

Lecture 1

1.

Multiply. Write your answer in lowest terms.

$$\frac{1}{7} \cdot \frac{1}{3}$$

2.

Multiply: $\frac{1}{8} \cdot \frac{2}{5}$

Give your answer as a fraction, reduced to lowest terms

3.

Add $\frac{3}{16} + \frac{5}{16}$

Reduce your answer to lowest terms

4.

Add and give your answer as a fraction (not mixed number), reduced to lowest terms.

$$\frac{2}{5} + \frac{3}{2} =$$

5.

Add $\frac{3}{3} + \frac{2}{6} + \frac{5}{12}$

Give your answer as a fraction (not mixed number) reduced to lowest terms, or integer.

6.

Subtract: $\frac{26}{35} - \frac{13}{25}$

Give your answer in reduced terms.

7.

Divide. Write your answer in lowest terms.

$$\frac{5}{7} \div \frac{11}{5}$$

8.

Divide, if possible. If not possible, put DNE as the answer.

$$\frac{9}{0} =$$

9.

Use the distributive property to simplify the expression:

$$5(a + 11) =$$

10.

Write the expression 'six times nine to the eighth power' using mathematical symbols. Do not evaluate the result.

In mathematical symbols the expression is

11.

Find each power

$$-4^2 =$$

$$(-4)^2 =$$

12.

In which set(s) of numbers would you find the number
0

- irrational number
 - natural number
 - whole number
 - real number
 - integer
 - rational number
-

13.

In which set(s) of numbers would you find the number
 $\sqrt{11}$

- real number
 - whole number
 - integer
 - rational number
 - natural number
 - irrational number
-

14.

Select all of the natural numbers on the list below:

- $-\frac{13}{4}$
- ∞
- $\sqrt{11}$
- -5
- 5.75
- 11
- π
- 0

Select all of the integers on the list below:

- 5.75
- ∞
- π
- 0
- $\sqrt{11}$
- -5
- $-\frac{13}{4}$
- 11

Select all of the rational numbers on the list below:

- $-\frac{13}{4}$
- 0
- π
- 5.75
- 11
- $\sqrt{11}$
- -5
- ∞

Select all of the irrational numbers on the list below:

- π
- 0
- ∞
- -5
- $\sqrt{11}$
- $-\frac{13}{4}$
- 5.75
- 11

Select all of the real numbers on the list below:

- 5.75
- -5
- $-\frac{13}{4}$
- 0
- 11
- $\sqrt{11}$
- π
- ∞

15.

Round 0.73 to the nearest tenth

16.

Round 0.69 to the nearest tenth

17.

Round 0.85 to the nearest tenth

18.

Round 0.723 to the nearest hundredth

19.

Is the statement shown below an expression or an equation?

$$-10x + 3 + 5x - 13$$

- Expression
 - Equation
-

20.

Is the statement shown below an expression or an equation?

$$-8x + 8 = 3x - 19$$

- Expression
 - Equation
-

21.

Use algebra to solve for x in the equation $3x + 5 = -9x + 2$.

If your answer is a fraction, write it in reduced, fractional form.

$x =$

22.

Evaluate.

$$|13| =$$

$$\left| -\frac{2}{3} \right| =$$

$$|0| =$$

23.

Simplify the expressions below assuming $x - 8 < 0$.

$$|x - 8| + 2$$

$$5x + 7 - 2|x - 8|$$

24.

Express each set as an interval.

$$4 \leq x < 19$$

$$x \geq 5$$

Lecture 2

Exp Rules/Simplifying, Radicals Simplifying/Combining, Rational Exp,
Rationalizing Denoms

1.

Simplify the following expression completely: $z^{12} \cdot z^{13}$

2.

Simplify the expression completely: $\frac{z^6}{z^{11}}$

3.

Simplify the following expression completely: $(3z^5)(11z^3)$

4.

Write the fraction in simplest form.

$$\frac{30d}{27d^2}$$

5.

Simplify: $\frac{12x^6y^7}{3x^5y^2}$

6.

a. The square roots of 81 are _____

b. $\sqrt{81} =$

7.

Evaluate $-\sqrt{100}$

Answer DNE if the result is not a real number.

8.

Evaluate $\sqrt{-4}$

Answer DNE if the result is not a real number.

9.

Evaluate $\sqrt{\frac{9}{100}}$

10.

Simplify without using a calculator.

$$\sqrt[4]{625} =$$

11.

Simplify without using a calculator.

$$\sqrt[5]{243} =$$

12.

Simplify $\sqrt{180}$

13.

Assuming x represents a positive value, simplify the expression below:

$$\sqrt[3]{x^{12}} =$$

14.

Assuming k represents a positive value, simplify the expression below:

$$\sqrt[3]{k^{32}} =$$

15.

Simplify $\sqrt{12a}$ where $a > 0$:

16.

If x and y are positive, then the expression $\sqrt{175x^8y^{11}}$ simplifies to...

17.

Simplify the expression, assuming all variables represent positive numbers.

$$\sqrt[3]{-8y^{33}} =$$

18.

Simplify the radical expression. Assume all variables represent positive values.

$$\sqrt[4]{\frac{625c^{28}}{16b^{16}}} =$$

19.

Simplify the radical expression. Assume all variables represent positive values.

$$\sqrt[3]{16x^4y^5}$$

20.

Simplify the radical expression $\sqrt[3]{128}$

21.

Combine the terms if possible or answer DNE if they cannot be combined:

$$7\sqrt{10} - 4\sqrt{90} =$$

22.

Combine the terms if possible or answer DNE if they cannot be combined:

$$13\sqrt{11} - 6\sqrt{160} =$$

23.

Combine the terms if possible or answer DNE if they cannot be combined:

$$-4\sqrt[3]{81} + 3\sqrt[3]{24} =$$

24.

A rectangle has a length of $\sqrt{96}$ meters and a width of $\sqrt{216}$ meters. Find its perimeter in exact and approximate forms, and then find its area.

The exact perimeter is _____ meters.

This is approximately _____ meters. (Round your answer to the nearest tenth)

The area of the rectangle is _____ square meters.

25.

Simplify:

$$3\sqrt{7} \cdot 5\sqrt{21}$$

26.

Simplify. $\frac{\sqrt{32k^2n^{10}}}{\sqrt{25k^8n^{12}}}$

27.

Rationalize the denominator: $\frac{2}{\sqrt{15}}$

The result can be expressed in the form $\frac{A}{B}$ where

$$A =$$

$$B =$$

28.

Rationalize the denominator: $\frac{1}{\sqrt{6}}$

The result can be expressed in the form $\frac{A}{B}$ where

$$A =$$

$$B =$$

29.

Rationalize the denominator: $\frac{-8}{\sqrt{2}}$

30.

Simplify. $\sqrt{\frac{15}{11}}$

31.

Rationalize the denominator. Give an exact answer.

$$\sqrt{\frac{5}{x^3}} =$$

32.

Rationalize the denominator:

$$\frac{15}{\sqrt{x}} =$$

33.

The radical expression $\sqrt[14]{x^9}$ can be rewritten in the rational exponent form $x^{\frac{a}{b}}$ where,

$$a =$$

$$b =$$

34.

Rewrite $x^{\frac{1}{4}}$ in radical form.

$$x^{\frac{1}{4}} =$$

35.

Evaluate and express your answer as a reduced fraction:

$$64^{-4/3} =$$

36.

Rewrite the expression $x^{-\frac{7}{10}}$ in radical form using positive exponents.

$$x^{-\frac{7}{10}} =$$

37.

Evaluate without a calculator:

$$125^{4/3}$$

38.

Simplify using the rules of exponents:

$$8^{\frac{1}{4}} \cdot 8^{\frac{5}{8}} =$$

Lecture 3

1.

Simplify

$$-(3x^2 - x + 6)$$

2.

Multiply and simplify:

$$3x^4(2x^2 - 5x)$$

3.

Add the two polynomials and simplify:

$$(8x^4 + 2x^3 + 9x) + (-4x^4 - x^3 + 10)$$

4.

Perform the indicated operations and simplify:

$$[(-7x^2 + 6x + 10) - (4x^2 + 14x + 17)] - (-19x^2 - 10x - 5)$$

5.

Multiply and simplify:

$$(x - 5)(2x - 2)$$

6.

Perform the following operation and simplify:

$$(7x - 5)^2$$

7.

Multiply and simplify:

$$6r(3r + 4)(r - 6) =$$

8.

Factor the following expression completely by pulling out the GCF. Factor out a negative number if the expression begins with a negative coefficient.

$$-6x^8 - 3x^5 - 9$$

9.

Factor the following expression completely by pulling out the GCF.

$$14x^9 + 6x^8 + 22x^6$$

10.

Factor the following expression completely:

$$x(x + 1) - 3(x + 1)$$

11.

Factor the following expression completely:

$$z^2 + 8z + 9z + 72$$

12.

Factor the following expression completely:

$$w^3 - 3w^2 + 10w - 30$$

13.

Factor the following expression completely:

$$y^2 + 2y - 8$$

14.

Factor the following expression completely:

$$6z^2 + 37z - 35$$

15.

Factor the following expression completely:

$$2w^3 - 8w^2 - 90w$$

16.

$$\text{Solve } (3y + 10)(5y + 7) = 0$$

$$y =$$

17.

$$\text{Solve the equation: } p^2 = 8p$$

$$p =$$

18.

$$\text{Solve } r^2 = 25$$

$$r =$$

19.

Solve the following equation:

$$(9c + 11)(c + 9) = 79$$

$$c =$$

20.

The product of two consecutive odd integers is 99. If x is the smallest of the integers, write an equation in terms of x that describes the situation, and then find all such pairs of integers.

The equation that describes the situation is _____

The positive set of integers is _____

The negative set of integers is _____

21.

Use factoring by grouping to solve the following equation:

$$a^3 - 4a^2 - 25a + 100 = 0$$

$$a =$$

22.

Use the square root property to determine all real solutions for each of the following equations.

$$3a^2 - 528 = 0$$

$$a =$$

$$5x^2 + 360 = 0$$

$$x =$$

Give exact solutions (don't use decimals), and separate multiple solutions with commas. If there are no real solutions, answer DNE.

23.

Find all real solutions of the equation.

$$(a + 2)^2 = 24$$

$$a =$$

Simplify your solutions.

24.

In order to solve an equation with the quadratic formula, the equation must be in which of these forms?

- $ax^2 + bx + c = 0$
- $a(x - h)^2 + k = 0$
- $y = m(x - x_1) + y_1$
- $(x - a)(x - b) = 0$

25.

Solve by the quadratic formula:

$$-2x^2 - 3x + 5 = 0$$

26.

Solve equation by using the quadratic formula

$$3m^2 - 1 = 5m$$

Simplify answers.

27.

Find the last term to make the trinomial into a perfect square:

$$x^2 + 18x + \underline{\hspace{2cm}}$$

Write the trinomial as a binomial squared:

28.

Add the same constant to both sides to make the left hand side a perfect square:

$$z^2 - 6z + \underline{\hspace{2cm}} = 3 + \underline{\hspace{2cm}}$$

29.

Consider the equation: $x^2 + 18x + 77 = 0$

- A. First, use the "completing the square" process to write this equation in the form $(x + A)^2 = B$ and enter your equation below.
 $x^2 + 18x + 77 = 0$ is equivalent to the equation _____
- B. Solve your equation and enter your answers below as a list of numbers, separated with a comma where necessary.
 $x =$

30.

Solve by completing the square:

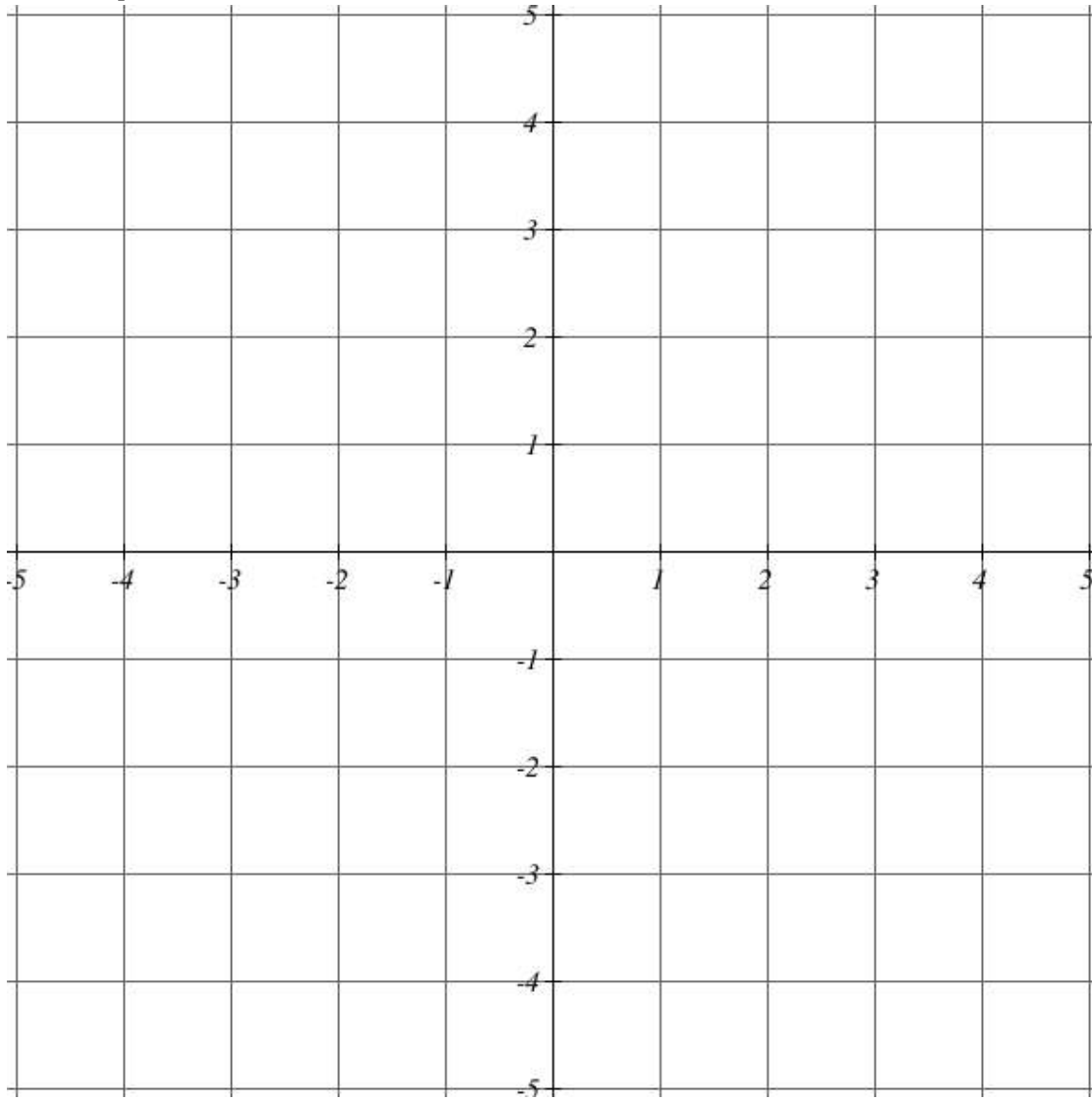
$$16z^2 - 24z = 216$$

$z =$

Lecture 4

1.

