|  | VS A | VS B |
| :--- | :--- | :--- |
| 1 | $D$ | $D$ |
| 2 | C | D |
| 3 | $D$ | $C$ |
| 4 | $D$ | $B$ |
| 5 | $B$ | $C$ |
| 6 | $A$ | $D$ |
| 7 | $C$ | $B$ |
| 8 | $A$ | $B$ |
| 9 | $D$ | $B$ |
| 10 | $D$ | $B$ |
| 11 | $D$ | $D$ |
| 12 | $A$ | $D$ |
| 13 | $B$ | $A$ |

## Multiple Questions are worth 4 points each.

Use the graph of $f(x)$ below to answer questions 1-3


1. Find $\lim _{x \rightarrow-4^{+}} f(x)$
A. 0
B. -1
C. 4
D. -2
E. DNE
2. How many discontinuities does $f(x)$ have?
A. 0
B. 1
C. 2
D. 3
E. 4
3. Find $\lim _{x \rightarrow 4} f(x)$
A. 0
B. 3
C. 2
D. 1
E. DNE
4. If $f(2)=3$ and $f^{\prime}(2)=-1$, what is the equation of the line tangent to the graph of $y=f(x)$ at $x=2$.
A. $y=-x+2$
B. $y=5-x$
C. $y=3 x-1$
D. $y=3 x+2$
E. $y=3-x$
5. Match the graph of $f(x)$ with it's derivative $f^{\prime}(x)$


B.



6. Calculate the limit

$$
\begin{aligned}
& \lim _{x \rightarrow 4} \frac{-x^{2}+6 x-8}{x-4} \\
& \begin{array}{lll}
\text { C. }-2 & \text { D. } 2 & \text { E. } 4
\end{array}
\end{aligned}
$$

A.
B. 2
2 DNE

7. Let

$$
\begin{array}{rlrl}
f^{\prime}(2) & =3 & f(2) & =4 \\
g(4) & =2 & g^{\prime}(4) & =1
\end{array}
$$

Let $h(x)=(f \circ g)(2 x)$. Find $h^{\prime}(2)$
A. 2
B. 6
C. -8
D. 3
E. 0
8. Which of the following functions have a root between -1 and 2 ?
i. $f(x)=x^{2}+1$
ii. $g(x)=e^{x}-1$
iii. $k(x)=x^{2}+x+2$
A. i only
B. ii only
C. i and iii
D. all of the above
E. none of the above
9. Let

$$
\begin{array}{rlll}
f^{\prime}(2)=3 & f(2)=4 & f(4)=-1 & f^{\prime}(4)=-4 \\
g(4)=2 & g^{\prime}(4)=1 & g(2)=-2 & g^{\prime}(2)=5
\end{array}
$$

Let $p(x)=\frac{x^{2} f(x)}{g(x)}$. Find $p^{\prime}(2)$
A. 6
B. -34
C. -8
D. $\frac{5}{4}$
E. $-\frac{5}{4}$
10. What value of $k$ will make $f(x)$ continous from $(-\infty, \infty)$ ?

$$
f(x)= \begin{cases}7 x-2 & x \leq 1 \\ k x^{2} & x>1\end{cases}
$$

A. -2
B. 5
C. 0
D. 7
E. 0
11. What is the value of the following limit: $\lim _{x \rightarrow \infty} \frac{x^{2}-6 x+3}{x+2 x^{2}+7}$ ?
A. 1
B. 0
C. $\infty$
D. $\frac{1}{2}$
E. DNE
12. State the equations of all horizontal asymptotes for a function $f(x)$ satisfying the following conditions.
$f(0)=0, \lim _{x \rightarrow \infty} f(x)=2, \lim _{x \rightarrow-\infty} f(x)=\infty, \lim _{x \rightarrow 3^{-}} f(x)=-\infty, \lim _{x \rightarrow 3^{+}} f(x)=\infty, \lim _{x \rightarrow-5} f(x)=$ $-\infty$
A. $x=3$ and $x=-5$
B. $y=3$ and $y=-5$
C. $x=2$ and $y=3$
D. $y=2$
E. $y=0$
13. Let $f$ be the piecewise function defined below. Which of the following statements about $f$ are true.

$$
f(x)= \begin{cases}\frac{x^{2}-4}{x-2} & \text { if } x \neq 2 \\ 1 & \text { if } x=2\end{cases}
$$

I. $\lim _{x \rightarrow 2} f(x)$ exists
II. $f(x)$ is continous at $x=2$
III. $f(x)$ is differentiable at $x=2$
A. I only
B. I and II
C. I and III
D. All of the Above
E. None of the Above
$\qquad$
$\qquad$
UP ID \# $\qquad$ Signature $\qquad$ YOU MUST SHOW ALL WORK TO RECEIVE FULL CREDIT.

