

MAC 2233: Exam 1 Review  
Unit 1 Exam Review covers Lectures 1 - 14

1. Solve for  $x$ :  $2(x+1)^{-1/3}x^{4/3} - (x+1)^{2/3}x^{-2/3} = 0$

$x \neq -1$

$$x^{2/3}(x+1)^{1/3} \left( \frac{2x^{4/3}}{(x+1)^{1/3}} - \frac{(x+1)^{2/3}}{x^{2/3}} \right) = 0 \cdot x^{2/3} \cdot (x+1)^{1/3}$$

$$2x^{4/3}x^{2/3} - (x+1)^{2/3}(x+1)^{1/3} = 0$$

$x = 1, -\frac{1}{2}$

$$2x^2 - (x+1) = 0$$

$$2x^2 - x - 1 = (2x+1)(x-1) = 0$$

2. Perform the operation and simplify the expression:

$$\frac{\frac{3x}{\sqrt{x^2+4}} - \sqrt{x^2+4}}{2\sqrt{x^2+4}}$$

$$\frac{\frac{3x}{\sqrt{x^2+4}} - \sqrt{x^2+4}}{2\sqrt{x^2+4}} \cdot \frac{\sqrt{x^2+4}}{\sqrt{x^2+4}} = \frac{3x - (x^2+4)}{2(x^2+4)} = \frac{3x - x^2 - 4}{2(x^2+4)}$$

3. Solve the inequality:  $\frac{x+4}{x-1} \leq 2$

$$\frac{x+4}{x-1} - 2 \leq 0$$

$$\frac{x+4 - 2(x-1)}{x-1} \leq 0$$

$$\frac{x+4 - 2x + 2}{x-1} \leq 0$$

$$\frac{-x+6}{x-1} \leq 0$$

critical numbers  
 $x = 6, 1$



$$(-\infty, 1) \cup [6, \infty)$$

4. Find and simplify  $\frac{f(x+h) - f(x)}{h}$  for

a)  $f(x) = 2x^2 - x - 3$  and b)  $f(x) = \frac{x}{x+4}$ .

$$\begin{aligned} \text{a) } \frac{f(x+h) - f(x)}{h} &= \frac{2(x+h)^2 - (x+h) - 3 - (2x^2 - x - 3)}{h} \\ &= \frac{2(x^2 + 2xh + h^2) - x - h - 3 - 2x^2 + x + 3}{h} \\ &= \frac{\cancel{2x^2} + 4xh + 2h^2 - \cancel{x} - h - \cancel{3} - \cancel{2x^2} + \cancel{x} + \cancel{3}}{h} \\ &= \frac{4xh + 2h^2 - h}{h} \\ &= \frac{\cancel{h}(4x + 2h - 1)}{\cancel{h}} \\ &= 4x + 2h - 1 \end{aligned}$$

$$b) f(x) = \frac{x}{x+4}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{\frac{x+h}{x+h+4} - \frac{x}{x+4}}{h}$$

$$\frac{(x+4)(x+h+4)}{(x+4)(x+h+4)}$$

$$= \frac{(x+h)(x+4) - x(x+h+4)}{h(x+4)(x+h+4)}$$

$$= \frac{\cancel{x^2} + \cancel{4x} + \cancel{hx} + \cancel{4h} - \cancel{x^2} - \cancel{xh} - \cancel{4x}}{h(x+4)(x+h+4)}$$

$$= \frac{\cancel{h}h}{\cancel{h}(x+4)(x+h+4)}$$

$$= \frac{4}{(x+4)(x+h+4)}$$

domain of  $f$   $(-\infty, 2) \cup (2, \infty)$

$g$   $(-\infty, 0) \cup (0, \infty)$

5. Let  $f(x) = \frac{x}{x-2}$  and  $g(x) = \frac{2}{x} + 1$ . Find the functions  $(f \circ g)(x)$

and  $(g \circ f)(x)$ . Include domains.

$$(f \circ g)(x) = (f(g(x))) = \frac{\frac{2}{x} + 1}{\frac{2}{x} + 1 - 2} \cdot \frac{x}{x} = \frac{2+x}{2-x} \quad \text{domain } x \neq 0, 2$$