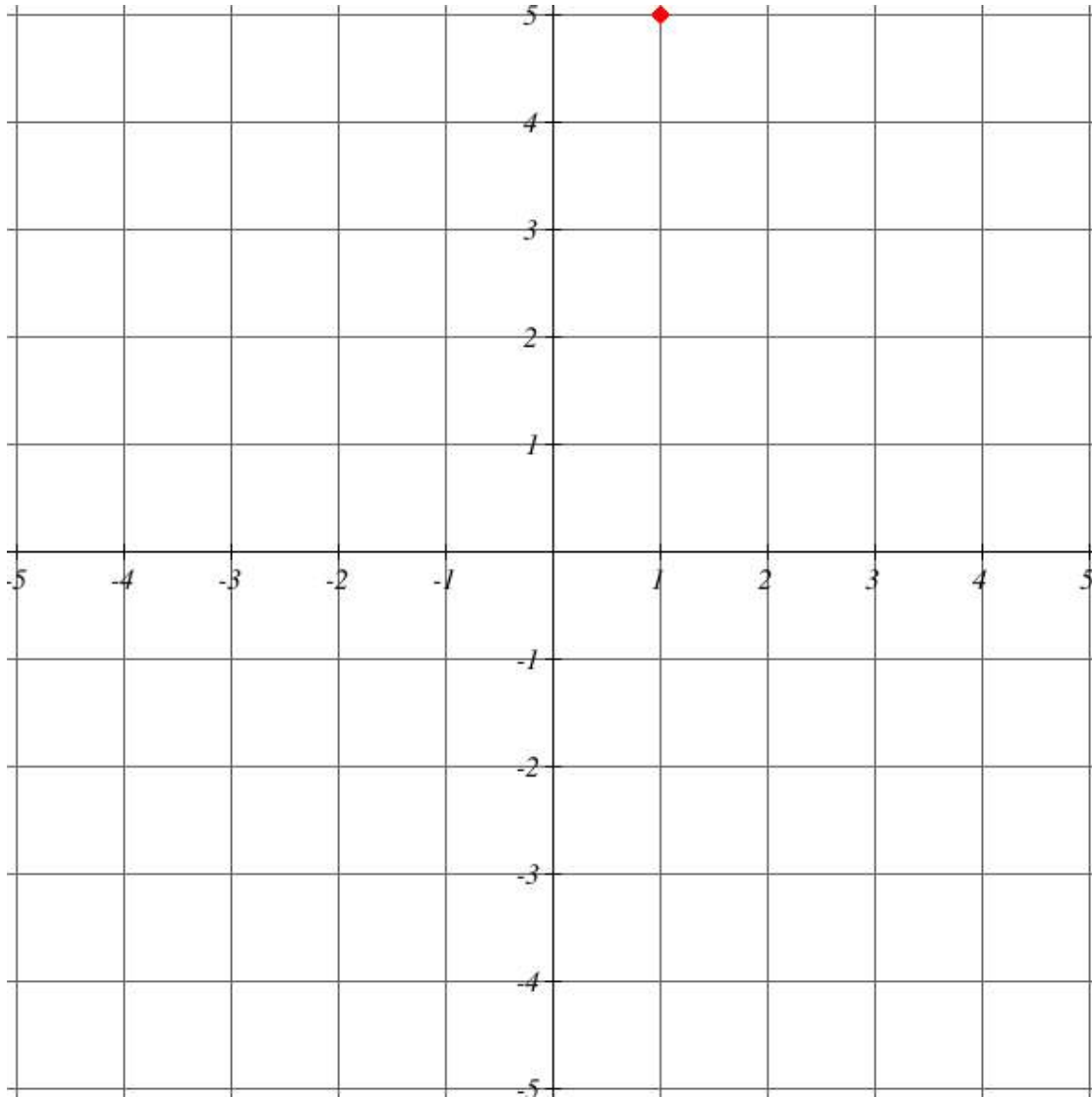
Plot the points $(0, -2)$, $(-3, 0)$, $(4, -5)$, $(-5, -5)$.



2.

The coordinate below represents the point $P(1,5)$

Plot another point by shifting the given point 1 units left and 4 units down



3.

Identify the quadrant or axis where each of the following points lies.

$(-4,1)$

$(0,-2)$

$(-3,0)$

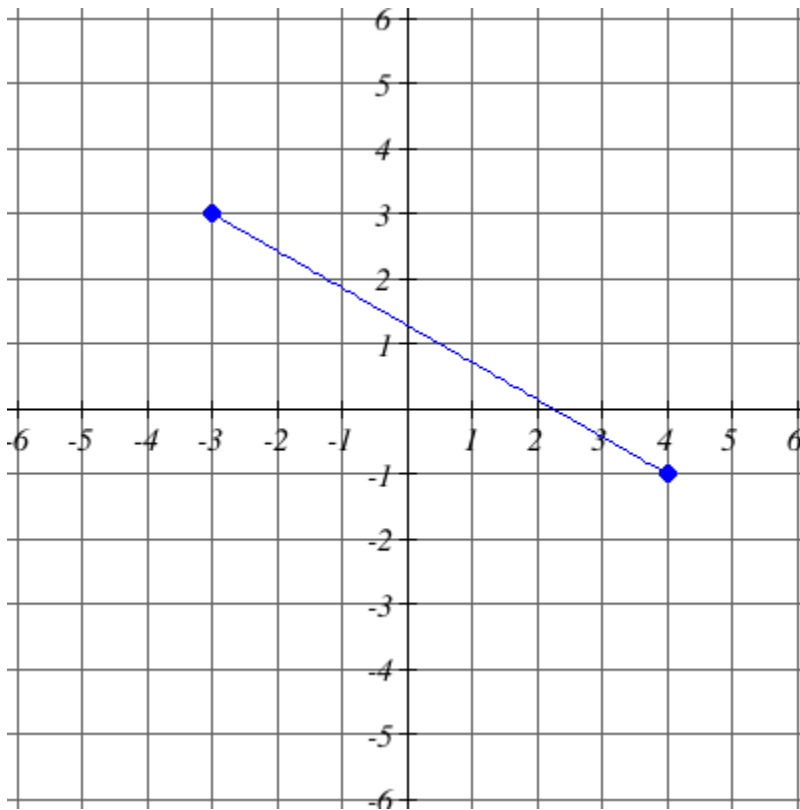
$(0,2)$

$(-1,-1)$

$(2,-4)$

4.

Find the midpoint of the line segment shown below.

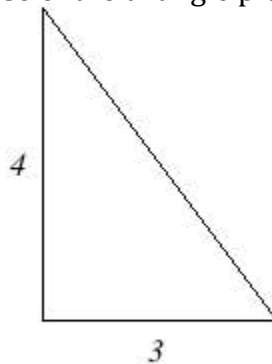


5.

Find the midpoint of the line segment with endpoints: $\left(\frac{10}{3}, -\frac{1}{8}\right)$ to $\left(-\frac{10}{3}, -\frac{9}{8}\right)$

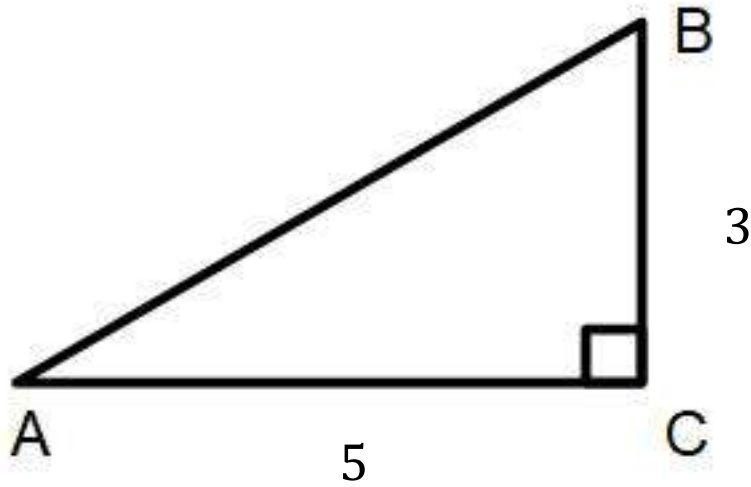
6.

Find the length of the hypotenuse of the triangle pictured below



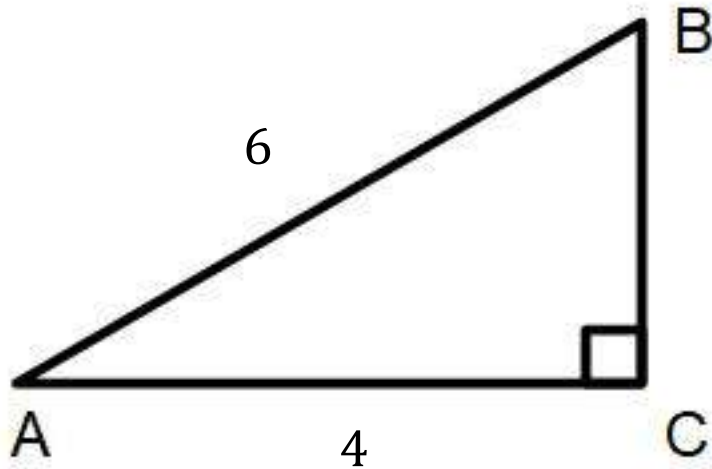
7.

In triangle ABC , $\angle ACB$ is a right angle. Find the length of the missing side.



8.

In triangle ABC , $\angle ACB$ is a right angle. Find the length of the missing side.

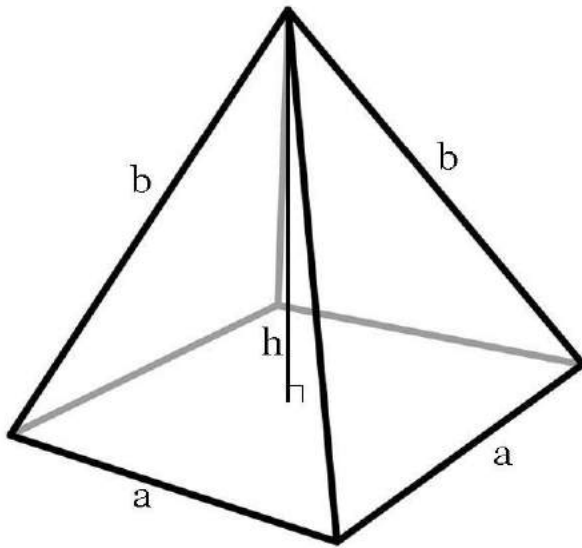


9.

Find the distance between the points $(7, 8)$ and $(-15, 12)$.

10.

Compute the **exact value** of the height h of the square-based straight pyramid, given that the base is a square with sides 24 inches long, and all other edges are 39 inches long.



11.

Write the equation of the circle centered at $(0,8)$ with radius 12.

12.

Find the standard form for the equation of a circle

$$(x - h)^2 + (y - k)^2 = r^2$$

with a diameter that has endpoints $(-6, -8)$ and $(4, -10)$.

$$h = \underline{\hspace{2cm}}$$

$$k = \underline{\hspace{2cm}}$$

$$r = \underline{\hspace{2cm}}$$

13.

Find the center and radius of the circle whose equation is $x^2 - x + y^2 + 6y - 20 = 0$.

The center of the circle is _____.

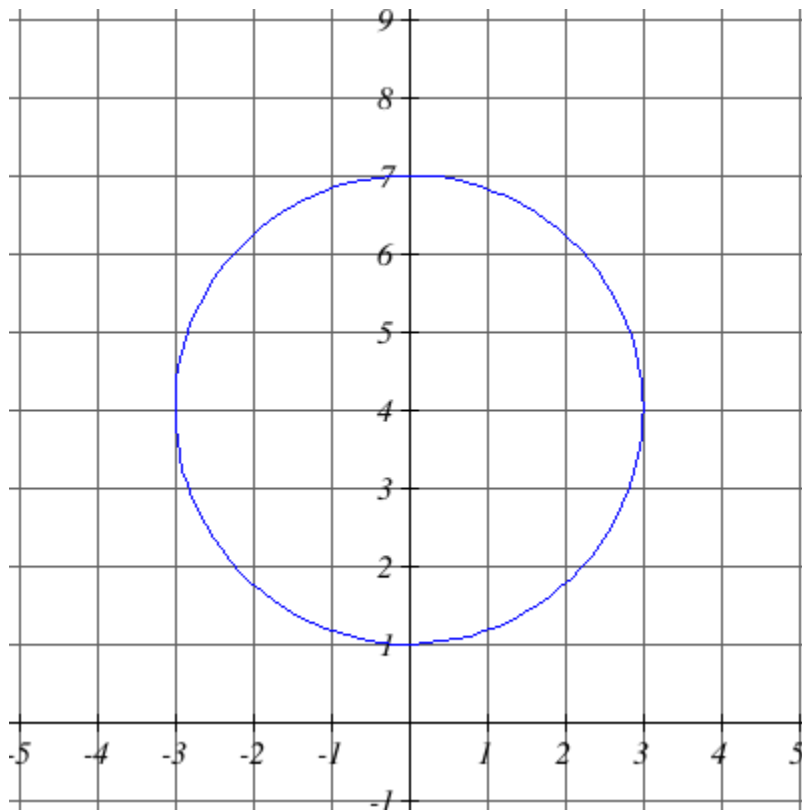
The radius of the circle is _____.

14.

Give the equation of the circle that has its center at $(-3,4)$ that intersects the y -axis exactly once. Hint: sketch the graph.

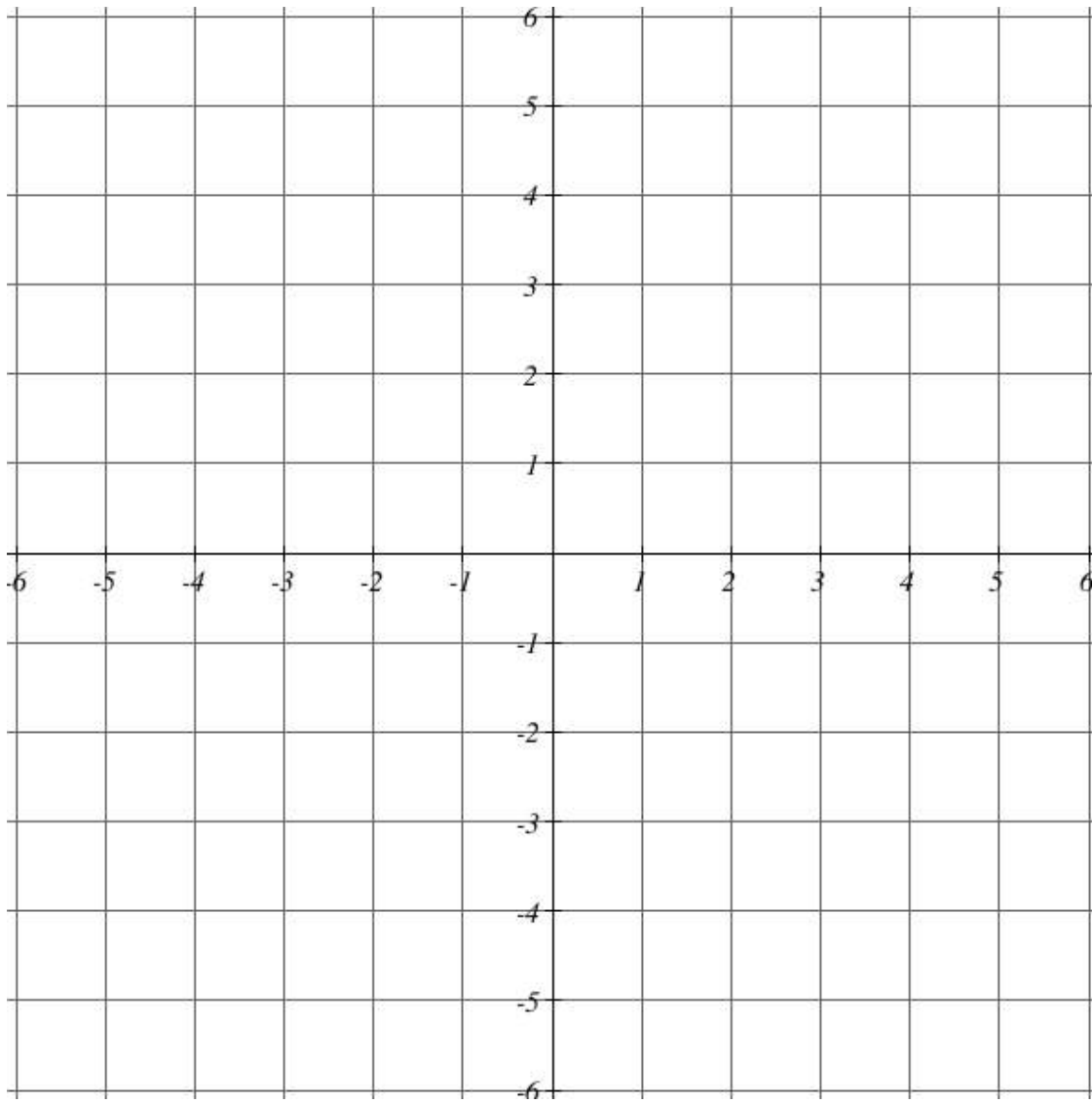
15.

Write the equation of the circle shown in standard form.



