Review 2: L6-L13

FORMULAS TO MEMORIZE:

The slope of the line through (x_1, y_1) and (x_2, y_2): $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.

Equation of the line with slope m through the point (x_1, y_1): $y - y_1 = m(x - x_1)$.

Equation of the line with $x-$intercept a and $y-$intercept b: $\frac{x}{a} + \frac{y}{b} = 1 \ \ (a \neq 0, b \neq 0)$

The vertex of a parabola $f(x) = ax^2 + bx + c$: (h, k), where $h = \frac{-b}{2a}$ and $k = f(h)$.

Identities: $a^{\log_a x} = x$ for $x > 0$ $\qquad$ $\log_a a^x = x$ for any real x

Properties of Logarithms ($x > 0,\ y > 0,\ a > 0,\ a \neq 1$, and $p \neq 0$):

(a) $\log_a (xy) = \log_a x + \log_a y$ $\qquad$ (b) $\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$

(c) $\log_a x^r = r \log_a x$ $\qquad$ (d) $\log_{a^p} x = \frac{1}{p} \log_a x$

(e) $\log_a a = 1$ $\qquad$ (f) $\log_a 1 = 0$

Change-of-Base Theorem: $\log_a x = \frac{\log_b x}{\log_b a}$ $\qquad$ $(a, b, x > 0 \quad a \neq 1 \quad b \neq 1)$

Compound Interest Formula: $A = P\left(1 + \frac{r}{m}\right)^{mt}$

Continuous Compounding: $A = Pe^{rt}$

Exponential Growth or Decay: $A(t) = A_0 e^{kt}$, $k \neq 0$

The half-life of a substance: $T = -\frac{\ln 2}{k}$

Effective rates for compound interest: $r_E = \left(1 + \frac{r}{m}\right)^m - 1$ or $r_E = e^r - 1$

Rules of 70 and 72. The time for the principal to double when r is compounded annually:

if $0.001 \leq r < 0.05$, then $t \approx \frac{70}{100r}$ years

if $0.05 \leq r < 0.12$, then $t \approx \frac{72}{100r}$ years

1. For the lines given below, find the slope m if it is defined:

(a) the line passes through the points $(1,3)$ and $(6,-5)$;

(b) the line passes through the points $(-2,3)$ and $(-2,5)$;

(c) the line is horizontal passing through the point $(4,-3)$;

(d) the line with x-intercept 1 and y-intercept 4;

(e) the line is vertical passing through the point $(3,5)$.

2. For each of the choices in #1(a-e), find an equation of the line. Give the final answer in the slope-intercept form where possible.

3. Find an equation of the line passing through the point $(-6,-3)$ and parallel to the line through $(-1,2)$ and $(\frac{1}{2},4)$. Give the final answer in a general form.

4. Find an equation of the line through the point $(\frac{3}{5},-2)$ perpendicular to the line $3x-2y=6$. Give the final answer in the slope-intercept form. What is the $y-$intercept?

5. Suppose the supply function for a certain good is given by

$$p = S(q) = 2q - 1$$

where q is a quantity in thousands and p is a price in dollars per unit. Suppose also that the equilibrium price is \$3 and the demand is 3 units when the price is \$2.5.

(a) Find an equation for the demand function assuming it is linear.

(b) For which values of the price p there will be a shortage of the good?

(c) For which p there will be a surplus?

(d) Express each the demand and the supply as a function of the price p.

6. Suppose the cost to produce x metal cans is given by $C(x)=\frac{1}{5}x+20$ and the break-even quantity is 150 cans. Assume that the price is a constant. Answer the questions below.

(a) What is the cost to produce one can?

(b) What is the fixed cost?

(c) What is the price at which a can is sold?

(d) Find the profit function.

(e) What is the marginal profit?

7. Which of the following relations define y as a function of x? When the relation defines a function, find the explicit representation as $y = f(x)$.

(a) $x-2y=35$ (b) $y^2-2=3x$ (c) $y=\sqrt{x^2+9}$ (d) $y^3-4x-1=0$.