$(-\infty, 0) \cup (0, 2) \cup (2, \infty)$

$$(g \circ f)(x) = g(f(x)) = \frac{2}{\frac{x}{x-2}} + 1 = \frac{2(x-2)}{x} + 1 = \frac{2x+4+x}{x} = \frac{3x+4}{x}$$

domain  $(-\infty, 0) \cup (0, 2) \cup (2, \infty)$   
 $x \neq 2, 0$

6. Let  $f(x) = \sqrt{x-1}$  and  $g(x) = \frac{x}{\sqrt{x-1}}$ . Find  $\frac{f}{g}(x)$  and its domain.

$$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-1}}{\frac{x}{\sqrt{x-1}}} \cdot \frac{\sqrt{x-1}}{x} = \frac{x-1}{x}$$

$x \neq 0$      $x-1 > 0$      $x > 1$

domain  $(1, \infty)$

$$x-1 > 0$$

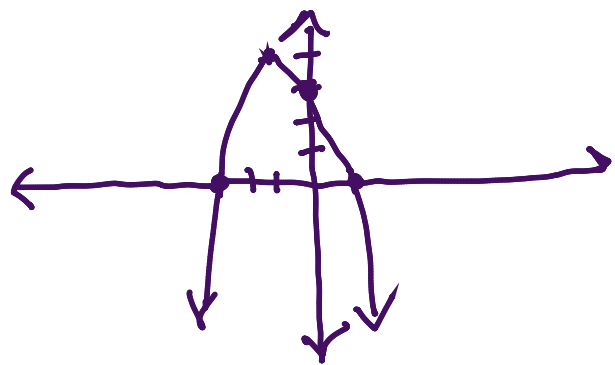
$$x^2 + 2x - 3 \quad (x+3)(x-1)$$

7. Sketch the graph of  $f(x) = 3 - 2x - x^2$  by using a formula to find the vertex. Show all intercepts. Confirm your work by writing your function in standard form  $f(x) = a(x-h)^2 + k$  by completing the square, and using translations to graph.

$$x = \frac{-b}{2a} = \frac{-(-2)}{2(-1)} = -1$$

$$f(-1) = 3 - 2(-1) - (-1)^2 = 3 + 2 - 1 = 4 \quad (-1, 4)$$

$$\begin{aligned} f(x) &= 3 - (x^2 + 2x) \\ &= 3 - (x^2 + 2x + 1) + 1 \\ &= 4 - (x+1)^2 \end{aligned}$$

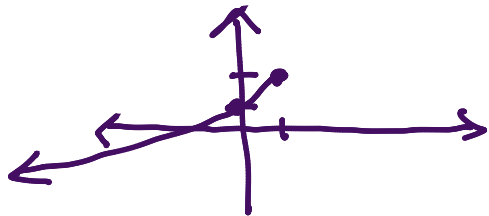




8. Sketch the graph of  $f(x) = 2 - \sqrt{1-x}$ . Starting with  $y = \sqrt{x}$ , list each translation used to graph  $f(x)$ .

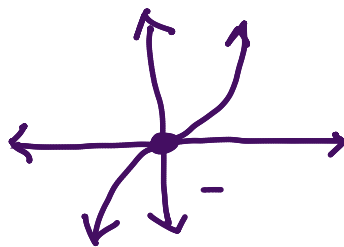
$$y = -\sqrt{-(x-1)} + 2$$

→ 1 reflect over x and y axis  
 ↑ 2



9. Use the definition of absolute value to write the function  $g(x) = x|x|$  as a piecewise defined function. Then sketch its graph.

$$g(x) = \begin{cases} x(-x) & x < 0 \\ x(x) & x \geq 0 \end{cases} \quad \begin{cases} -x^2 & x < 0 \\ x^2 & x \geq 0 \end{cases}$$



10. Find the inverse of  $f(x) = \sqrt{4-x}$ . Be sure to include domain.

$$x = \sqrt{4-y}$$

$$x \geq 0$$

$$(0, \infty)$$

$$x^2 = 4 - y$$

$$y = 4 - x^2$$

$$f^{-1}(x) = 4 - x^2$$

11. Find the inverse of one-to-one function  $f(x) = \frac{x+2}{x-3}$ . Use that inverse function to find the range of  $f(x)$ . Then find the horizontal asymptote of  $f(x)$  if possible.

$$x = \frac{y+2}{y-3}$$

$$x(y-3) = y+2$$

$$xy - 3x = y + 2$$

$$xy - y = 3x + 2$$

$$y(x-1) = 3x+2$$

$$f^{-1}(x) = \frac{3x+2}{x-1}$$

domain of  $f^{-1}$  is equal to the range of  $f$   $(-\infty, 1) \cup (1, \infty)$

horizontal asymptote  $y=1$

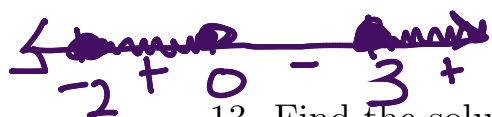
12. Find the domain of the following functions:

(a)  $f(x) = \sqrt{x^3 - x^2 - 6x}$       (b)  $f(x) = \ln\left(\frac{8}{x} - 2\right)$

$$x^3 - x^2 - 6x \geq 0$$

$$x(x^2 - x - 6) \geq 0$$

$$x(x-3)(x+2) \geq 0$$



$$[-2, 0] \cup [3, \infty)$$

$$\frac{8}{x} - 2 > 0$$

$$\frac{8-2x}{x} > 0$$



13. Find the solution set of each of the following equations:

(a)  $\log_3(2x^2 - 5) - \log_3 x = 1$       (b)  $4^{3-x^2} = \left(\frac{1}{8}\right)^{x+1}$

(c)  $\ln(x+8) + \ln(x-2) = \ln(3x+2)$

a)  $\log_3\left(\frac{2x^2-5}{x}\right) = 1$

$$3^1 = \frac{2x^2-5}{x}$$

$$0 = 2x^2 - 3x - 5$$

$$0 = 2x^2 - 5x + 2x - 5$$

$$= x(2x-5) + (2x-5)$$

$$(2x-5)(x+1)$$

$$x = 5/2, -1$$

b)  $(2^2)^{3-x^2} = (2^{-3})^{x+1}$

$$2(3-x^2) = -3(x+1)$$

$$6 - 2x^2 = -3x - 3$$

$$0 = 2x^2 - 3x - 9$$

$$0 = (2x+3)(x-3)$$

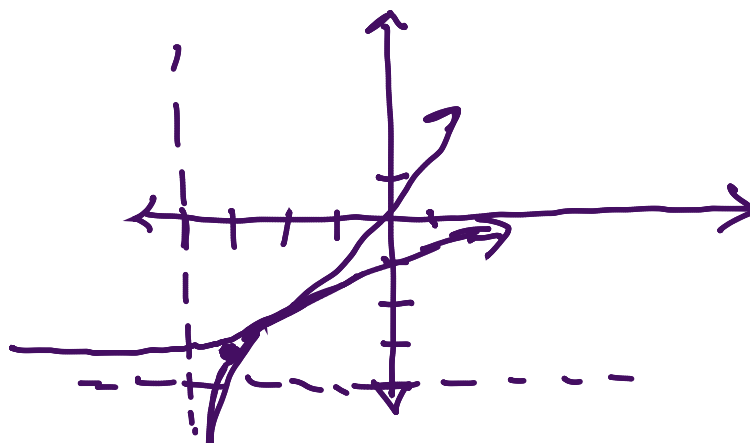
14. Find the inverse of  $f(x) = e^{x+3} - 4$ . Sketch the graph of  $f$  and  $f^{-1}$  on the same axes. Include at least one point and any asymptotes of each function.

$$x = e^{y+3} - 4$$

$$x + 4 = e^{y+3}$$

$$\ln(x+4) = y+3$$

$$f^{-1}(x) = \ln(x+4) - 3$$



15. Let  $f(x) = \begin{cases} x + 4 & x < -2 \\ 2 - |x| & -2 \leq x < 2 \\ \ln(x - 1) & x > 2 \end{cases}$ .

(a) Find if possible:  $f(-4)$ ,  $f(-2)$ ,  $f(0)$ ,  $f(2)$ ,  $f(e + 1)$ .

(b) Sketch the graph of  $y = f(x)$ . (c) Use your graph to evaluate the following

a)