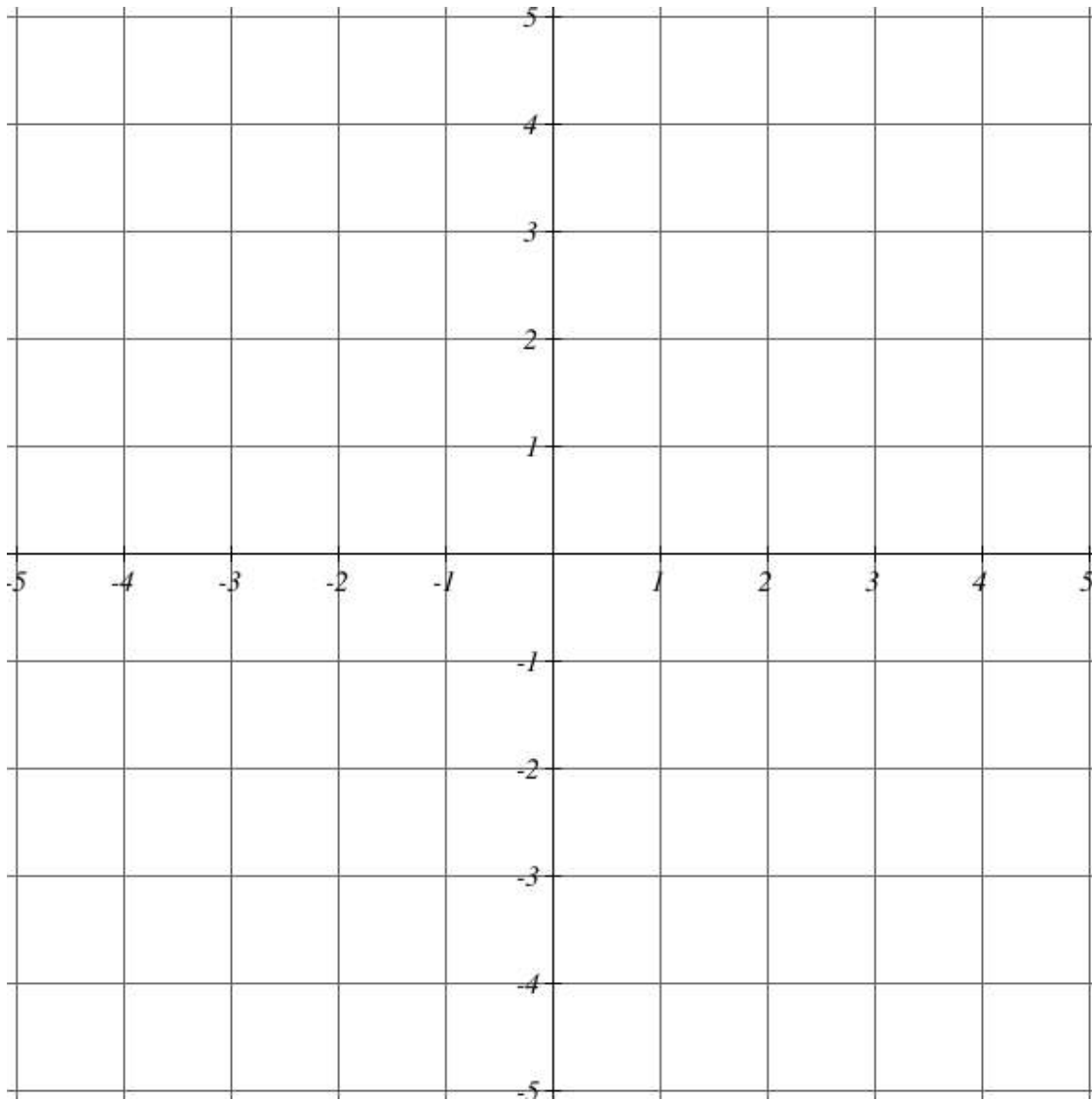
16.

Draw the circle $(x + 2)^2 + (y + 3)^2 = 9$.



17.

Draw a circle with an equation of $x^2 + 2x + y^2 - 2y = 7$.



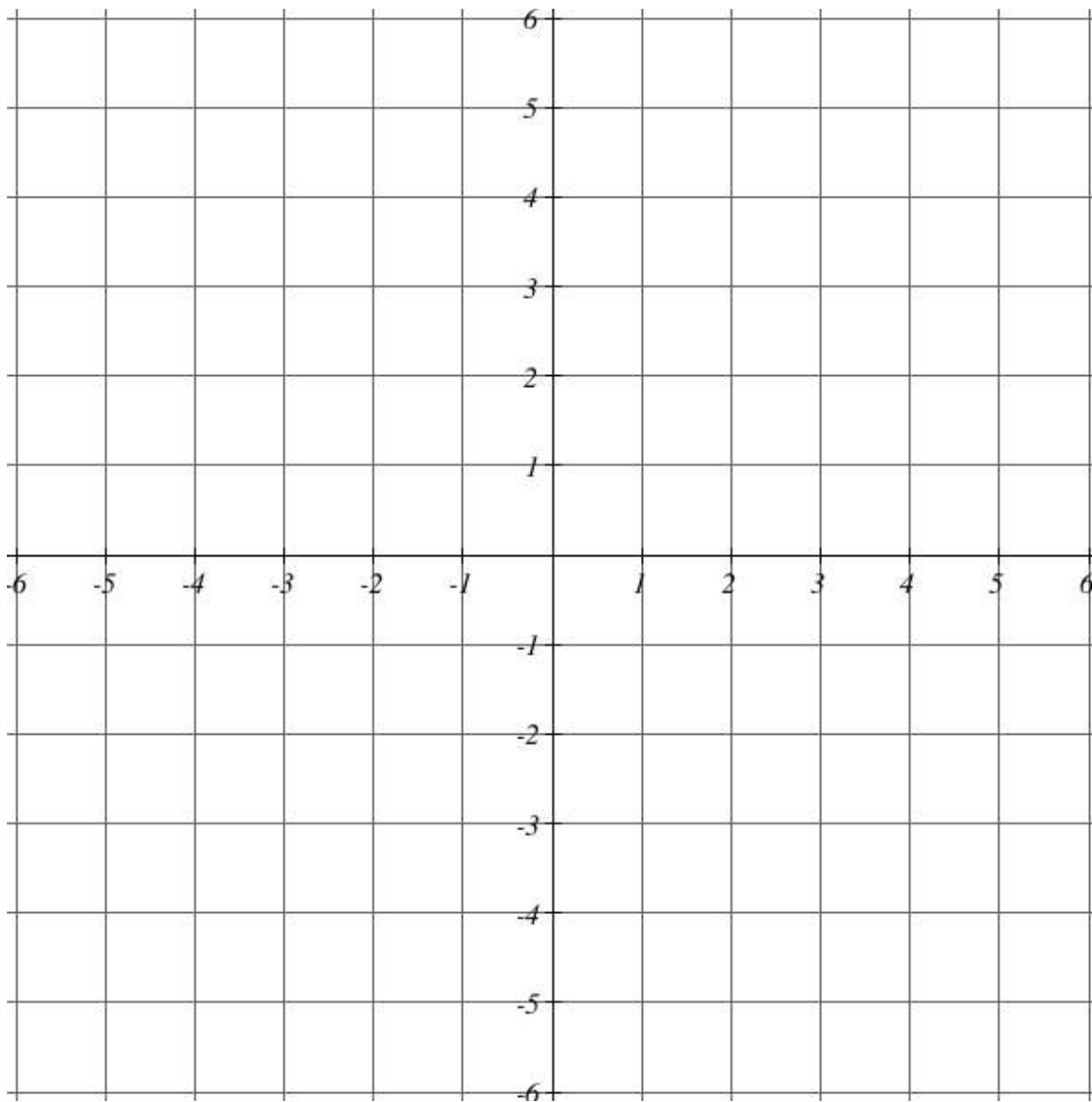
18.

For the equation $-3x + y = 3$

a) Complete the table:

x	y
	0
0	

b) Plot the two points you found in the table.



19.

Find the x -intercepts of the graph of the equation

$$x^2 - 3x + y^2 + 6y = 28$$

20.

Find the y -intercepts of the graph of the equation

$$x^2 - 8x + y^2 + 5y = 50$$

21.

Find the x -intercept of the equation $(-8x - 10)y = -6x - 1$

22.

Find the y -intercept of the equation $-8xy - 2y = 4x + 8$

23.

Suppose the point $P = (12, 2)$ is reflected across the x -axis to the point P' .

The coordinates of P' are _____

24.

Suppose the point $P = (-8, -3)$ is reflected across the y -axis to the point P' .

The coordinates of P' are _____

25.

Suppose the point $P = (7, -9)$ is reflected across the origin to the point P' .

The coordinates of P' are _____

26.

Consider the curve given by the equation

$$2x^2 - 5xy - 4y^2 = 88$$

Which of the symmetries below does this curve display?

- symmetry about the x -axis
- symmetry about the y -axis
- symmetry about the origin
- no symmetry

27.

Consider the curve given by the equation

$$4x^2 - 5x + 5y^2 = 86$$

Which of the symmetries below does this curve display?

- symmetry about the x -axis
- symmetry about the y -axis
- symmetry about the origin
- no symmetry

28.

Consider the curve given by the equation

$$2x^2 - 3y + 5y^2 = 75$$

Which of the symmetries below does this curve display?

- symmetry about the x -axis
- symmetry about the y -axis
- symmetry about the origin
- no symmetry

29.

Consider the curve given by the equation

$$5x^2 - 2y^2 = 82$$

Which of the symmetries below does this curve display?

- symmetry about the x -axis
- symmetry about the y -axis
- symmetry about the origin
- no symmetry

30.

Consider the curve given by the equation

$$-2x^3 + 3y^3 = 130$$

Which of the symmetries below does this curve display?

- symmetry about the x -axis
- symmetry about the y -axis
- symmetry about the origin
- no symmetry

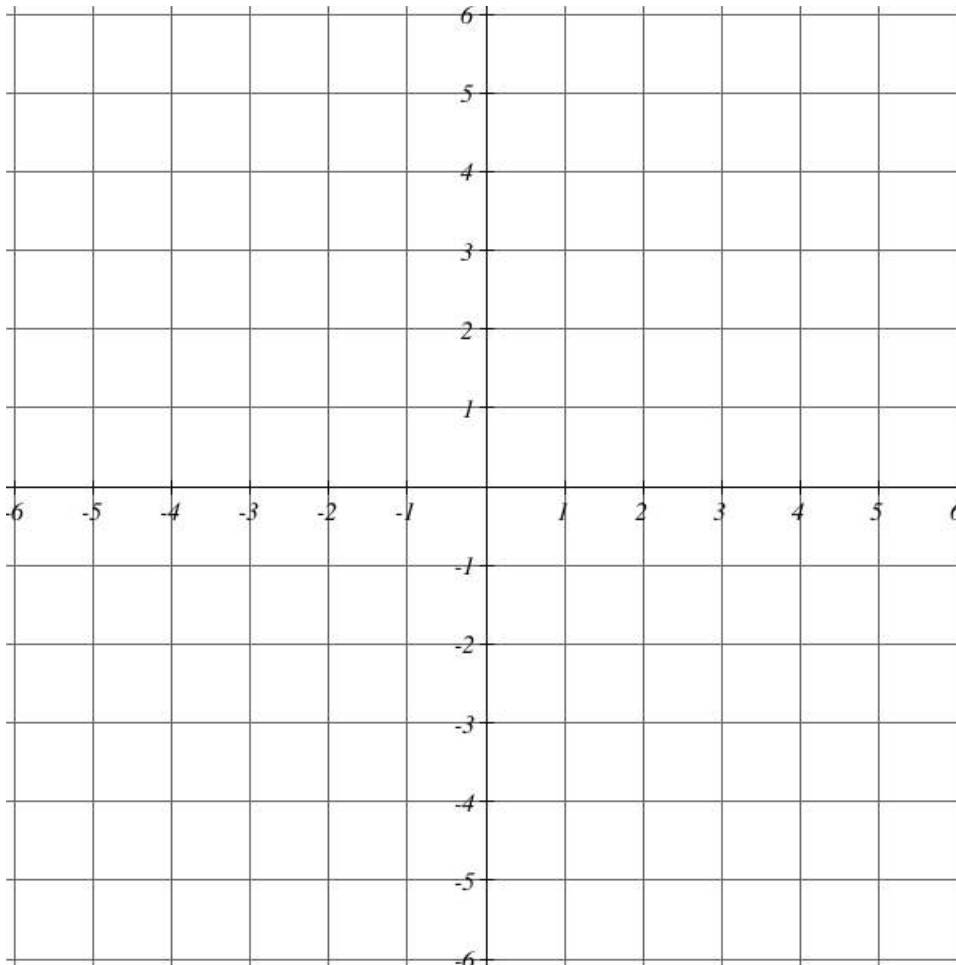
31.

For the equation $y = -|x - 1|$

a) Complete the table:

x	y
-1	
0	
1	
2	
3	

b) Plot the points you found in the table.



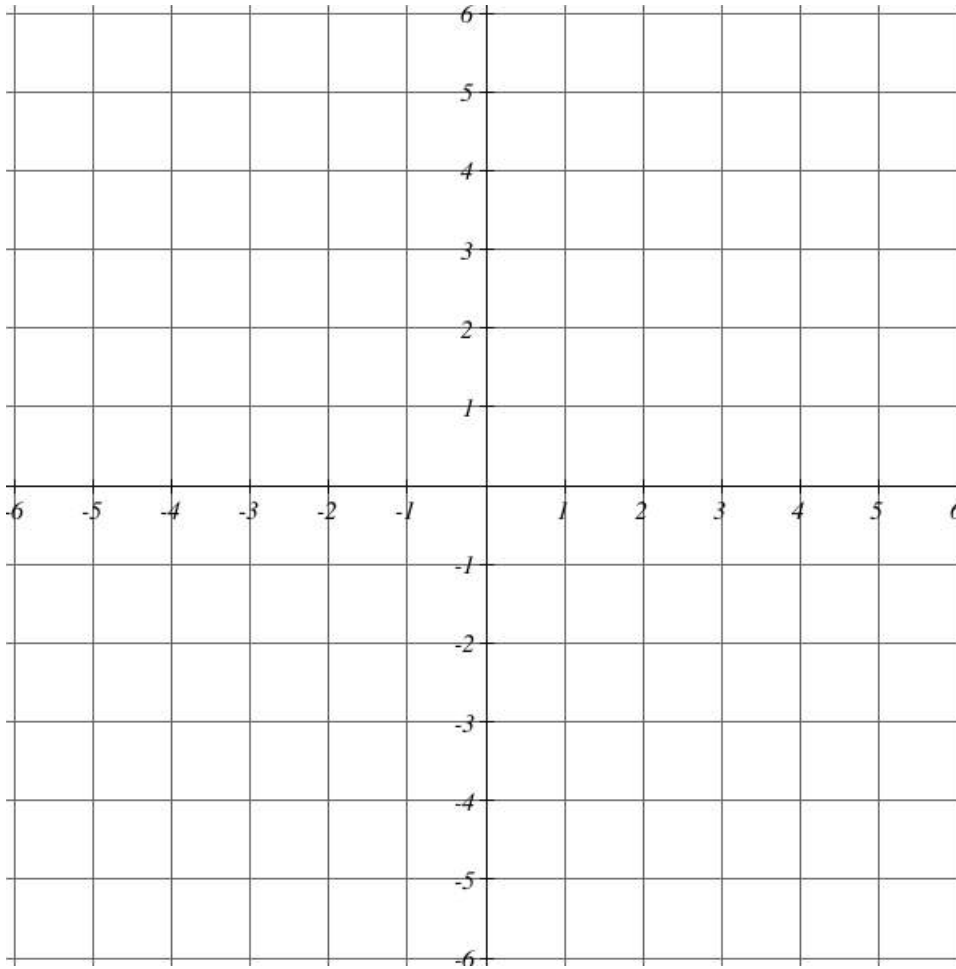
32.

For the equation $y = -\sqrt{x - 2}$

a) Complete the table, rounding to three decimal places where needed.

x	y
2	
3	
4	

b) Plot the points you found in the table.



Lecture 5

Relations, functions, function notation, evaluation, graphing, domain/range, piecewise

1.

Find the domain and range of the relation R given below:

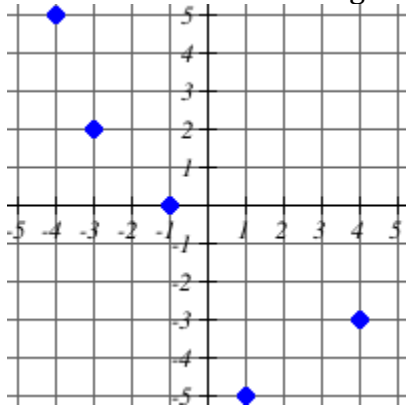
$$R = \{(10, -18), (-21, -21), (-14, 19), (24, -29)\}$$

The domain is:

The range is:

2.