1. Let $f(x)=\sqrt{2+3 x}$. Use the limit definition of the derivative to find $f^{\prime}(1)$. (NOTE: No credit will be given if another method is used.)

$$
\begin{aligned}
& \lim _{h \rightarrow 0} \frac{\sqrt{2+3(x+h)}-\sqrt{2+3 x}}{h} \cdot \frac{\sqrt{2+3 x+3 h}+\sqrt{2+3 x}}{\sqrt{2+3 x+3 h}+\sqrt{2+3 x}} \\
& =\lim _{h \rightarrow 0} \frac{2+3 x+3 h-(2+3 x)}{h(\sqrt{2+3 x+3 h}+\sqrt{2+3 x})} \\
& =\lim _{n \rightarrow 0} \frac{3 x}{1 x(\sqrt{2+3 x+3 n}+\sqrt{2+3 x})} \\
& =\lim _{h \rightarrow 0} \frac{3}{\sqrt{2+3 x+3 n}+\sqrt{2+3 x}} \\
& =\frac{3}{2 \sqrt{2+3 x}}=f^{\prime}(x) \quad f^{\prime}(1)=\frac{3}{2 \sqrt{5}} \\
& \left.\lim _{x \rightarrow 1} \frac{\sqrt{2+3 x}-\sqrt{5}}{x-1} \cdot \frac{\sqrt{2+3 x}+\sqrt{5}}{\sqrt{2+3} x+\sqrt{5}} \right\rvert\,=\lim _{x \rightarrow 1} \frac{3(x-1)}{(x-1)(\sqrt{2+3 x}+\sqrt{5})} \\
& =\lim _{x \rightarrow 1} \frac{2+3 x-5}{(x-1)(\sqrt{2+3 x}+\sqrt{5})} \\
& =\lim _{x \rightarrow 1} \frac{3 x-3}{(x-1)(\sqrt{2+3 x}+\sqrt{5})}
\end{aligned}
$$

2. Let $f(x)=\frac{x^{2}+3 x-1}{x-2}$
(a) Find $f^{\prime}(x)$

$$
\begin{aligned}
f^{\prime}(x) & =\frac{(2 x+3)(x-2)-\left(x^{2}+3 x-1\right)}{(x-2)^{2}} \\
& =\frac{2 x^{2}-4 x+3 x-6-x^{2}-3 x+1}{(x-2)^{2}} \\
& =\frac{x^{2}-4 x-5}{(x-2)^{2}}
\end{aligned}
$$

(b) Find the equation of the tangent line to $f(x)$ at $x=3$

$$
f^{\prime}(3)=\frac{3^{2}-4(3)-5}{1}=-8 \quad f(3)=\frac{9+9-1}{7}=17
$$

3. Let $f(x)=\sqrt{x^{2}-4 x+13}$.
(a) At which $x$ values does $f(x)$ have horizontal tangent lines?

$$
\begin{aligned}
f^{\prime}(x) & =\frac{1}{2}\left(x^{2}-4 x+13\right)^{-\frac{1}{2}}(2 x-4) \\
0 & =\frac{2 x-4}{2\left(x^{2}-4 x+13\right)^{\frac{1}{2}}} \quad \begin{aligned}
2 x & =4 \\
x & =2
\end{aligned}
\end{aligned}
$$

(b) Write the equation of the horizontal tangent line (s).

$$
f(2)=\sqrt{2^{2}-4(2)+13}=3 \quad y=3
$$

4. Consider the function

$$
f(x)=\frac{3 x+x^{2}}{x^{2}-2 x}=\frac{\chi(3+X)}{\chi(x-2)}=\frac{3+x}{x-2}
$$

(a) List each value of $x$ at which $f(x)$ is discontinous, and describe each as removable, jump or infinite discontinuity.
$x=0$ removable
$x=2$ non removable (infinite)
(b) List all of the vertical and horizontal asymptotes. You MUST support your answers with limits.

$$
\begin{aligned}
& \lim _{x \rightarrow \infty} \frac{3+x}{x-2}=1 \quad \Rightarrow \text { horizontal asymptote } y=1 \\
& \lim _{x \rightarrow 2^{-}} \frac{3+x}{x-2}=-\infty \\
& \lim _{x \rightarrow 2^{+}} \frac{3+x}{x-2}=\infty
\end{aligned} \quad \Rightarrow \text { vertical asymptote } x=2
$$