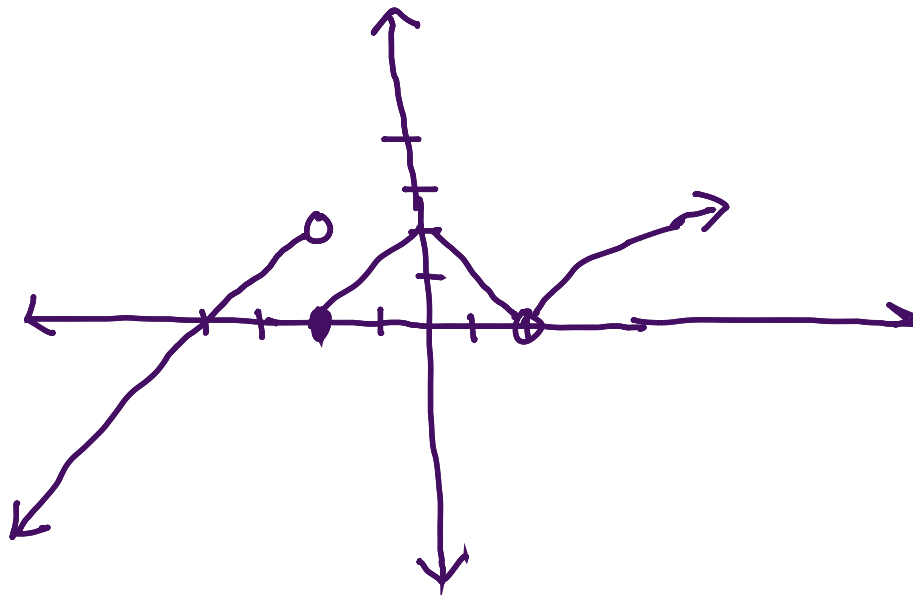
$$f(-4) = -4 + 4 = 0$$

$$b) f(-2) = 2 - |-2| = 0$$

$$c) f(0) = 2 - |0| = 2$$

d)  $f(2)$  is undefined

$$e) f(e+1) = \ln(e+1-1) = \ln(e) = 1$$



limits if they exist:

$$1) \lim_{x \rightarrow -2} f(x)$$

$$= \text{DNE}$$

$$2) \lim_{x \rightarrow 0} f(x)$$

$$= 2$$

$$3) \lim_{x \rightarrow 2} f(x)$$

$$= 0$$

16. Let  $f(x) = \frac{x^2 - 4}{x^2 - 2x - 8}$ . Find:

- (a) domain of  $f$   $(-\infty, -2) \cup (2, 4) \cup (4, \infty)$   
 (b) all intercepts (express as ordered pairs)  
 (c) all vertical and horizontal asymptotes  
 (d) Sketch the graph of  $y = f(x)$ . Include the coordinates of any holes in the function.  
 (e) Use your graph to find  $\lim_{x \rightarrow -2} f(x)$ .

$$f(x) = \frac{(x-2)(x+2)}{(x-4)(x+2)} = \frac{x-2}{x-4}$$

b) X-intercept

$$0 = \frac{x-2}{x+4}$$

$$0 = x - 2$$

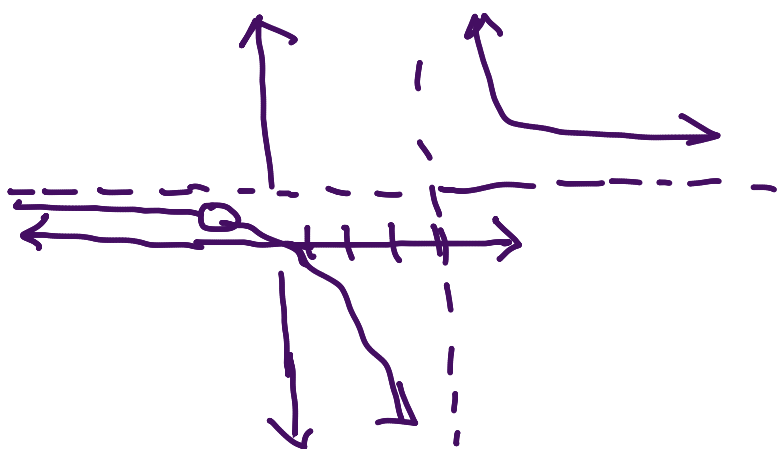
$$x = 2 \quad (2, 0)$$

Y-intercept

$$f(0) = \frac{0-2}{0-4} = \frac{1}{2} \quad (0, \frac{1}{2})$$

$$V.A \quad x = 4$$

$$H.A \quad y = 1$$



$$\lim_{x \rightarrow -2} \frac{x-2}{x-4} = \frac{-2-2}{-2-4} = \frac{-4}{-6} = \frac{2}{3}$$

17. There is a linear relationship between temperature in degrees Celsius  $C$  and degrees Fahrenheit  $F$ . Water freezes at  $0^\circ C$  ( $32^\circ F$ ) and boils at  $100^\circ C$  ( $212^\circ F$ ). Write the model expressing  $C$  as function of  $F$ . What is the temperature in degrees Fahrenheit if the temperature is  $30^\circ C$ ? What does the slope of the line tell you?

$$(32, 0), (212, 100)$$

$$m = \frac{0 - 100}{32 - 212} = \frac{-100}{-180} = \frac{5}{9}$$

$$C = mF + b$$

$$0 = \frac{5}{9}(32) + b$$

$$b = -\frac{160}{9}$$

$$C = \frac{5}{9}F - \frac{160}{9}$$

temperature increases  $5^\circ C$  as Fahrenheit temp increases by  $9^\circ$

18. The demand and supply functions for a given product are given by

$p = D(q) = 60 - 2q^2$  and  $p = S(q) = q^2 + 9q + 30$  where  $q$  is quantity in thousands and  $p$  is the unit price. Find the equilibrium quantity and price.