State the domain and range of the relation graphed below:



Domain:

Range:

3.

State the domain and range of the relation given in the table below, and determine if it is a function.

x	-18	-20	7	-3	-7
y	12	-18	-9	16	-8

Domain:

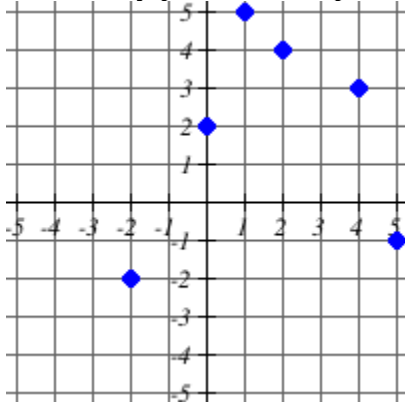
Range:

Is the relation a function?

- Yes
- No

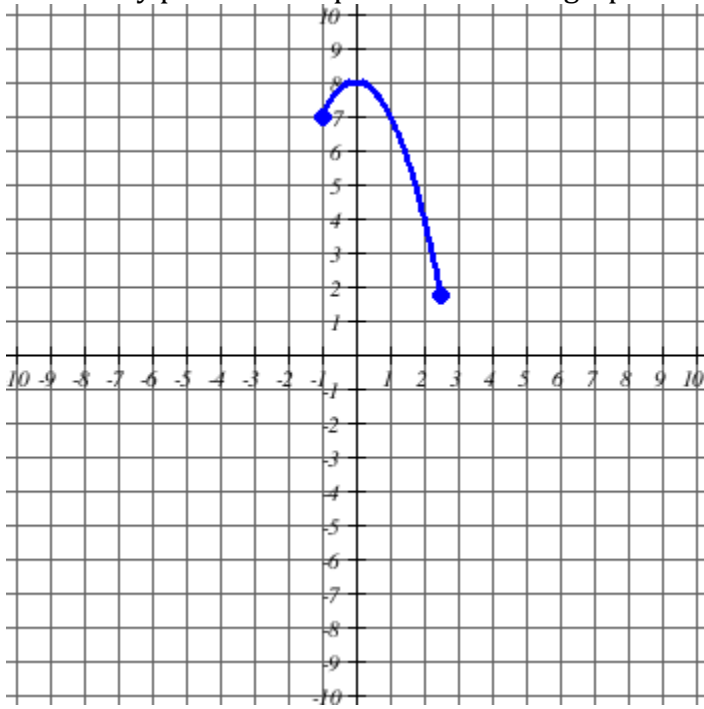
4.

How many points are represented in the graph below?



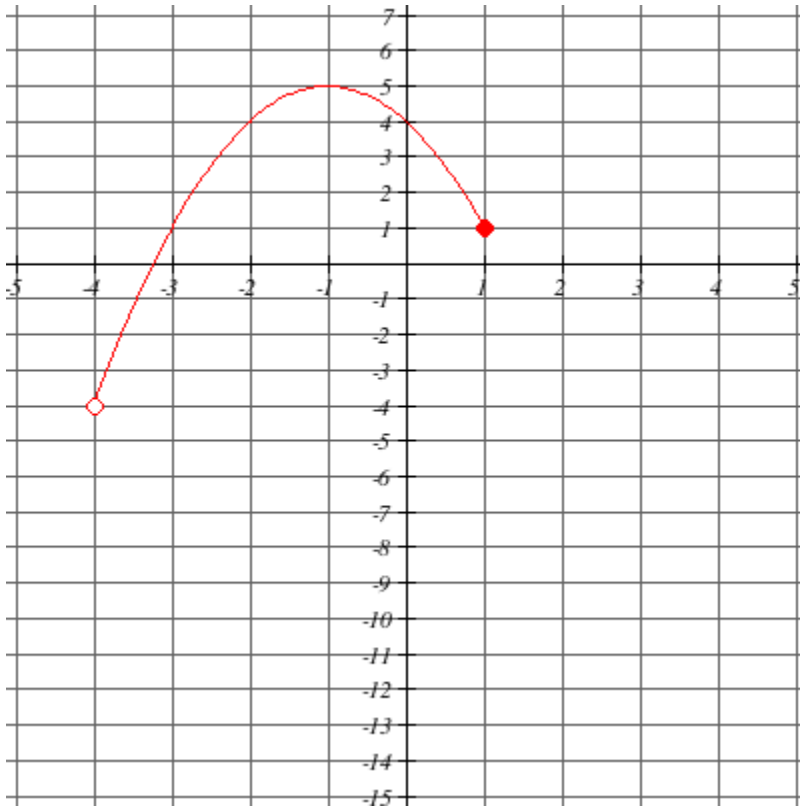
5.

How many points are represented in the graph below?



6.

Determine the domain and range of the function using the graph below. Give your answer as an inequality using the appropriate variables.

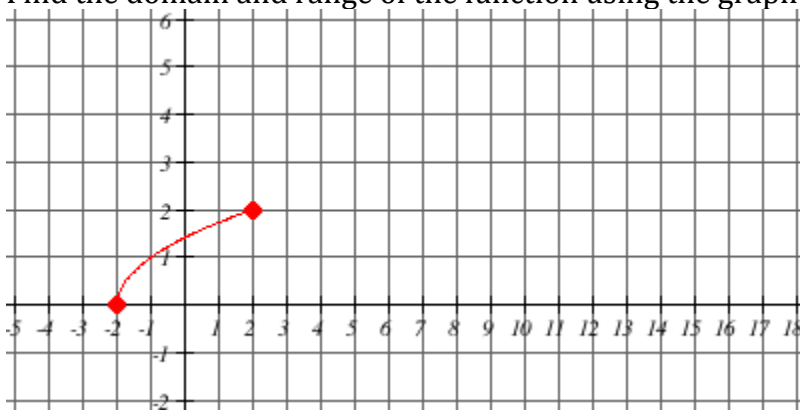


Domain:

Range:

7.

Find the domain and range of the function using the graph below.

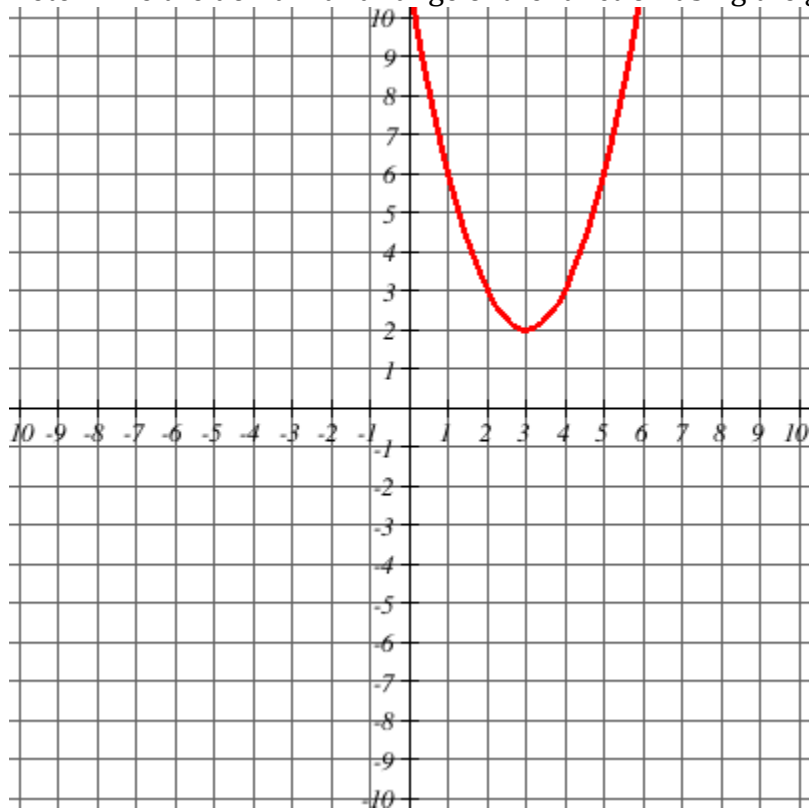


Domain: $_ \leq x \leq _$

Range: $_ \leq y \leq _$

8.

Determine the domain and range of the function using the graph below.



The domain is:

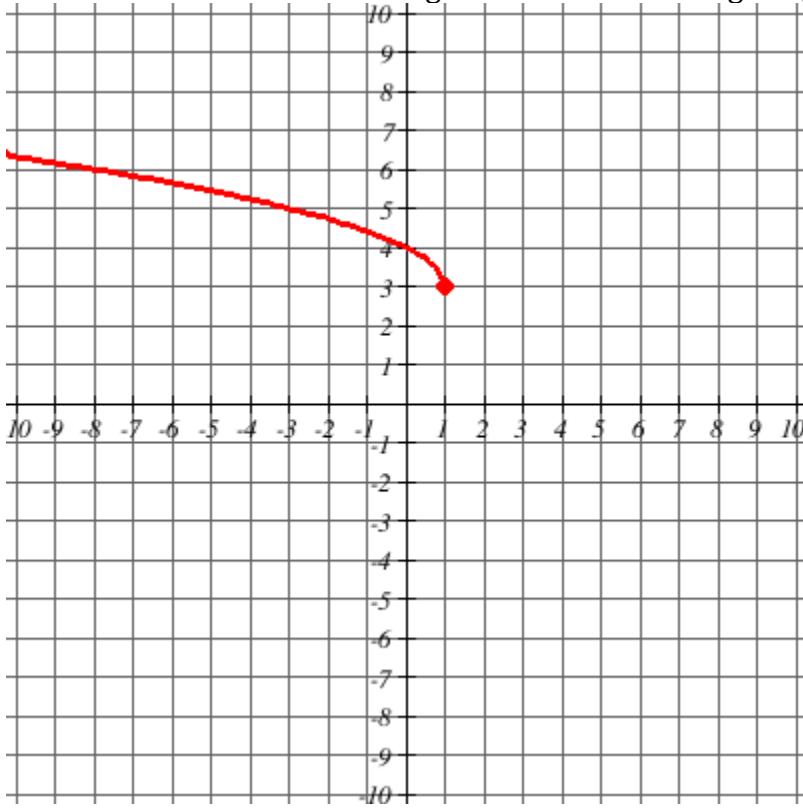
- $x \geq 2$
- $x \leq 2$
- $x \leq 0$
- All real numbers

The range is:

- $y \geq 2$
- All real numbers
- $y \leq 0$
- $y \leq 2$

9.

Determine the domain and range of the function using the graph below.



The domain is:

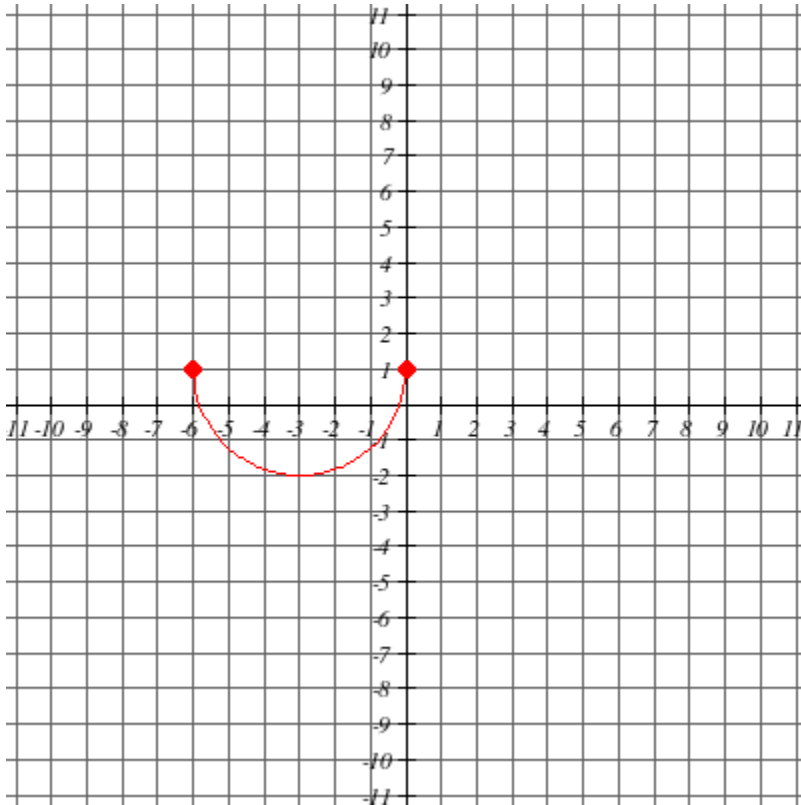
- All real numbers
- $x \leq 0$
- $x \leq 1$
- $x \geq 1$

The range is:

- $y \leq 3$
- $y \geq 3$
- All real numbers
- $y \leq 0$

10.

Give the domain and range for the function shown below. State your answers as inequalities.



Domain:

Range:

11.

When the definition of a function involves a fraction, the function is undefined at any value that would make the denominator of the function _____

- the same as the numerator
- 1
- your best friend
- 0

12.

The domain of the function $f(x) = \frac{50}{5x-27}$ is all real numbers x except for _____

13.

Find the domain of the function given below:

$$f(t) = \frac{t}{t^2 + 81}$$

The domain is:

- $t \neq -9$
- $t \leq 81$
- $t \geq 9$
- All real numbers
- $t \geq 9$
- $t \leq 9$

14.

If $f(-7) = -8$, then the point ____ is on the graph of f .

15.

Given the function $f(x) = 6x^2 - 6x + 3$. Calculate the following values:

$$f(-2) =$$

$$f(-1) =$$

$$f(0) =$$

$$f(1) =$$

$$f(2) =$$

16.

Evaluate the function $g(x) = -3x^2 + 7$ at two different inputs and state the corresponding points. You choose the inputs.

17.

Express the rule "Subtract 17, then square" as a function of x .

$$f(x) = .$$

18.

For the function $g(x) = \frac{-5x-9}{10x+7}$, evaluate $g(1)$.

$$g(1) =$$

19.

Evaluating Functions

Use the function $f(x) = -8x - 6$ to answer the following questions

Evaluate $f(0)$:

$$f(0) =$$

For what value(s) of x does $f(x) = -78$?

$$x =$$

20.

Evaluating Functions

Use the table to answer the following questions. Separate multiple answers with commas if needed.

x	0	1	2	3	4	5	6	7	8	9
$h(x)$	-37	-2	-25	-32	-95	-2	-99	-79	33	7

Evaluate $h(7)$:

$$h(7) =$$

For what value(s) of x does $h(x) = -2$?

$$x =$$

21.

Construct a table of values for a function $f(x)$ which specifies that $f(2) = 10$.

x				
$f(x)$				

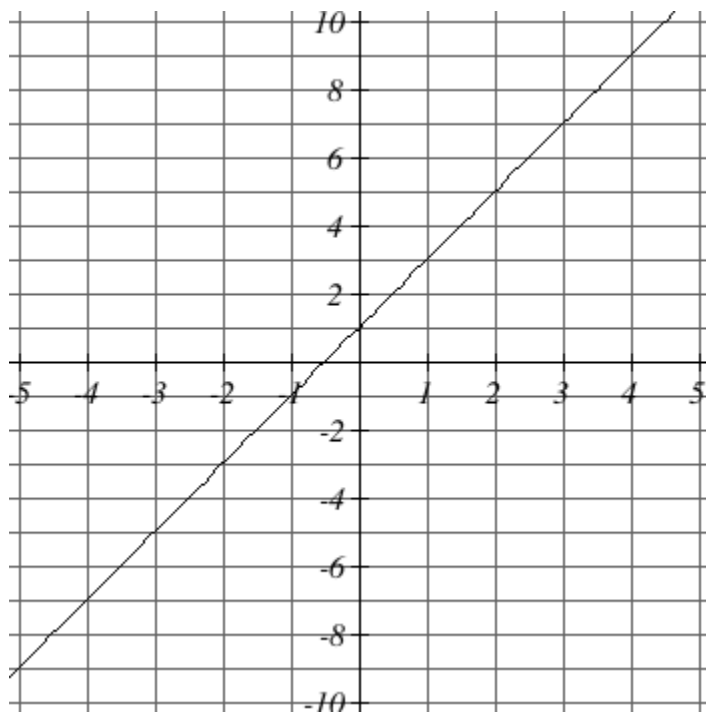
22.

Create a formula for a function $f(x)$ that has $f(4) = 10$. Do not give a simple constant function (like $f(x) = 10$) as your answer.

$$f(x) =$$

23.

