3}$

4. B or D

5. (a)  $(-\infty, -2) \cup 0) \cup (0, 3) \cup (3, 4) \cup (4, \infty)$

(b)  $0$

(c)  $4$

(d)  $2, 3$

$$6. f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h} = \frac{1}{\sqrt{2x-1}}$$

$$7. (a) f'(x) = \lim_{h \rightarrow 0} \frac{\frac{x+h}{2x+2h-1} - \frac{x}{2x-1}}{h} = -\frac{1}{(2x-1)^2}$$

$$(c) y = -\frac{1}{9}x + \frac{2}{9}$$

$$(b) \left(-\infty, \frac{1}{2}\right), \left(\frac{1}{2}, \infty\right)$$

8. (a) False (consider  $f(x) = |x|$  at  $x = 0$  for example)

(b) True

(c) True

9. (a) use one-sided limits to show  $\lim_{x \rightarrow 0} f(x) = f(0) = 2$

(b)  $f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0}$ . Use one-sided limits to show that the limit does not exist, so  $f$  is not differentiable at  $x = 0$ .

10. (a) 32 feet per second      (b)  $IV = \lim_{t \rightarrow 1} \frac{h(t) - h(1)}{t - 1} = 32$  feet per second

11.  $x = -2$  or  $x = 3$

12.  $a = 15$

13.  $f'(x) = 7x^{5/2} + 3x^2 - 6\sqrt{x} - 2$

14.  $x = -\frac{2}{5}$ ,  $x = \frac{2}{9}$ ,  $x = 1$

15.  $f'(x) = \frac{\sqrt{x} - 1}{x^2}$ ;  $y = \frac{1}{16}x$

16.  $y = 5x - 14$

17.  $f'(x) = \frac{-4x - 1}{x^2(6x + 1)^{2/3}}$

(a) horizontal tangent line at  $x = -\frac{1}{4}$ ; equation  $y = \frac{4}{\sqrt[3]{2}} = 2^{5/3}$

(b) vertical tangent line at  $x = -\frac{1}{6}$ ; equation  $x = 0$

18. (a) 4      (b)  $-\frac{5}{4}$

19. Graph should cross  $x$ -axis at  $x = -1, 1$ , have vertical asymptote at  $x = 2$ , be negative (below the  $x$ -axis) on intervals  $(-\infty, -1)$ ,  $(1, 2)$ , and be positive (above  $x$ -axis) on intervals  $(-1, 1)$ ,  $(2, \infty)$

20. (a)  $f'(x) = 2 \cdot 3^{2x-1}(\ln 3)$       (b)  $f'(x) = \frac{2x - 1}{(x^2 - x)(\ln 4)}$

21.  $m = -\frac{1}{e}$

22.  $m = -1$

23.  $f'(x) = \frac{2x}{x^2 + 1}$ ;  $f''(x) = \frac{2 - 2x^2}{(x^2 + 1)^2}$ ;  $f'''(x) = \frac{4x(x^2 - 3)}{(x^2 + 1)^3}$

24.  $f'(x) = \frac{1}{x-3} - \frac{x}{x^2+3} = 0$  when  $x = -1$  but  $-1$  is not in the domain of  $f(x)$   
so the graph has no horizontal tangent lines.

25.  $1 + \frac{1}{6+3x} - \frac{6}{3x+1}$

26.  $8 + 4 \ln 4$

27. horizontal tangent lines at  $x = -1$ ,  $x = 0$  and  $x = 2$   
relative maximum at  $x = 0$ ; relative minima at  $x = -1$  and  $x = 2$   
absolute maximum on  $[-2, 1]$ :  $\frac{8}{3} = f(-2)$  and  
absolute minimum on  $[-2, 1]$ :  $-\frac{13}{12} = f(1)$

28. critical number:  $x = 4$  only  
relative maximum value is  $f(4) = -\frac{3}{16}$ , no relative minima

29. (a)  $f'(x) = \frac{2-2x}{x^2(3x-2)^{2/3}}$

(b) HTL:  $y = 1$ , VTL:  $x = \frac{2}{3}$

(c)  $x = 1$  and  $x = \frac{2}{3}$  ( $f(x)$  has a vertical asymptote at  $x = 0$  so not a critical number)

(d) local maximum:  $f(1) = 1$ , no local minima

30. (a)  $\frac{dC}{dt} = 600$  so cost is increasing by \$600 per day

(b) Average cost  $\bar{C}(x) = \frac{C(x)}{x}$  is decreasing on interval  $(0, 80)$  and increasing for  $x > 80$  so average cost is minimized when 80 items are produced.

31.  $P(x) = -0.02x^2 + 300x - 300,000$

(a) When  $x = 2000$ ,  $MP = 220$  so the profit from the 2001st item is approximately \$220.

(b)  $\Delta P = P(2001) - P(2000) = 219.98$

(c) increasing:  $(0, 7500)$  and decreasing:  $(7500, 20,000)$

Profit is maximized when 7500 items are sold at a unit price of \$250.

32.  $f'(x) = \frac{10x - 10}{3x^{1/3}}$

relative maximum is  $f(0) = 0$ ; relative minimum is  $f(1) = -3$

on  $[-8, 0]$ : absolute maximum is  $f(0) = 0$  and absolute minimum is  $f(-8) = -84$

33. maximum:  $1 = f(0)$ , minimum:  $\frac{1}{e^{16}} = f(2)$

34. maximum:  $1 = f(1)$ , minimum:  $4 - 8 \ln 2 = f(2)$