How many items will the supplier provide if the unit price of the product is \$40? What will be the demand for the product when the unit price is \$40? What should happen to the price of the product?

$$60 - 2q^2 = q^2 + 9q + 30$$

$$0 = 3q^2 + 9q - 30$$

$$0 = 3(q^2 + 3q - 10)$$

$$= 3(q + 5)(q - 2) \quad q = 2$$

$$p = D(2) = 60 - 2(2)^2 = 52 \quad \$52$$

$$40 = q^2 + 9q + 30$$

$$0 = q^2 + 9q - 10$$

$$= (q + 10)(q - 1) \quad q = 1$$

(1000 items)

$$40 = 60 - 2q^2 \quad 2q^2 = -20$$

$$q = \sqrt{10}$$

price will rise

19. A financial manager at Target has made the following observations about a certain product in one of its districts: an average of 250 units will sell in a month when the price is \$15, but an average of 50 more will sell if the price is reduced by \$1. Assuming the demand function is linear,

- Express  $p$  as a function of  $x$ .
- Find the revenue function  $R(x)$ . Find the production level  $x$  that will maximize revenue. What is the maximum revenue?
- If fixed costs are \$800 and the marginal cost is \$10 per item, find each value of  $x$  at which the company will break even. What is the profit for those values?
- Find the profit function  $P(x)$ . What price should the manager charge to maximize profit on this item?

$$a) m = \frac{15-14}{250-300} = -\frac{1}{50}$$

$$15 = -\frac{1}{50}(250) + b$$

$$15 = -5 + b \quad b = 20$$

$$p = -\frac{1}{50}x + 20$$

$$b) R(x) = xp = x\left(-\frac{1}{50}x + 20\right)$$

$$= -\frac{1}{50}x^2 + 20x$$

$$x = -\frac{b}{2a} = \frac{-20}{2\left(-\frac{1}{50}\right)} = \frac{-20}{-\frac{1}{25}} = 500$$

$$R(500) = -\frac{1}{50}(500)^2 + 20(500)$$

$$= -5000 + 10,000 = 5000 \quad \leftarrow \text{maximum revenue}$$

$$c.) \text{ revenue} = \text{cost}$$

$$-\frac{1}{50}x^2 + 20x = 10x + 800$$

$$0 = \frac{1}{50}x^2 - 10x + 800$$

$$= x^2 - 500x + 40,000$$

$$(x - 400)(x - 100)$$

$$x = 400, 100$$

profit is 0

d) Profit = Revenue - Cost

$$P(x) = -\frac{1}{50}x^2 + 20x - (10x + 800)$$

$$= -\frac{1}{50}x^2 + 10x - 800$$

$$x = \frac{-b}{2a}$$

$$= \frac{-10}{2\left(-\frac{1}{50}\right)}$$

$$= 250$$

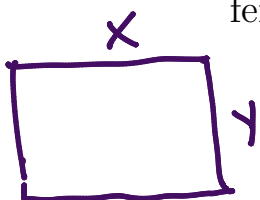
$$p = -\frac{1}{50}x + 20$$

$$p = -\frac{1}{50}(250) + 20$$

$$= -5 + 20$$

$$= 15$$

20. A farmer plans to spend \$6000 to enclose a rectangular field with two kinds of fencing. Two opposite sides will require heavy-duty fencing that costs \$3 per linear foot, while the other two sides can be constructed with standard fencing that costs \$2 per foot. Express the area of the field,  $A$ , as a function of  $x$ , the length of a side that requires the more expensive fence. Find the value of  $x$  that will maximize the area of the field, and the length of a side that uses standard fencing.