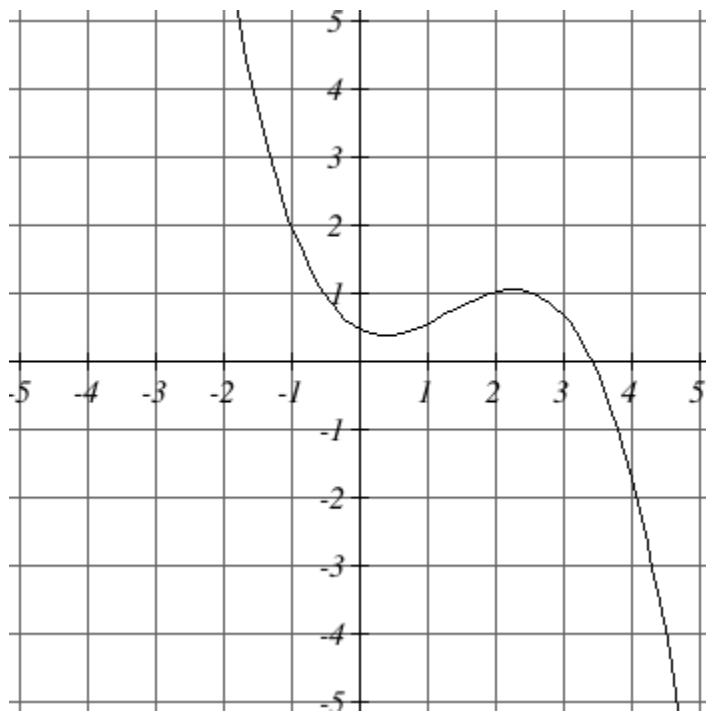
The plot below represents the function $f(x)$



Evaluate $f(-2)$

24.

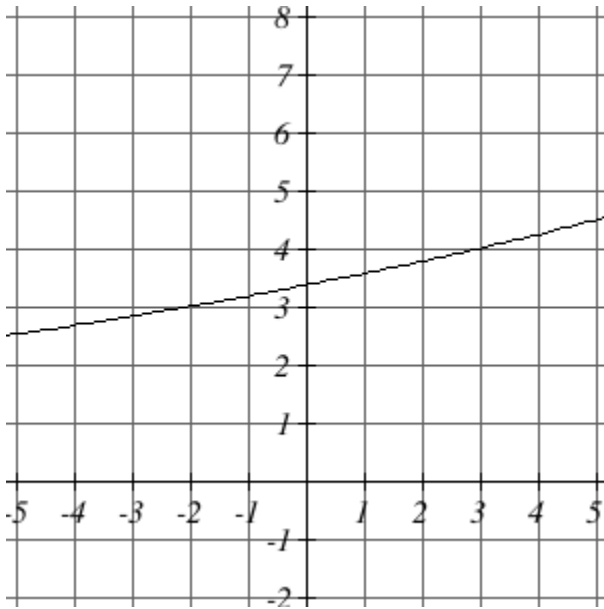
The plot below represents the function $f(x)$



Evaluate $f(2)$

25.

The plot below represents the function $f(x)$



Evaluate $f(3)$

$$f(3) =$$

Solve $f(x) = 3$

$$x =$$

26.

The function f multiplies its input by 11 and then subtracts 20 to produce its output. What is the symbolic representation of this operation?

$$f(x) =$$

27.

Give a verbal description of what the function $f(n) = (n + 8)^2$ does to the input n .

- n is squared and 8 is added to the result
- 8 is squared and the result is added to n
- 8 is added to n and the result is squared
- 8 and n are both squared and the results are added together

28.

Give a verbal description of what the function $f(x) = x^2 + 5$ does to the input x .

- 5 is squared and the result is added to x
- 5 is added to x and the result is squared
- 5 and x are both squared and the results are added together
- x is squared and 5 is added to the result

29.

You are studying meteorology and collect weather data for Gainesville, FL for the months of April, May, and June 2015. The function $T(x) = .18x + 80.25$ gives an estimate of the daily high temperature during this period where x is the number of days after April 1, 2015. Evaluate $T(29)$ (rounded to one decimal place) and then state its physical interpretation.

$$T(29) = \underline{\hspace{2cm}}$$

The physical interpretation of $T(29)$ is:

- $T(29)$ is the estimated high temperature on June 30
- $T(29)$ is the estimated high temperature on April 30
- $T(29)$ is the estimated number of days that have passed since the high temperature was 29
- $T(29)$ is the estimated number of days it will be until the high temperature is 29

30.

The amount of garbage, G , in tons per week, produced by a city with population p , measured in thousands of people, is given by $G = f(p)$

The town of Tola has a population of 45,000 and produces 5 tons of garbage each week. Express this information in terms of the function f

31.

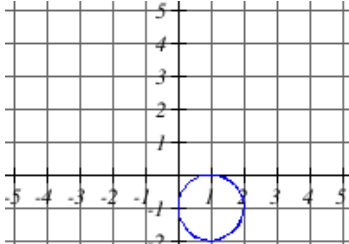
For the equation shown below, solve for y as a function of x and express the result in function notation. Use f for the name of the function.

$$-20x + 4y = 28$$

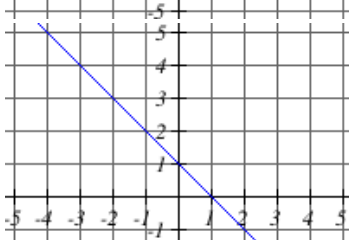
The function is _____

32.

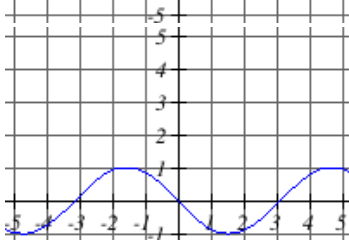
Select all of the following graphs which represent y as a function of x .



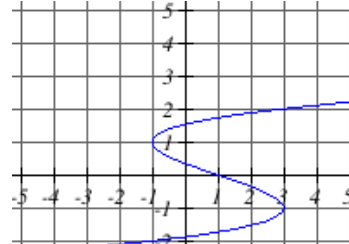
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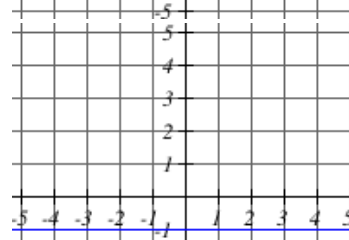
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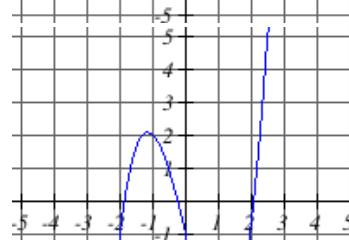
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•



•

33.

Select all of the following tables which represent y as a function of x .

x	y
0	-1
1	2
4	2
8	9
11	10

x	y
-2	-4
3	2
6	5
7	8
14	15

x	y
-4	-2
3	2
5	2
8	7
3	10

x	y
-3	-5
3	2
3	5
9	8
14	12

34.

Based on the table below,

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	26	5	78	100	2	52	22	36	37	73

Evaluate $f(2)$:

$$f(2) =$$

Solve $f(x) = 5$:

$$x =$$

35.

Suppose $f(x) = -2x^2 + 10x - 9$. Compute the following:

A.) $f(-3) + f(5) =$

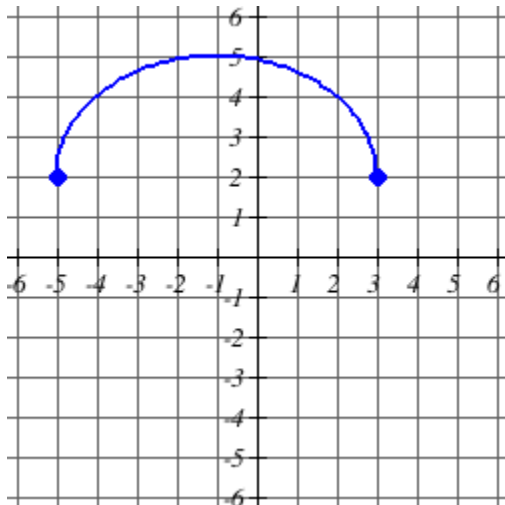
B.) $f(-3) - f(5) =$

36.

Find the the domain of the function $f(x) = \frac{x-4}{x^2+9x-10}$.

- $\{x \mid x \neq -10\}$
- $\{x \mid x \neq -10 \text{ and } x \neq 1\}$
- $\{x \mid x \neq -4\}$
- $\{x \mid x \neq 10 \text{ and } x \neq 4\}$
- $\{x \mid x \neq 10\}$

37.



Write the domain of the function using interval notation.

38.

Given the function:

$$f(x) = \begin{cases} 2x + 1 & x < 0 \\ 2x + 2 & x \geq 0 \end{cases}$$

Calculate the following values:

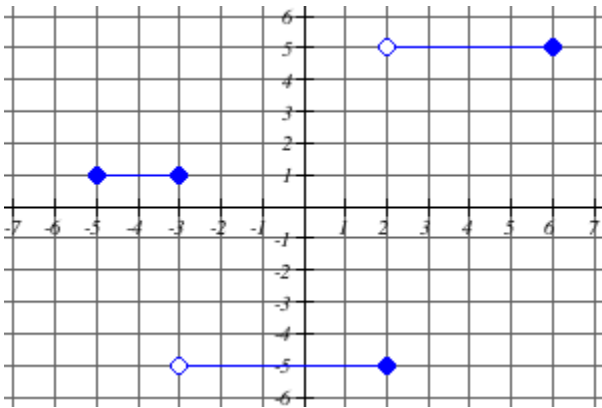
$$f(-1) =$$

$$f(0) =$$

$$f(2) =$$

39.

Complete the description of the piecewise function graphed below. Use interval notation to indicate the intervals.



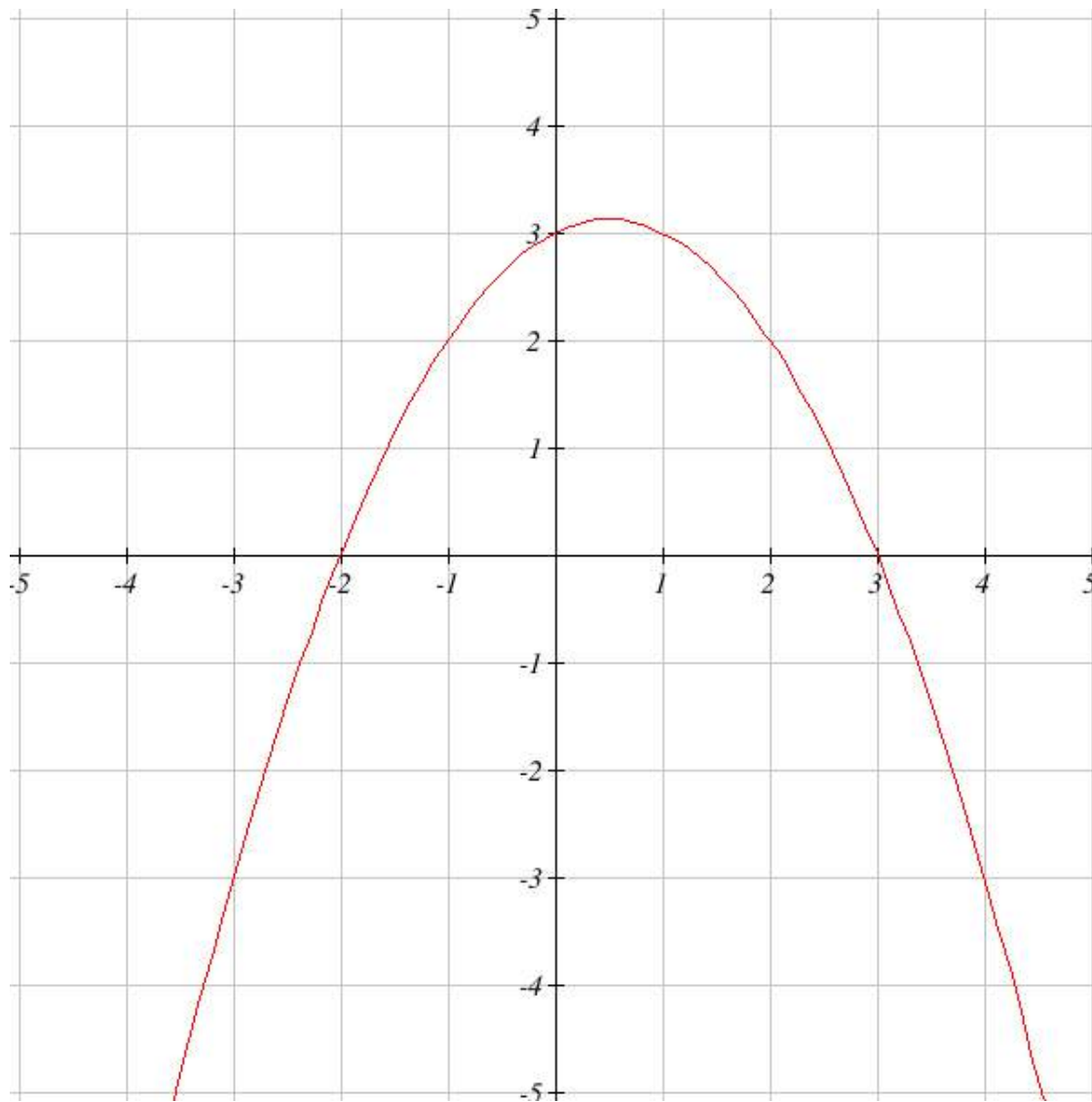
$$f(x) = \begin{cases} 1 & \text{if} \\ -5 & \text{if} \\ 5 & \text{if} \end{cases}$$

Lecture 6

Vertical Line Test, Zeros, Positive/Negative, Increasing/Decreasing, Concavity, Relative min/max, Avg Rate of Change, Even/odd

1.

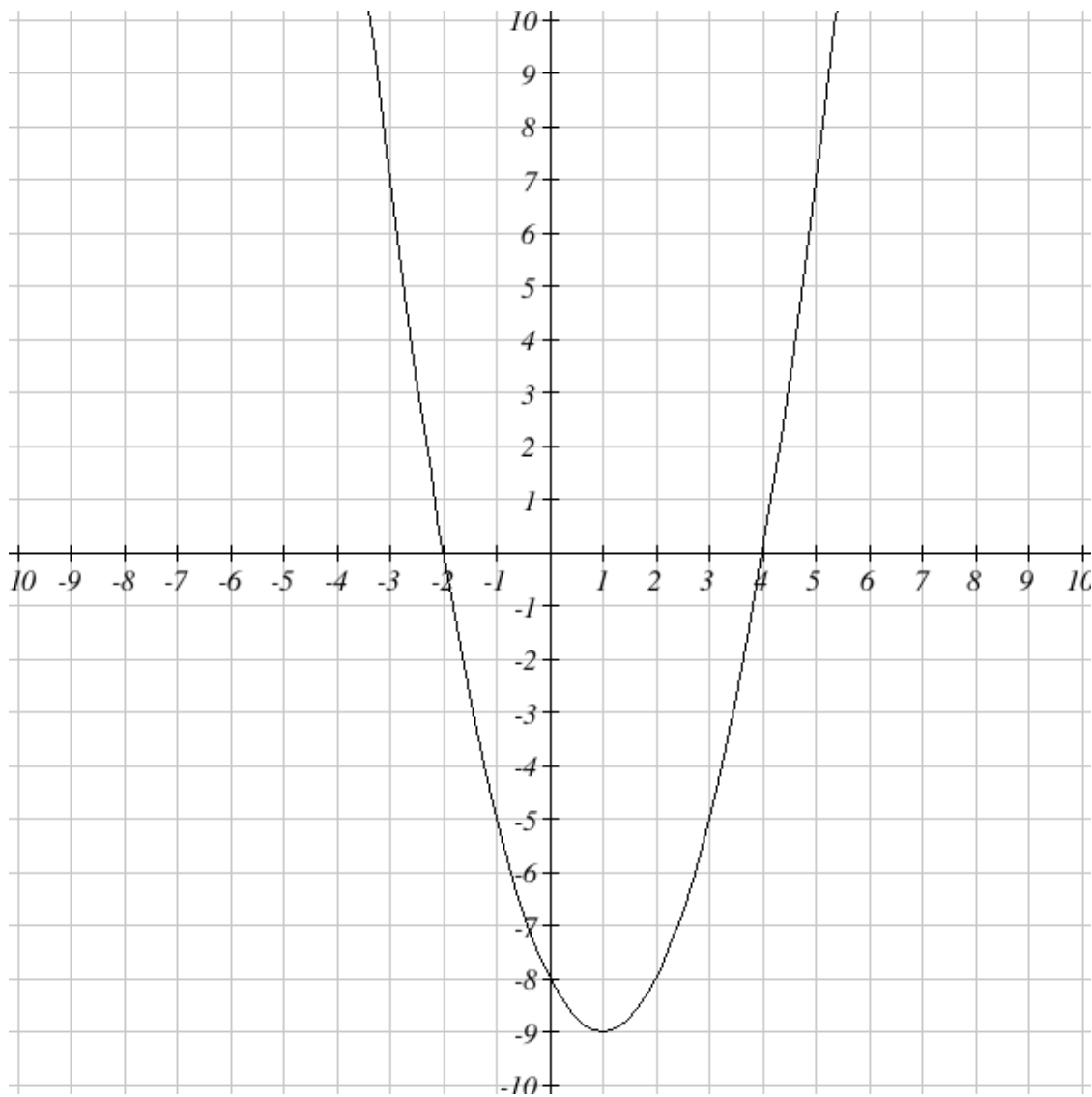
Put dots on the graph where the "zeros" can be found. Then list the zeros.



