MAC 2233: Exam 1 Review
Unit 1 Exam Review covers Lectures 1 – 14

1. Solve for x: $2(x+1)^{-1/3}x^{4/3} - (x+1)^{2/3}x^{-2/3} = 0$

2. Perform the operation and simplify the expression: $\dfrac{\dfrac{3x}{\sqrt{x^2+4}} - \sqrt{x^2+4}}{2\sqrt{x^2+4}}$

3. Solve the inequality: $\dfrac{x+4}{x-1} \leq 2$

4. Find and simplify $\dfrac{f(x+h)-f(x)}{h}$ for

a) $f(x) = 2x^2 - x - 3$ and b) $f(x) = \dfrac{x}{x+4}$.

5. Let $f(x) = \dfrac{x}{x-2}$ and $g(x) = \dfrac{2}{x} + 1$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$. Include domains.

6. Let $f(x) = \sqrt{x-1}$ and $g(x) = \dfrac{x}{\sqrt{x-1}}$. Find $\dfrac{f}{g}(x)$ and its domain.

7. Sketch the graph of $f(x) = 3 - 2x - x^2$ by using a formula to find the vertex. Show all intercepts. Confirm your work by writing your function in standard form $f(x) = a(x-h)^2 + k$ by completing the square, and using translations to graph.

8. Sketch the graph of $f(x) = 2 - \sqrt{1-x}$. Starting with $y = \sqrt{x}$, list each translation used to graph $f(x)$.

9. Use the definition of absolute value to write the function $g(x) = x|x|$ as a piecewise defined function. Then sketch its graph.

10. Find the inverse of $f(x) = \sqrt{4-x}$. Be sure to include domain.

11. Find the inverse of one-to-one function $f(x) = \dfrac{x+2}{x-3}$. Use that inverse function to find the range of $f(x)$. Then find the horizontal asymptote of $f(x)$ if possible.

12. Find the domain of the following functions:

(a) $f(x) = \sqrt{x^3 - x^2 - 6x}$ (b) $f(x) = \ln\left(\dfrac{8}{x} - 2\right)$

13. Find the solution set of each of the following equations:

(a) $\log_3(2x^2 - 5) - \log_3 x = 1$ (b) $4^{3-x^2} = \left(\dfrac{1}{8}\right)^{x+1}$

(c) $\ln(x+8) + \ln(x-2) = \ln(3x+2)$

14. Find the inverse of $f(x) = e^{x+3} - 4$. Sketch the graph of f and f^{-1} on the same axes. Include at least one point and any asymptotes of each function.

15. Let $f(x) = \begin{cases} x+4 & x < -2 \\ 2 - |x| & -2 \leq x < 2 \\ \ln(x-1) & x > 2 \end{cases}.$

(a) Find if possible: $f(-4)$, $f(-2)$, $f(0)$, $f(2)$, $f(e+1)$.

(b) Sketch the graph of $y = f(x)$. (c) Use your graph to evaluate the following

limits if they exist:

1) $\lim_{x \to -2} f(x)$ 2) $\lim_{x \to 0} f(x)$ 3) $\lim_{x \to 2} f(x)$

16. Let $f(x) = \dfrac{x^2 - 4}{x^2 - 2x - 8}$. Find:

(a) domain of f

(b) all intercepts (express as ordered pairs)

(c) all vertical and horizontal asymptotes

(d) Sketch the graph of $y = f(x)$. Include the coordinates of any holes in the function.

(e) Use your graph to find $\lim_{x \to -2} f(x)$.

17. There is a linear relationship between temperature in degrees Celsius C and degrees Fahrenheit F. Water freezes at $0°C\,(32°F)$ and boils at $100°C\,(212°F)$. Write the model expressing C as function of F. What is the temperature in degrees Fahrenheit if the temperature is $30°C$? What does the slope of the line tell you?

18. The demand and supply functions for a given product are given by

    $p = D(q) = 60 - 2q^2$ and $p = S(q) = q^2 + 9q + 30$ where q is quantity in thousands and p is the unit price. Find the equilibrium quantity and price.

    How many items will the supplier provide if the unit price of the product is \$40? What will be the demand for the product when the unit price is \$40? What should happen to the price of the product?

19. A financial manager at Target has made the following observations about a certain product in one of its districts: an average of 250 units will sell in a month when the price is \$15, but an average of 50 more will sell if the price is reduced by \$1. Assuming the demand function is linear,

(a) Express p as a function of x.

(b) Find the revenue function $R(x)$. Find the production level x that will maximize revenue. What is the maximum revenue?

(c) If fixed costs are \$800 and the marginal cost is \$10 per item, find each value of x at which the company will break even. What is the profit for those values?

(d) Find the profit function $P(x)$. What price should the manager charge to maximize profit on this item?

20. A farmer plans to spend \$6000 to enclose a rectangular field with two kinds of fencing. Two opposite sides will require heavy-duty fencing that costs \$3 per linear foot, while the other two sides can be constructed with standard fencing that costs \$2 per foot. Express the area of the field, A, as a function of x, the length of a side that requires the more expensive fence. Find the value of x that will maximize the area of the field, and the length of a side that uses standard fencing.