$$A = xy$$

$$A(x) = x \left( 1500 - \frac{3}{2}x \right)$$

$$= 1500x - \frac{3}{2}x^2$$

$$2(3x) + 2(2y) = 6000$$

$$3x + 2y = 3000$$

$$2y = 3000 - 3x$$

$$y = 1500 - \frac{3}{2}x$$

$$x = -\frac{b}{2a} = \frac{-1500}{2(-\frac{3}{2})} = 500$$

$$y = 1500 - \frac{3}{2}(500)$$

$$= 750$$

21. Rewrite the expression as the sum, difference, or multiple of logarithms:

(a)  $\log \frac{x^2}{1000}$

(b)  $\ln \sqrt[3]{\frac{e^{x+1}(x-2)^4}{x^6}}$

$$(a) = \log x^2 - \log 1000$$

$$= 2 \log x - 3$$

$$(b) \ln \sqrt[3]{\frac{e^{x+1}(x-2)^4}{x^6}} = \frac{1}{3} \ln \left( \frac{e^{x+1}(x-2)^4}{x^6} \right)$$

$$= \frac{1}{3} \left[ \ln e^{x+1} + \ln (x-2)^4 - \ln x^6 \right]$$

$$= \frac{1}{3} (x+1 + 4 \ln(x-2) - 6 \ln x)$$



22. Mr. Jones invested \$2500 at 5.5% compounded continuously. How long will it take his account to grow to \$4000 if he adds no new funds to the account?

$$\frac{4000}{2500} = \frac{2500}{2500} e^{.055t} \quad t = \frac{\ln(8/5)}{.055} \approx 8.5 \text{ years}$$

$$\frac{8}{5} = e^{.055t}$$

$$\ln\left(\frac{8}{5}\right) = .055t$$

23. How much money must be invested now at 3 1/4% compounded quarterly in order to have \$6000 in three years?

$$6000 = P_0 \left(1 + \frac{.0325}{4}\right)^{4 \cdot 3}$$

$$P_0 = \frac{6000}{\left(1 + \frac{.0325}{4}\right)^{12}} \approx \$5444.76$$

24. Iodine - 131 has a half-life of 8 days. Suppose some hay was contaminated with ten times the allowable amount of I-131. How long must the hay be stored before it can be fed to cattle? Hint: the hay must have one-tenth of its current amount of I-131.

$$\frac{1}{2} Q_0 = Q_0 e^{k(8)}$$

$$\frac{1}{2} = e^{8k}$$

$$\ln\left(\frac{1}{2}\right) = 8k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{8}$$

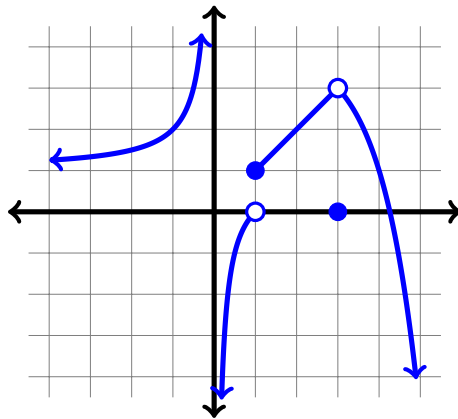
$$Q(t) = Q_0 e^{\frac{\ln\left(\frac{1}{2}\right)}{8} t}$$

$$\frac{1}{10} Q_0 = Q_0 e^{\frac{\ln\left(\frac{1}{2}\right)}{8} t}$$

$$\ln\left(\frac{1}{10}\right) = \frac{\ln\left(\frac{1}{2}\right)}{8} t$$

$$t = \frac{8 \ln\left(\frac{1}{10}\right)}{\ln\left(\frac{1}{2}\right)} \approx 26.57$$

25. Use the following graph of a function  $f(x)$  to evaluate the limits and function value if possible. If the limit does not exist, write "dne".



- a)  $\lim_{x \rightarrow 0^-} f(x) = \infty$       b)  $\lim_{x \rightarrow 0^+} f(x) = -\infty$       c)  $\lim_{x \rightarrow 0} f(x) = \text{DNE}$   
d)  $\lim_{x \rightarrow 1^+} f(x) = 1$       e)  $\lim_{x \rightarrow 1^-} f(x) = 0$       f)  $\lim_{x \rightarrow 1} f(x) = \text{DNE}$   
g)  $\lim_{x \rightarrow 3} f(x) = 3$       h)  $f(3) = 0$       i)  $\lim_{x \rightarrow -1} f(x) = 2$

26. Use the properties of limits to evaluate  $\lim_{x \rightarrow a} \frac{(fg)(x)}{\sqrt[3]{g(x)} - 1}$  if  $\lim_{x \rightarrow a} f(x) = -\frac{1}{3}$  and  $\lim_{x \rightarrow a} g(x) = 9$ .

$$\lim_{x \rightarrow a} \frac{f(x)g(x)}{\sqrt[3]{g(x)} - 1} = \frac{-\frac{1}{3}(9)}{\sqrt[3]{9-1}} = \frac{-3}{\sqrt[3]{8}} = -\frac{3}{2}$$