If more than one zero, separate with a comma. The zero's are $x = \underline{\hspace{2cm}}$.

2.

Consider the function graphed below.

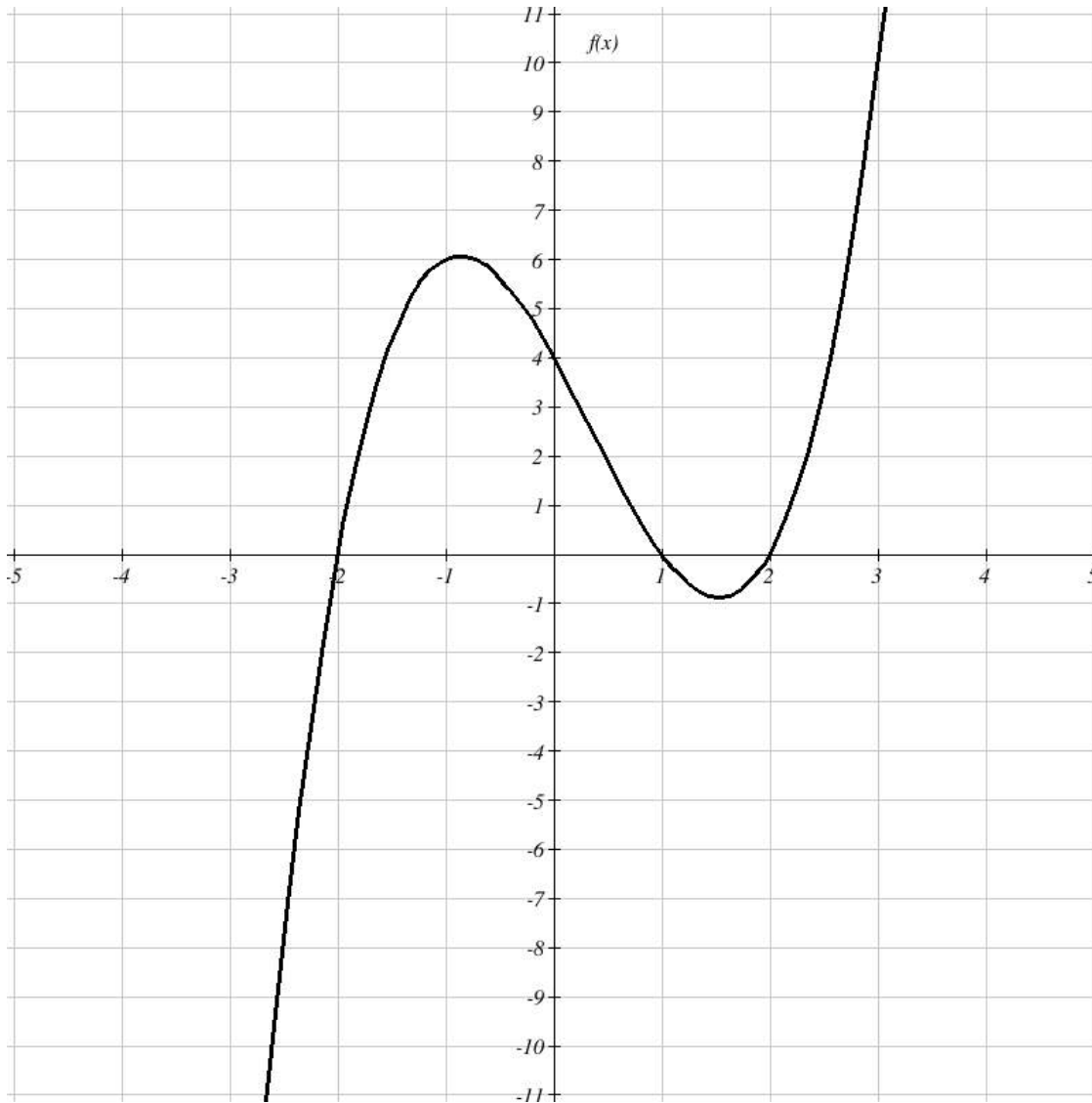


What are the X -intercepts of the function?

What are the zeros of the function?

3.

For the graph of $f(x)$ below, identify the zeros of $f(x)$ and the intervals where $f(x)$ is positive/negative. Answer DNE if there is no such interval.



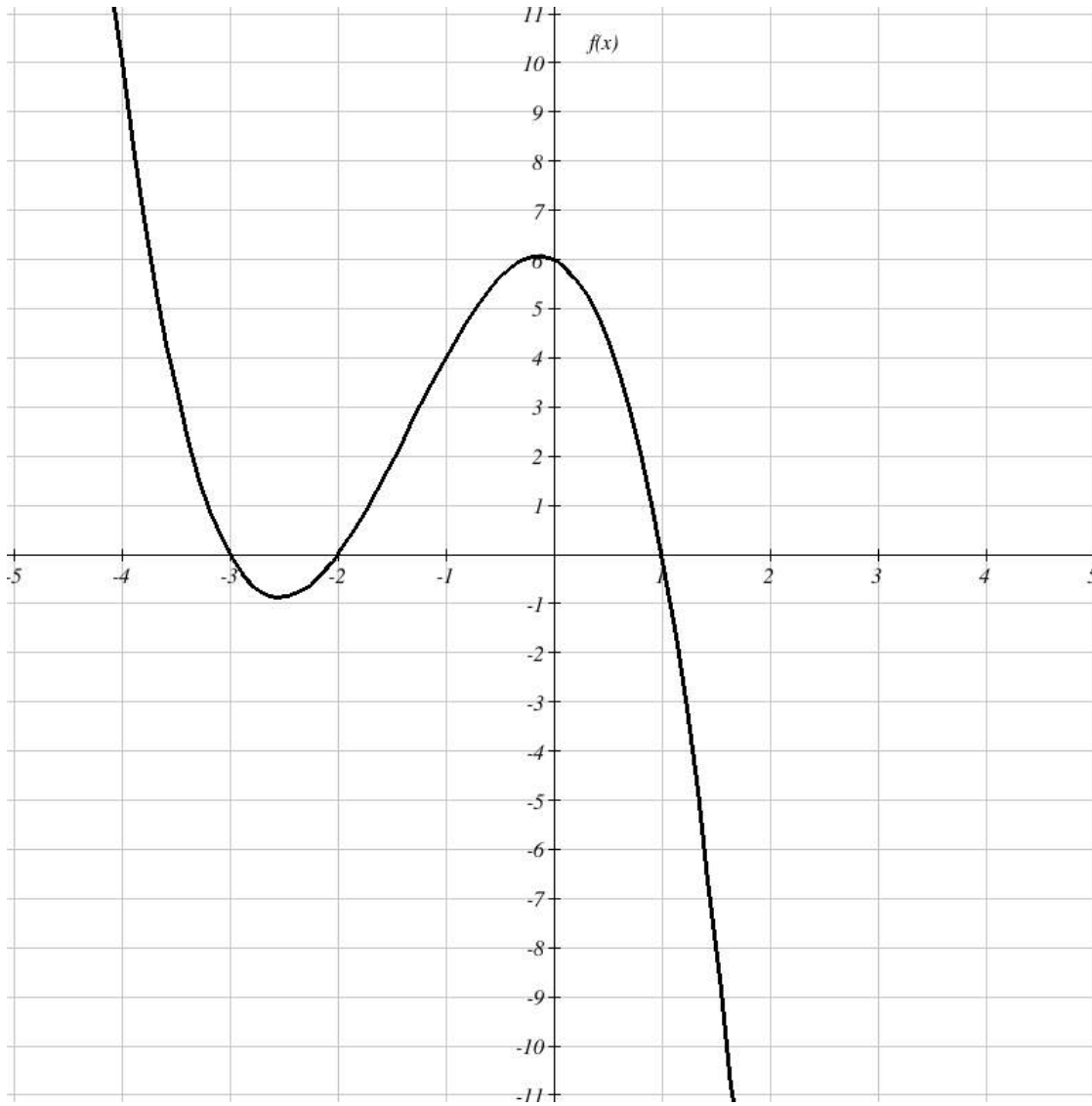
The zeros of $f(x)$ are _____

$f(x)$ is positive on the interval(s) _____

$f(x)$ is negative on the interval(s) _____

4.

For the graph of $f(x)$ below, identify the zeros of $f(x)$ and the intervals where $f(x)$ is positive/negative. Answer DNE if there is no such interval



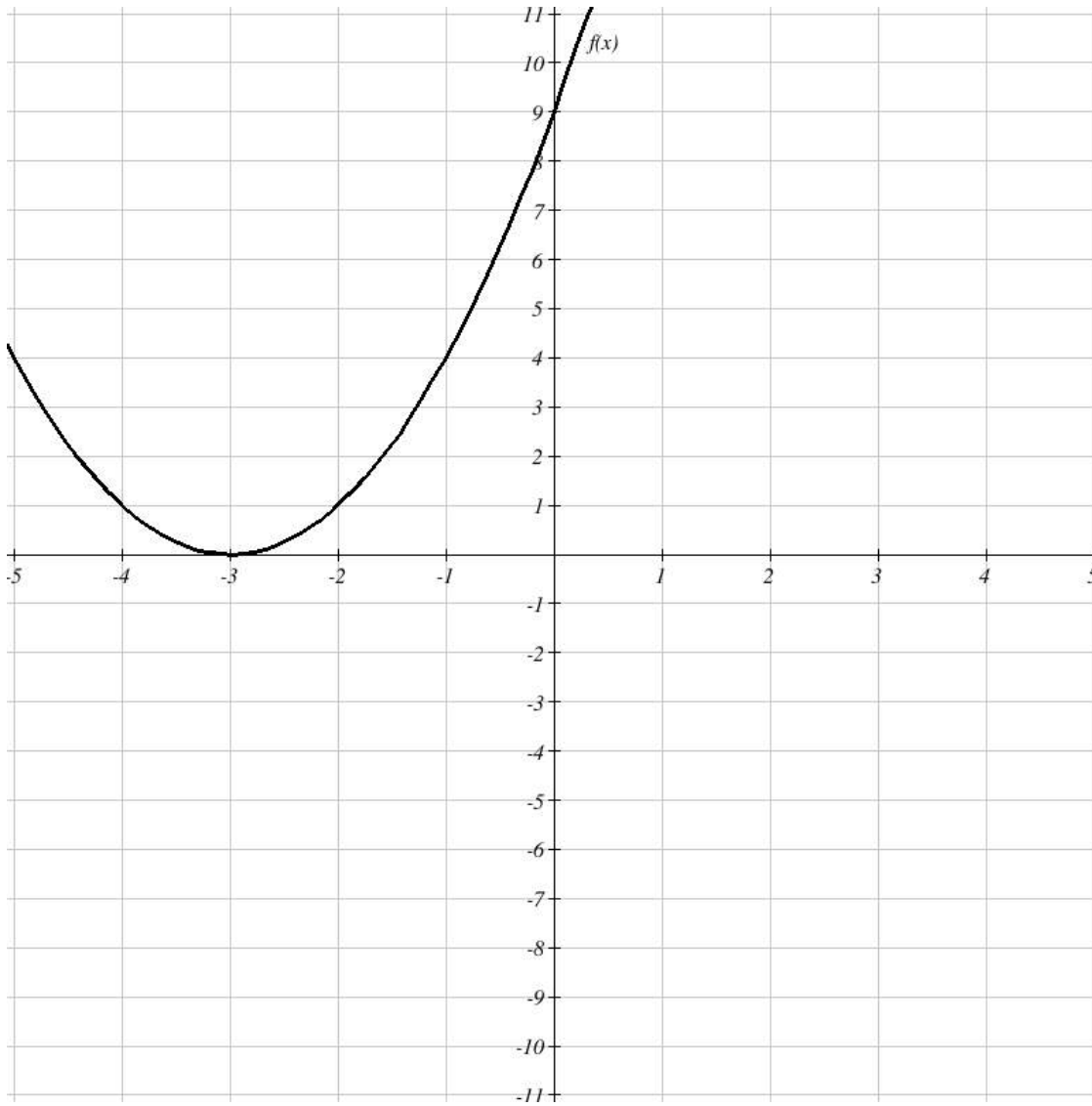
The zeros of $f(x)$ are _____

$f(x)$ is positive on the interval(s) _____

$f(x)$ is negative on the interval(s) _____

5.

For the graph of $f(x)$ below, identify the zeros of $f(x)$ and the intervals where $f(x)$ is positive/negative. Answer DNE if there is no such interval.



The zeros of $f(x)$ are _____

$f(x)$ is positive on the interval(s) _____

$f(x)$ is negative on the interval(s) _____

6.

The table below gives the annual sales (in millions) of a product.

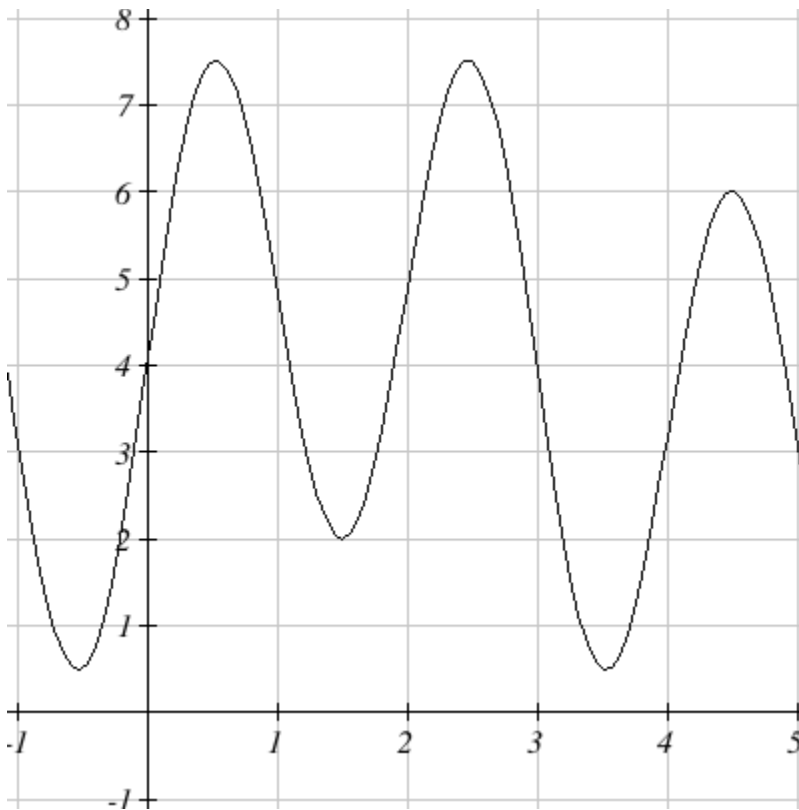
year	1998	1999	2000	2001	2002	2003	2004	2005	2006
sales	243	297	339	369	387	393	387	369	339

What was the average rate of change of annual sales

a) between 1999 and 2000? _____ millions of dollars/year

b) between 1999 and 2006? _____ millions of dollars/year

7.