8. If $f(x)=4x^2-3x$, find and simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$ ($h \neq 0$).

9. Given a function: $f(x)=\begin{cases} x-3 & \text{if } x \le -2 \\ -1 & \text{if } -2 < x < 1. \\ x^2+1 & \text{if } x > 1 \end{cases}$

Find: $f(-2)$, $f(0)$, $f(1)$, $f(2)$

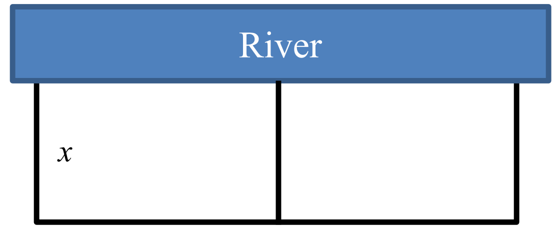
10. Classify the following functions as even, odd, or neither even nor odd:

(a) $f(x)=x^3-2x^2$ (b) $f(x)=\frac{1}{\sqrt{x^2+5}}$ (c) $f(x)=\frac{x^2+1}{x^3-x}$ (d) $f(x)=|x|$

11. Find the domain of each function. For the functions in (a) and (b), find the range.

(a) $f(x)=\frac{x^2-4}{x-2}$ (b) $f(x)=\frac{4}{x^2+4}$ (c) $f(x)=\sqrt{\frac{x-2}{3-x}}$

12. Mary wants to construct a rectangular pasture on her property of the shape of two equal adjacent rectangles parallel to the river and she has 600 feet of fencing (no fencing is needed along the river). Let x represents the length of the side perpendicular to the river (see the chart below).



(a) Express the area of the pasture A as a function of x.
(b) Find the domain of the function $A(x)$.

13. Determine the transformations (such as reflections and translations) and their sequence that are needed to graph the following function starting with $y = \sqrt[3]{x}$.

$$f(x) = -\sqrt[3]{2-x} - 1$$

14. Find the vertex, axis, and intercepts and graph the quadratic function.

$$f(x) = 3x^2 - 4x - 7$$

15. Find the quadratic function whose graph has the vertex at $(2, 4)$ and passes through the point $(-1, 1)$.

16. For the given a polynomial $f(x) = x^4 - x^3 - 6x^2$, determine the following:
(a) the end behavior;
(b) real zeros (x-intercepts) and their multiplicities;
(c) number of turning points;
(d) symmetry, if any.

17. For the given rational function $R(x) = \dfrac{2(x^2-1)(x-3)}{(x-1)(x^2+x-6)}$, determine the following:
(a) domain;
(b) real zeros (x-intercepts) and their multiplicities;
(c) vertical asymptotes;
(d) holes;
(e) horizontal asymptote;
(f) symmetry.

18. Find the equation of a rational function $R(x)$ which has the following features:
x-intercepts: $x = -3$ (where it crosses the x-axis) and $x = 4$ (where it touches the x-axis);
vertical asymptotes: $x = -2$ (where it does not change the sign) and $x = 1$ (where it changes the sign), $x = 0$ (where it does not change the sign);
horizontal asymptote: $y = 0$;
hole: $(-4, 0)$;
degree of the numerator: 5;
degree of the denominator: 6;
$R(x) < 0$ as $x \to +\infty$.

19. Consider the function $f(x)=2-e^{-x+1}$.

(a) Find the domain and the asymptote. Evaluate the function at $x=-1, 0, 1, 2$. Sketch the graph of the function.

(b) Give the range of f. Is the function increasing or decreasing?

(c) Describe all transformations (in a correct order) that could be applied to the graph of $y=e^x$ to obtain the graph of $f(x)$.

(d) What function (used in applications) does the function $f(x)$ for $x>0$ represent?

20. Solve: $16^{x+2}=8^{1-x}$.

21. Write the statements below in equivalent exponential form:

(a) $\log_5 125=3$ (b) $\ln 2=x$ (c) $\log\frac{1}{1000}=-3$.

22. Write the statements below in equivalent logarithmic form:

(a) $e^0=1$ (b) $10^{2x}=4$ (c) $\left(\frac{1}{2}\right)^{-4}=16$ (d) $\frac{1}{y}=2^{1-x}$

23. Evaluate without using a calculator:

(a) $\log_{25} 5$ (b) $\log_2 \frac{1}{2}$ (c) $\log 100$ (d) $\ln e^3$ (e) $\log_{1/3} 81$

24. (a) Find the domain, range, asymptote, intercepts and sketch the graph of the function $f(x)=2-2\log_2(x+1)$.