27. Evaluate (a)  $\lim_{x \rightarrow -1} \frac{x + \sqrt{x+2}}{x+1}$  and (b)  $\lim_{x \rightarrow 2} \frac{\frac{2}{x} - 1}{x-2}$ .

$$\begin{aligned} \text{a) } \lim_{x \rightarrow -1} \frac{x + \sqrt{x+2}}{x+1} & \cdot \frac{x - \sqrt{x+2}}{x - \sqrt{x+2}} = \lim_{x \rightarrow -1} \frac{x^2 - (x+2)}{(x+1)(x - \sqrt{x+2})} \\ & = \lim_{x \rightarrow -1} \frac{(x-2)(x+1)}{\cancel{(x+1)}(x - \sqrt{x+2})} \\ & = \lim_{x \rightarrow -1} \frac{x-2}{x - \sqrt{x+2}} \\ & = \frac{-1-2}{-1 - \sqrt{-1+2}} = \frac{-3}{-2} = \frac{3}{2} \end{aligned}$$

$$\begin{aligned} \text{(b) } \lim_{x \rightarrow 2} \frac{\frac{2}{x} - 1}{x-2} & \cdot \frac{x}{x} = \lim_{x \rightarrow 2} \frac{2-x}{x(x-2)} \\ & = \lim_{x \rightarrow 2} \frac{-(-2+x)}{x(\cancel{x-2})} \\ & = \lim_{x \rightarrow 2} -\frac{1}{x} = -\frac{1}{2} \end{aligned}$$

28. If  $f(x) = \begin{cases} \frac{x^2 - 16}{x^2 + 3x - 4} & x \neq -4 \\ 0 & x = -4 \end{cases}$

find  $p = \lim_{x \rightarrow -4} f(x)$  and  $q = \lim_{x \rightarrow 1^-} f(x)$ .

$$\frac{x^2 - 16}{x^2 + 3x - 4} = \frac{(x-4)(x+4)}{(x+4)(x-1)} = \frac{x-4}{x-1}$$

$$\lim_{x \rightarrow -4} \frac{x-4}{x-1} = \frac{-8}{-5}$$

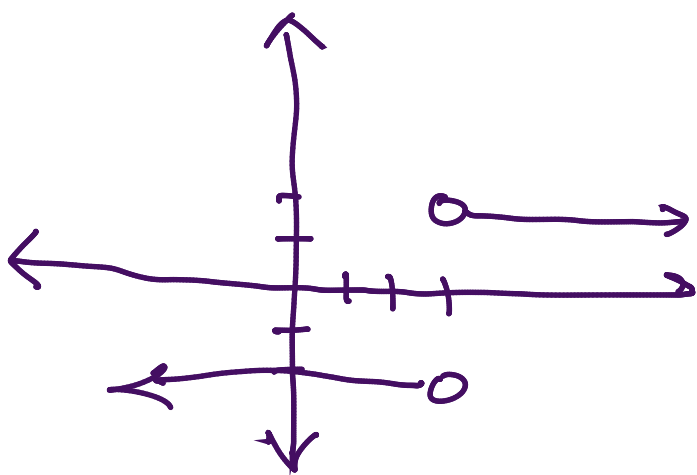
$$\lim_{x \rightarrow 1^-} \frac{x-4}{x-1} \begin{array}{l} \text{approaches } -5 \\ \text{approaches } 0 \text{ and} \\ \text{negative} \end{array} = -\infty$$

29. Sketch the graph of  $f(x) = \frac{|6-2x|}{x-3}$ . Hint: rewrite as a piecewise function without absolute value bars.

Use the graph to find: (a)  $\lim_{x \rightarrow 3^-} f(x)$ , (b)  $\lim_{x \rightarrow 3^+} f(x)$ , and (c)  $\lim_{x \rightarrow 3} f(x)$ .

Now find those limits algebraically without using the graph.

$$\begin{cases} \frac{-(6-2x)}{x-3} & 6-2x < 0 & \begin{cases} 2 & x > 3 \\ -2 & x < 3 \end{cases} \\ \frac{6-2x}{x-3} & 6-2x > 0 \end{cases}$$



$$\lim_{x \rightarrow 3^-} -2 = -2$$

$$\lim_{x \rightarrow 3^+} 2 = 2$$

30. If  $f(x) = \frac{x^3 + 3x^2 + 2x}{x - x^3}$ , find a)  $\lim_{x \rightarrow 0^+} f(x)$  b)  $\lim_{x \rightarrow -1^+} f(x)$ , c)  $\lim_{x \rightarrow 1^-} f(x)$

and d)  $\lim_{x \rightarrow -\infty} f(x)$ . Find each vertical and horizontal asymptote of  $f(x)$ .