Based on the graph above, estimate (to one decimal place) the average rate of change from $x = 1$ to $x = 3$

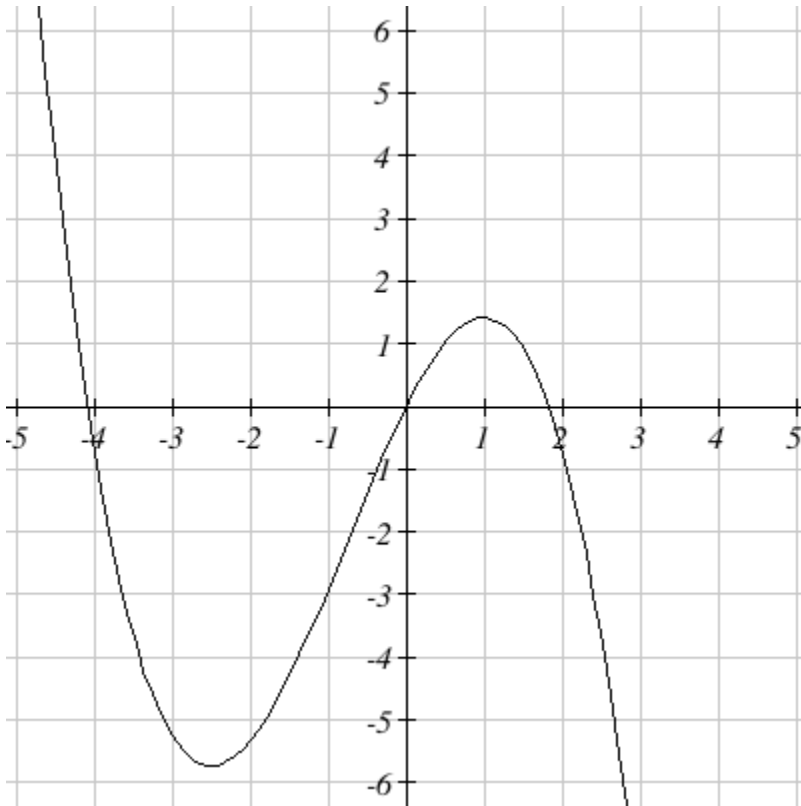
8.

Find the average rate of change of $g(x) = -2x^3 - 2$ from $x = -3$ to $x = 1$.

9.

Find the average rate of change of $f(x) = 4x^2 - 9$ on the interval $[5, t]$.
Your answer will be an expression involving t .

10.



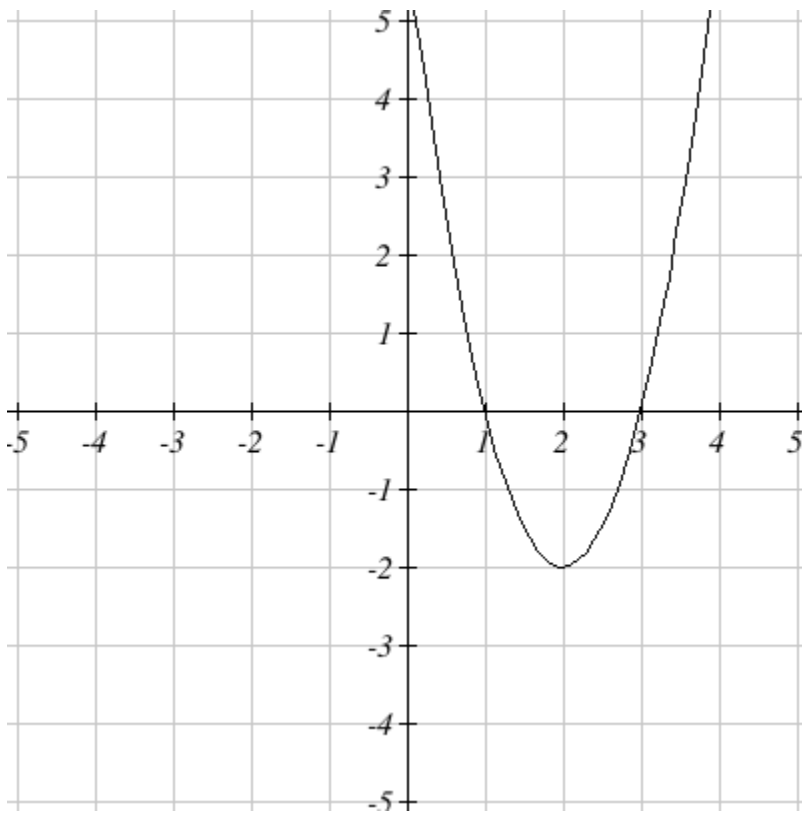
The function graphed above is:

Increasing on the interval(s) _____

Decreasing on the interval(s) _____

11.

Consider the function graphed below.

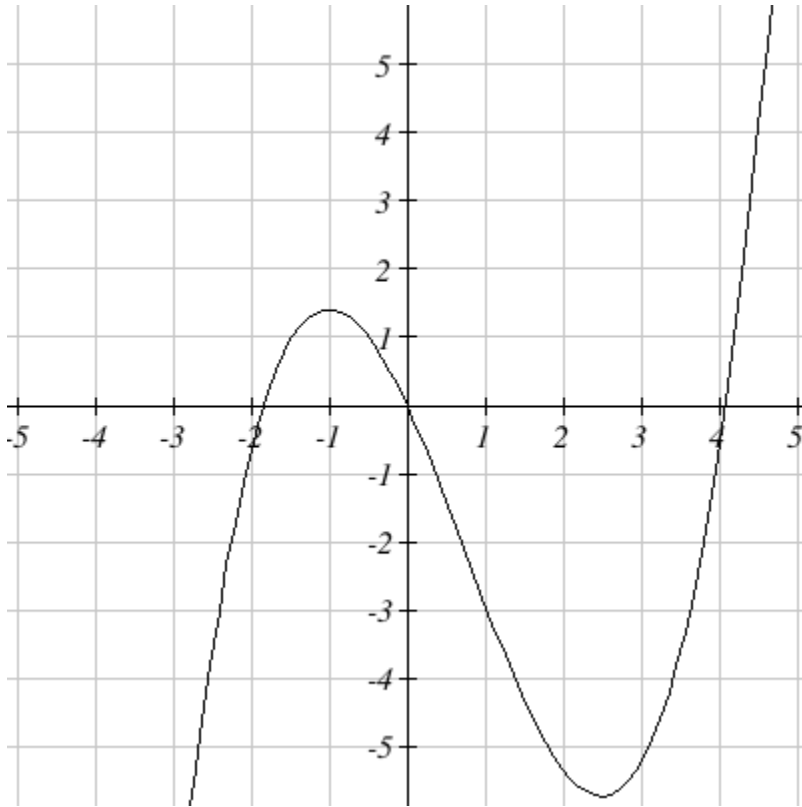


The function has a minimum/maximum (choose one) of _____ at $x =$ _____

The function is increasing on the interval(s): _____

The function is decreasing on the interval(s): _____

12.

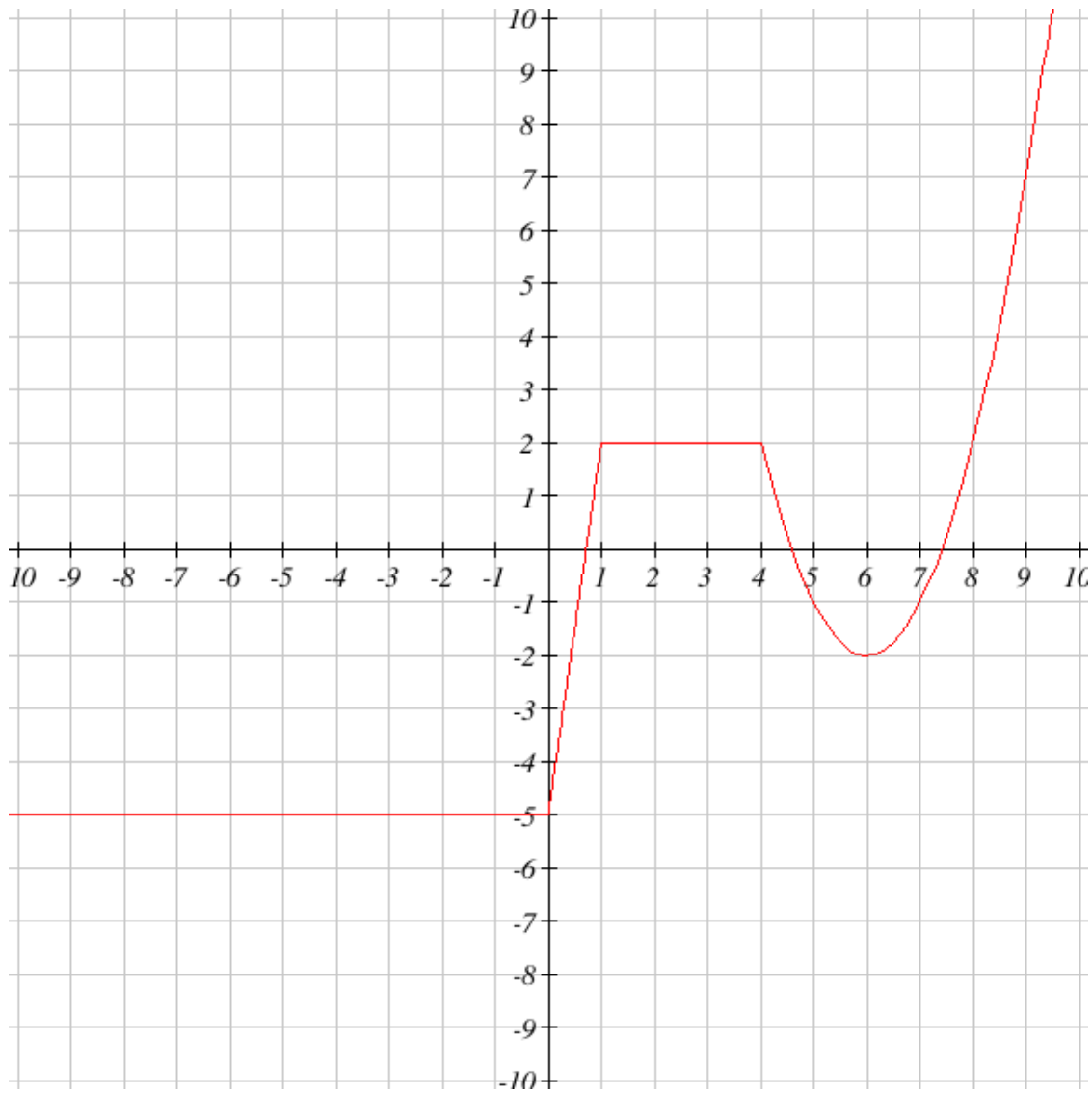


The function graphed above is decreasing on the interval

_____ < x < _____

13.

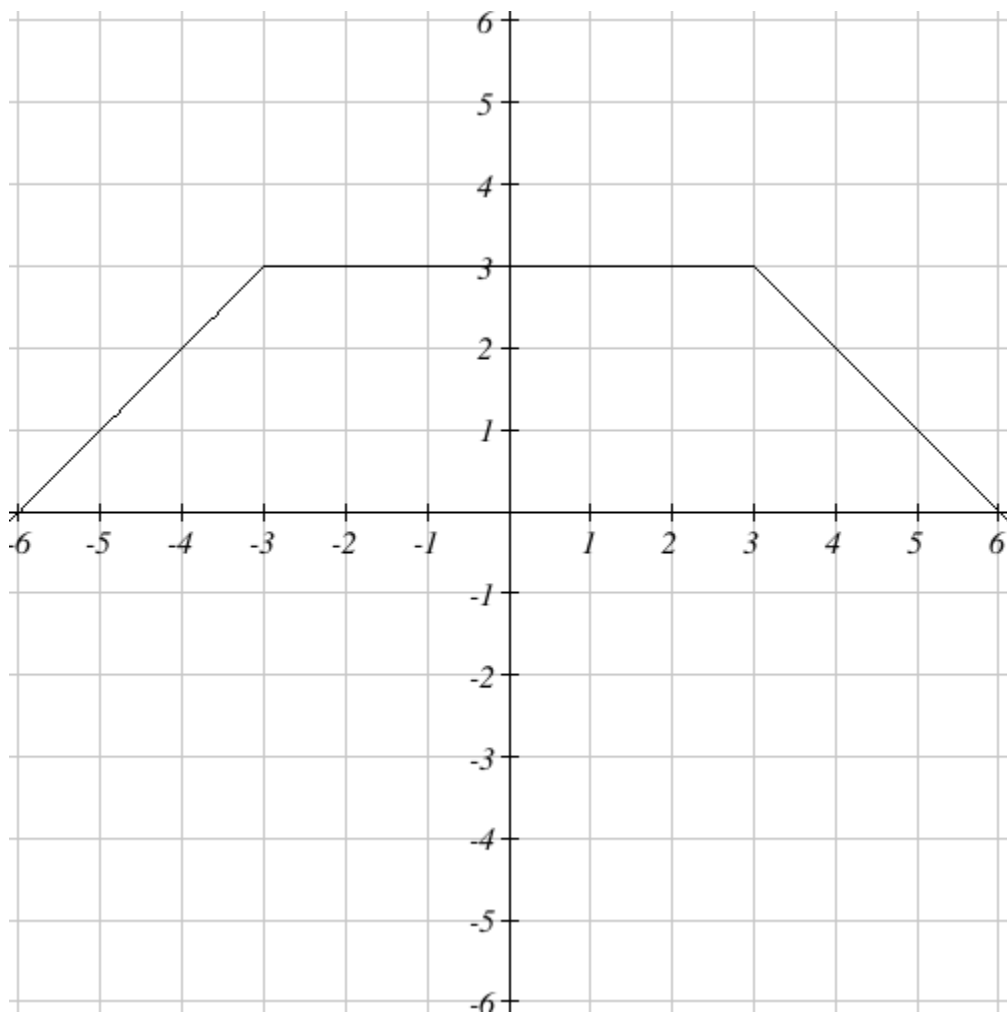
Consider the function graphed below



Give the interval(s) where the function is increasing and join multiple intervals with a union, U .

14.

Consider the function graphed below.



The function is increasing on the interval(s): _____

The function is decreasing on the interval(s): _____

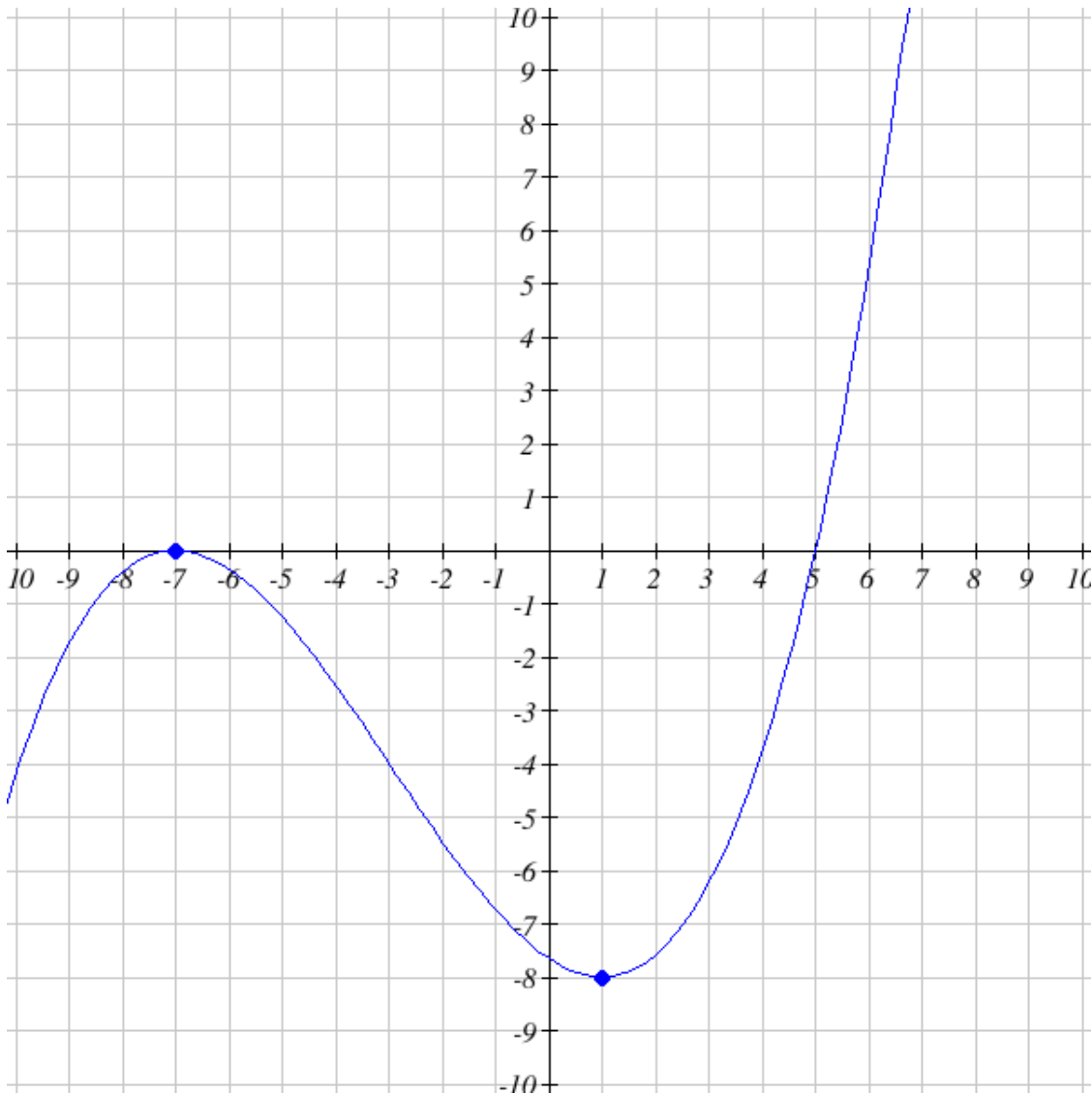
The function is constant on the interval(s): _____

The domain of the function is: _____

The range of the function is: _____

15.