(b) Describe all transformations (in a correct order) that should be applied to the graph of $y=\log_2 x$ in order to obtain the graph of $f(x)$.

25. Find the domain of the logarithmic function.

$$f(x)=\ln\frac{x-4}{x+3}$$

26. Which of the following statements is/are true?

A. $\log_3 0=1$

B. $-\log_3 x=\log_{\frac{1}{3}} x$

C. $\log_a b=\frac{1}{\log_b a}$ $(a,b>0;\ a,b\neq 1)$

D. $\frac{\log x}{\log y}=\log(x-y)$ $(x, y, x-y>0$

E. $\log_a\left(\frac{1}{b}\right)=-\log_a b$ $(a,b>0, a\neq 1)$

F. $2^x=e^{x\ln 2}$

27. Solve the exponential equations. Simplify your answers where possible and leave them in term of logarithms.

(a) $3\left(2^{x^2}\right)=99$; (b) $e^{3x}=125$; (c) $5^{x-1}=4^{x+1}$ (use natural logarithm); (d) $e^{\frac{x}{3}\ln 4}=6$.

28. Solve the logarithmic equations:

(a) $\log_2(x-3)^2 = 4$; (b) $\log_3(2x+1)-\log_3(x-1)=2$;

(c) $\ln x + \ln(x-2) = \ln 3$; (d) $\log_3\sqrt{x-6} = 2$.

29. Simplify, where it is possible. Give all restrictions on the variables:

(a) $\dfrac{\log_3 9}{\log_5 25}$; (b) $e^{3\ln x}$; (c) $\log 10^{2x^2}$; (d) $\log_4 2^{x^2}$; (e) $5^{\log_3 x}$; (f) $\ln\left(x^2+y^2\right)$.

30. Write as a logarithm of a single quantity: $5\log 2 - 3\log x + 2\log y$ ($x, y > 0$).

31. Write the single logarithm as a sum and/or difference: $\log_6 \dfrac{6\sqrt{x}\, y\sqrt{y}}{z^2}$ ($x, y, z > 0$).

32. Use Change-of-Base Formula for Logarithms to rewrite each logarithm with a new base b. Simplify where it is possible.

(a) $\log_2 e$, $b = e$; (b) $\log_{100} x$, $b = 10$; (c) $\log_{36} 216$, $b = 6$; (d) $\log_5 26, b = e$.

33. Use Change-of-Base Formula for Exponentials to rewrite each expression with a new base b. Simplify where it is possible.

(a) 5^x, $b = e$; (b) $e^{\frac{5}{2}t\ln 2}$, $b = 2$.

34. George wants to buy a \$30,000 car. He has saved \$27,000. How many years will be needed for \$27,000 to increase to \$30,000 at 4% compounded

(a) quarterly; (b) continuously?

35. How long will it take a certain bacteria population to double if it grows according to the formula

$$N(t) = N_0 e^{3t(\ln 2)/2}$$

where t is in hours?

36. A radioactive substance is decaying so that the number of grams present after t days is given by

$$A(t) = 2000e^{-0.02t}.$$

(a) Find the amount of the substance present after 100 days.

(b) Find the half-life of the substance, T.

37. A fossilized leaf contains 27% of its normal amount of carbon 14. How old is the fossil? Use 5600 years as the half-life of carbon 14.

38. Suppose Sally has \$5,000 on her account. What interest rate does she need to have \$7,500 in 10 years if the interest is compounded

(a) monthly; (b) continuously?

39. Suppose a certain bacteria population is growing according to the formula

$$N(t) = 100 - 80e^{-t/10},$$

where N is in thousands and t is in hours.

(a) What is the original size of the population?

(b) How long does it take for the population to reach 90,000?

(c) What is the limiting size of the population?

40. Suppose the stated rate of interest is 6%. Find the effective rate of interest if it is compounded

(a) quarterly; (b) continuously.

41. Approximate the time of the amount to double at an interest rate of 3.5% using the rule of 70 or 72.