21. Rewrite the expression as the sum, difference, or multiple of logarithms:

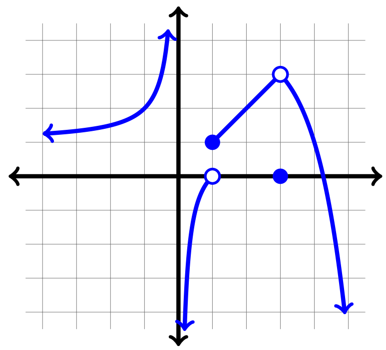
(a) $\log \dfrac{x^2}{1000}$ (b) $\ln \sqrt[3]{\dfrac{e^{x+1}(x-2)^4}{x^6}}$

22. Mr. Jones invested \$2500 at 5.5% compounded continuously. How long will it take his account to grow to \$4000 if he adds no new funds to the account?

23. How much money must be invested now at 3 1/4% compounded quarterly in order to have \$6000 in three years?

24. Iodine - 131 has a half-life of 8 days. Suppose some hay was contaminated with ten times the allowable amount of I-131. How long must the hay be stored before it can be fed to cattle? Hint: the hay must have one-tenth of its current amount of I-131.

25. Use the following graph of a function $f(x)$ to evaluate the limits and function value if possible. If the limit does not exist, write "dne".



a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$

d) $\lim_{x\to 1+} f(x)$ e) $\lim_{x\to 1^-} f(x)$ f) $\lim_{x\to 1} f(x)$

g) $\lim_{x\to 3} f(x)$ h) $f(3)$ i) $\lim_{x\to -1} f(x)$

26. Use the properties of limits to evaluate $\lim_{x\to a} \dfrac{(fg)(x)}{\sqrt[3]{g(x)} - 1}$ if $\lim_{x\to a} f(x) = -\dfrac{1}{3}$ and $\lim_{x\to a} g(x) = 9$.

27. Evaluate (a) $\lim_{x \to -1} \dfrac{x + \sqrt{x+2}}{x+1}$ and (b) $\lim_{x \to 2} \dfrac{\frac{2}{x} - 1}{x-2}$.

28. If $f(x) = \begin{cases} \dfrac{x^2 - 16}{x^2 + 3x - 4} & x \neq -4 \\ 0 & x = -4 \end{cases}$

find $p = \lim_{x \to -4} f(x)$ and $q = \lim_{x \to 1^-} f(x)$.

29. Sketch the graph of $f(x) = \dfrac{|6-2x|}{x-3}$. Hint: rewrite as a piecewise function without absolute value bars.

Use the graph to find: (a) $\lim_{x \to 3^-} f(x)$, (b) $\lim_{x \to 3^+} f(x)$, and (c) $\lim_{x \to 3} f(x)$.

Now find those limits algebraically without using the graph.

30. If $f(x) = \dfrac{x^3 + 3x^2 + 2x}{x - x^3}$, find a) $\lim_{x \to 0^+} f(x)$ b) $\lim_{x \to -1^+} f(x)$, c) $\lim_{x \to 1^-} f(x)$ and d) $\lim_{x \to -\infty} f(x)$. Find each vertical and horizontal asymptote of $f(x)$.

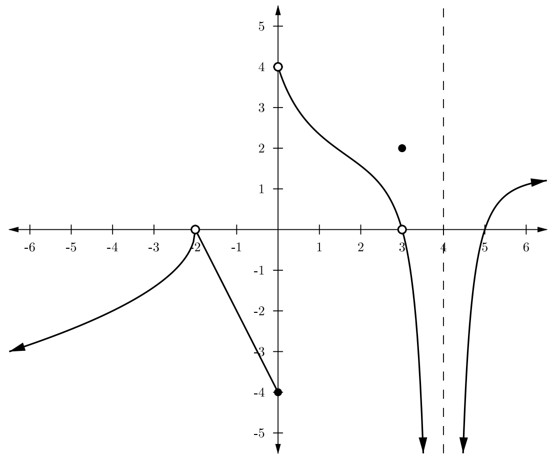
31. If $f(x) = \dfrac{2}{e^{-x} - 3}$, find if possible:

1) $\displaystyle\lim_{x \to -\infty} f(x)$ 2) $\displaystyle\lim_{x \to +\infty} f(x)$ 3) Each asymptote of the graph of $f(x)$.

32. The Intermediate Value Theorem guarantees that the function $f(x) = x^3 - \dfrac{1}{x} - 5x + 3$ has a zero on which of the following intervals?

a) $[-1, 1]$ b) $[1, 3]$ c) $[3, 5]$ d) $[-3, -2]$

33. Consider a function $f(x)$ which has the following graph.



(a) On which interval(s) is $f(x)$ continuous?

(b) $f(x)$ has a jump discontinuity at $x =$________.

(c) $f(x)$ has an infinite discontinuity at $x =$________.

(d) $f(x)$ has a removable discontinuity at $x =$________.

(e) How would you define or redefine $f(x)$ at the point(s) in part (d) in order to make $f(x)$ continuous?