$$\begin{aligned} f(x) &= \frac{x^3 + 3x^2 + 2x}{x - x^3} = \frac{x(x^2 + 3x + 2)}{x(1 - x^2)} \\ &= \frac{(x+2)(x+1)}{(1-x)(1+x)} \\ &= \frac{x+2}{1-x} \end{aligned}$$

$$a) \lim_{x \rightarrow 0^+} f(x) = 2$$

$$b) \lim_{x \rightarrow -1^+} f(x) = \frac{1}{2}$$

$$c) \lim_{x \rightarrow 1^-} f(x) = \infty$$

$$d) \lim_{x \rightarrow -\infty} \frac{1 + \frac{2}{x}}{1 - \frac{1}{x}} = \frac{1}{-1} = -1$$

31. If  $f(x) = \frac{2}{e^{-x} - 3}$ , find if possible:

- 1)  $\lim_{x \rightarrow -\infty} f(x)$    2)  $\lim_{x \rightarrow +\infty} f(x)$    3) Each asymptote of the graph of  $f(x)$ .

$$\lim_{x \rightarrow -\infty} \frac{2}{\frac{1}{e^x} - 3} = \lim_{x \rightarrow -\infty} \frac{2e^x}{1 - 3e^x} = 0$$

$$\lim_{x \rightarrow +\infty} \frac{2}{\frac{1}{e^x} - 3} = -\frac{2}{3}$$

$$e^{-x} = 3$$

$$\frac{1}{3} = e^x$$

$$\ln\left(\frac{1}{3}\right) = x$$

asymptotes  
 $y = 0$   
 $y = -\frac{2}{3}$   
 $x = \ln\left(\frac{1}{3}\right)$

32. The Intermediate Value Theorem guarantees that the function

$f(x) = x^3 - \frac{1}{x} - 5x + 3$  has a zero on which of the following intervals?

- a)  $[-1, 1]$    b)  $[1, 3]$    c)  $[3, 5]$    d)  $[-3, -2]$

b)

$$f(1) = (1)^3 - \frac{1}{1} - 5(1) + 3 = -2 < 0$$

$$f(3) = 3^3 - \frac{1}{3} - 5(3) + 3 > 0$$

} changes signs

d)

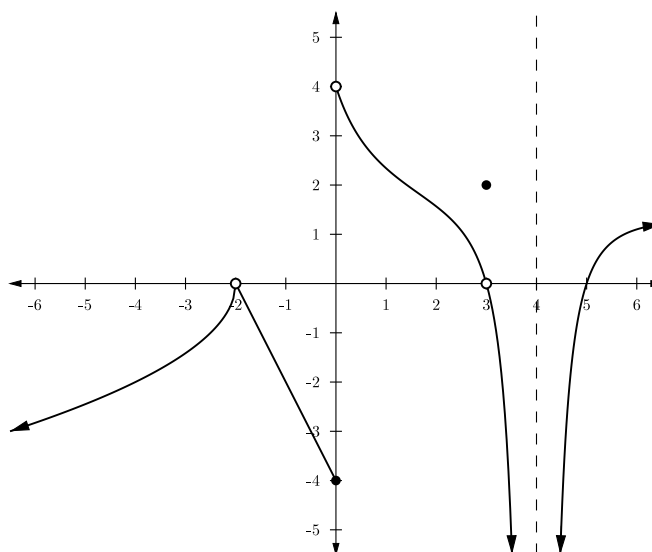
$$f(-3) = (-3)^3 - \frac{1}{-3} - 5(-3) + 3 < 0$$

$$f(-2) = (-2)^3 - \frac{1}{-2} - 5(-2) + 3$$

$$8 + \frac{1}{2} \quad 10 + 3 > 0$$

} changes signs

33. Consider a function  $f(x)$  which has the following graph.



- (a) On which interval(s) is  $f(x)$  continuous?  $(-\infty, -2), (-2, 0), (0, 3), (3, 4), (4, \infty)$
- (b)  $f(x)$  has a jump discontinuity at  $x = \underline{0}$ .
- (c)  $f(x)$  has an infinite discontinuity at  $x = \underline{4}$ .
- (d)  $f(x)$  has a removable discontinuity at  $x = \underline{-2, 3}$ .
- (e) How would you define or redefine  $f(x)$  at the point(s) in part (d) in order to make  $f(x)$  continuous?

$$f(-2) = 0$$

$$f(3) = 0$$