Consider the function in the graph below.



The function has a maximum of _____ at $x =$ _____

The function has a minimum of _____ at $x =$ _____

The function is increasing on the interval(s): _____

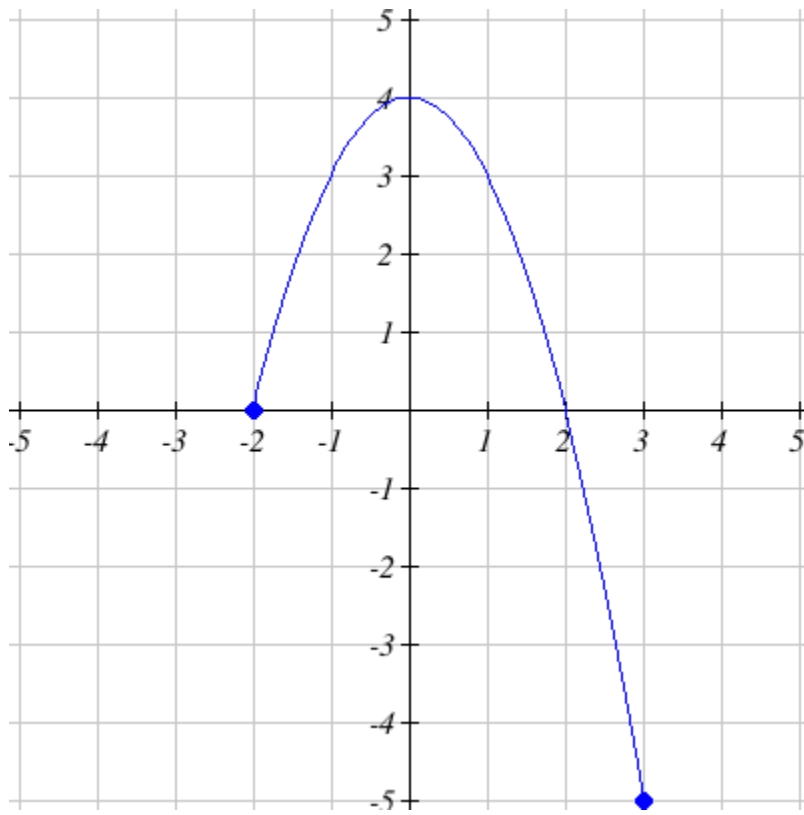
The function is decreasing on the interval(s): _____

The domain of the function is: _____

The range of the function is: _____

16.

Find the absolute maximum and minimum for the given graph. Give your answer as an ordered pair.

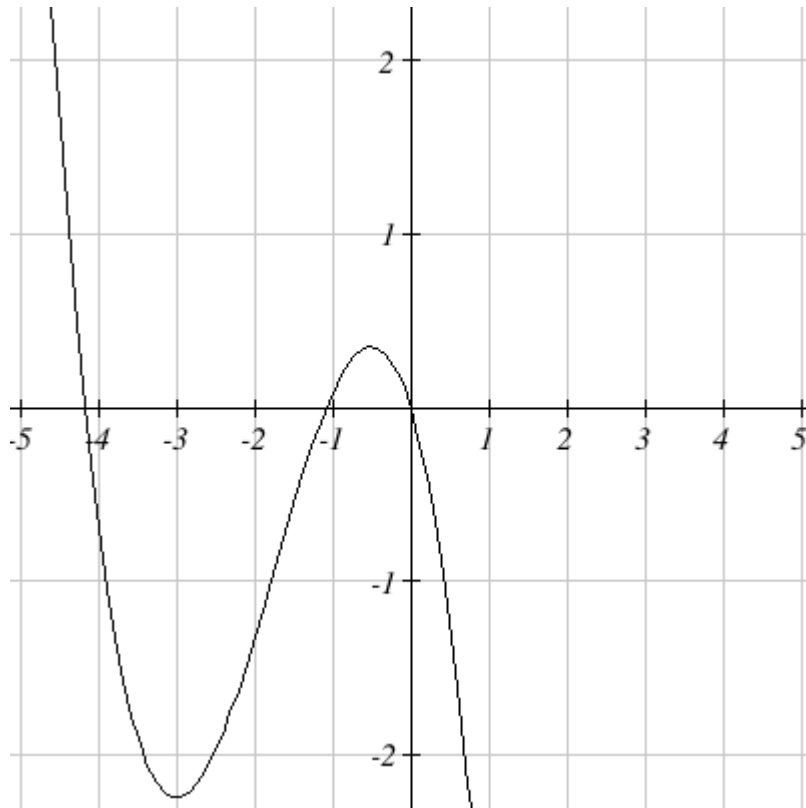


Absolute maximum: _____

Absolute minimum: _____

17.

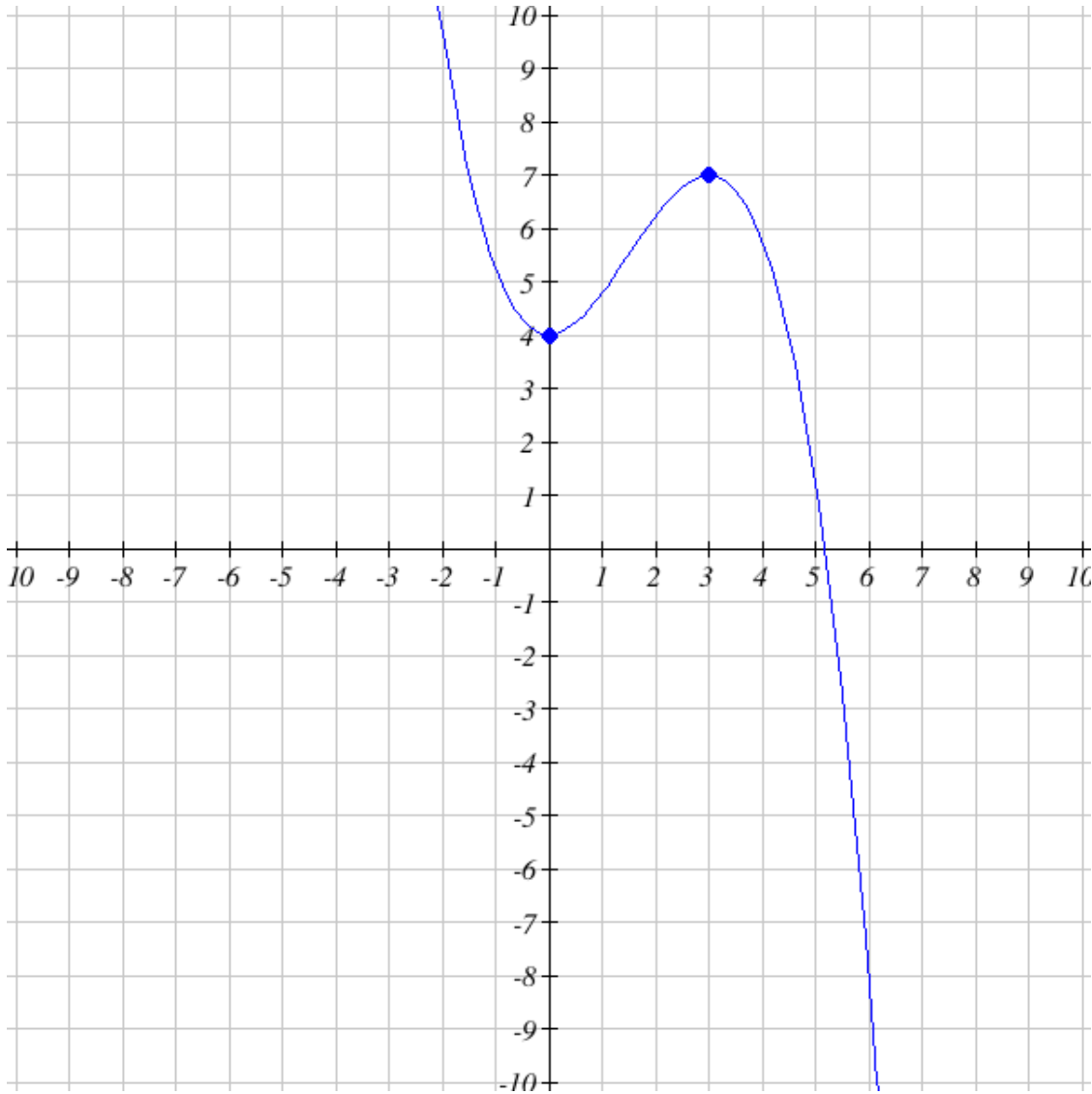
Consider the function graphed below.



The function has local extrema at approximately _____
what point(s), accurate to within .2 units? _____

18.

Consider the function in the graph below.



The function has a relative maximum of at $x =$ _____

The function has a relative minimum of at $x =$ _____

The function is increasing on the interval(s): _____

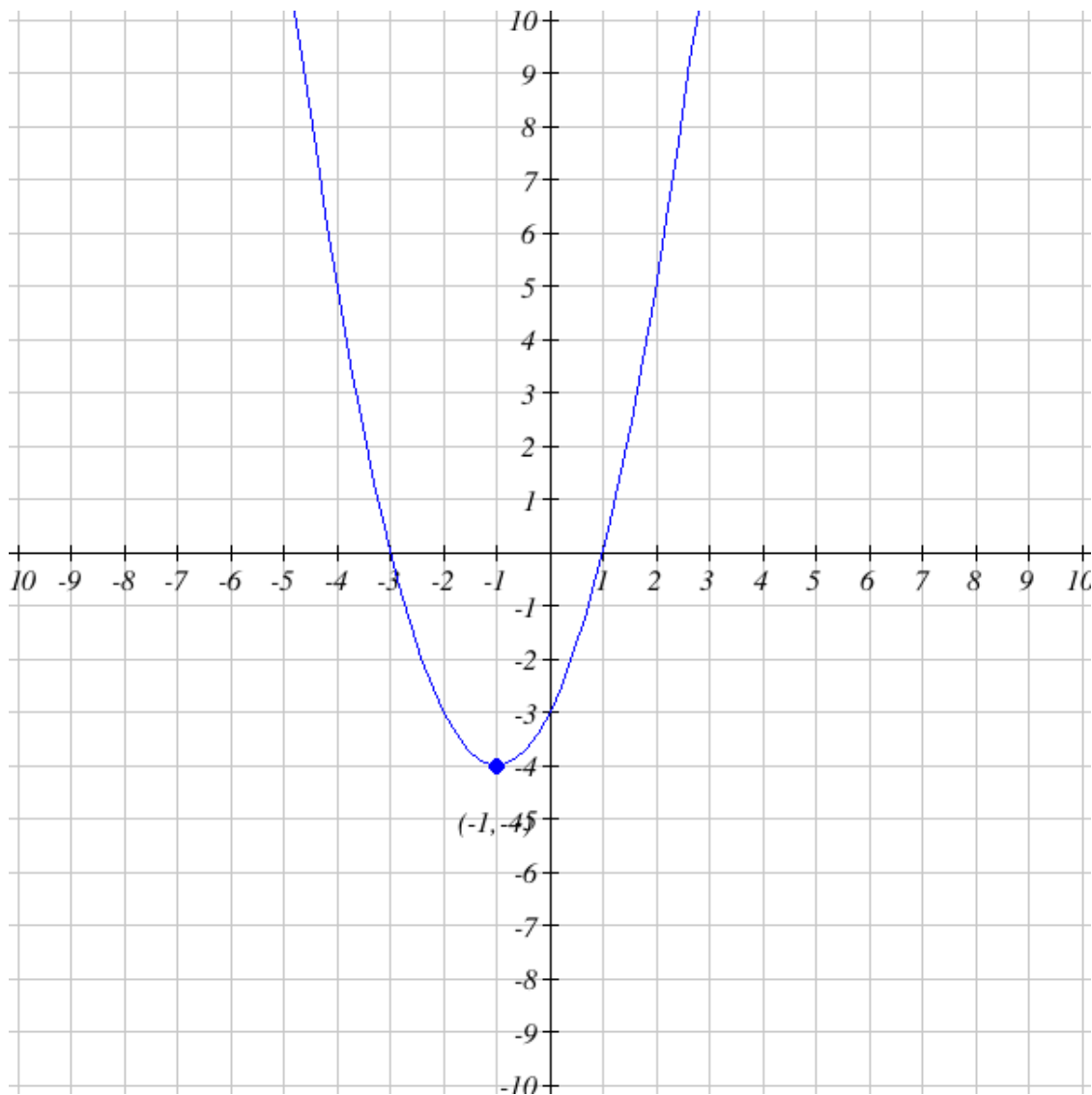
The function is decreasing on the interval(s): _____

The domain of the function is: _____

The range of the function is: _____

19.

Consider the function in the graph below.



The function has a minimum/maximum (choose one) of _____ at $x =$ _____

The function is increasing on the interval(s): _____

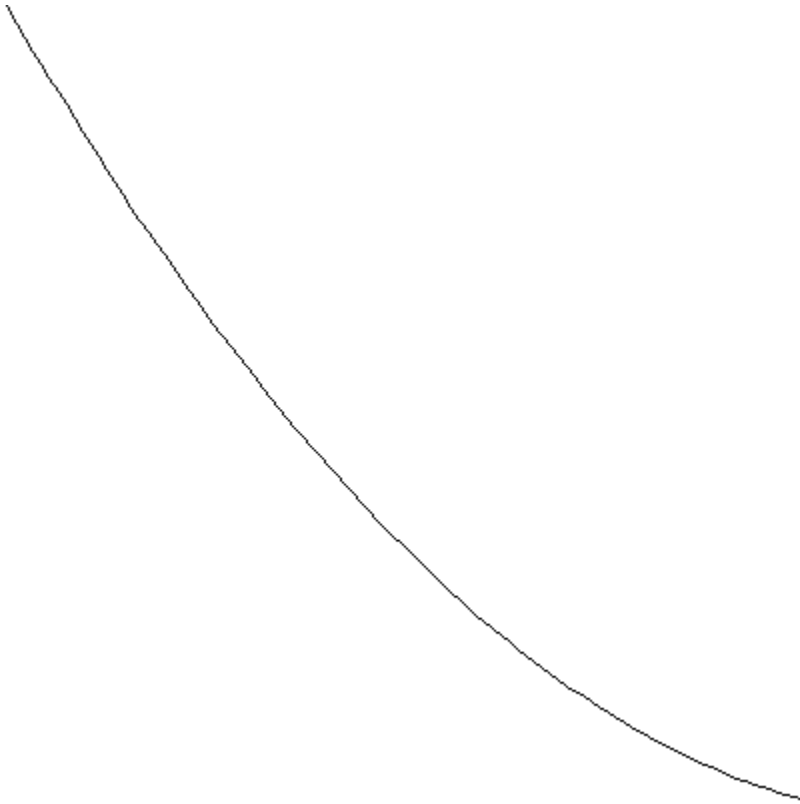
The function is decreasing on the interval(s): _____

The domain of the function is: _____

The range of the function is: _____

20.

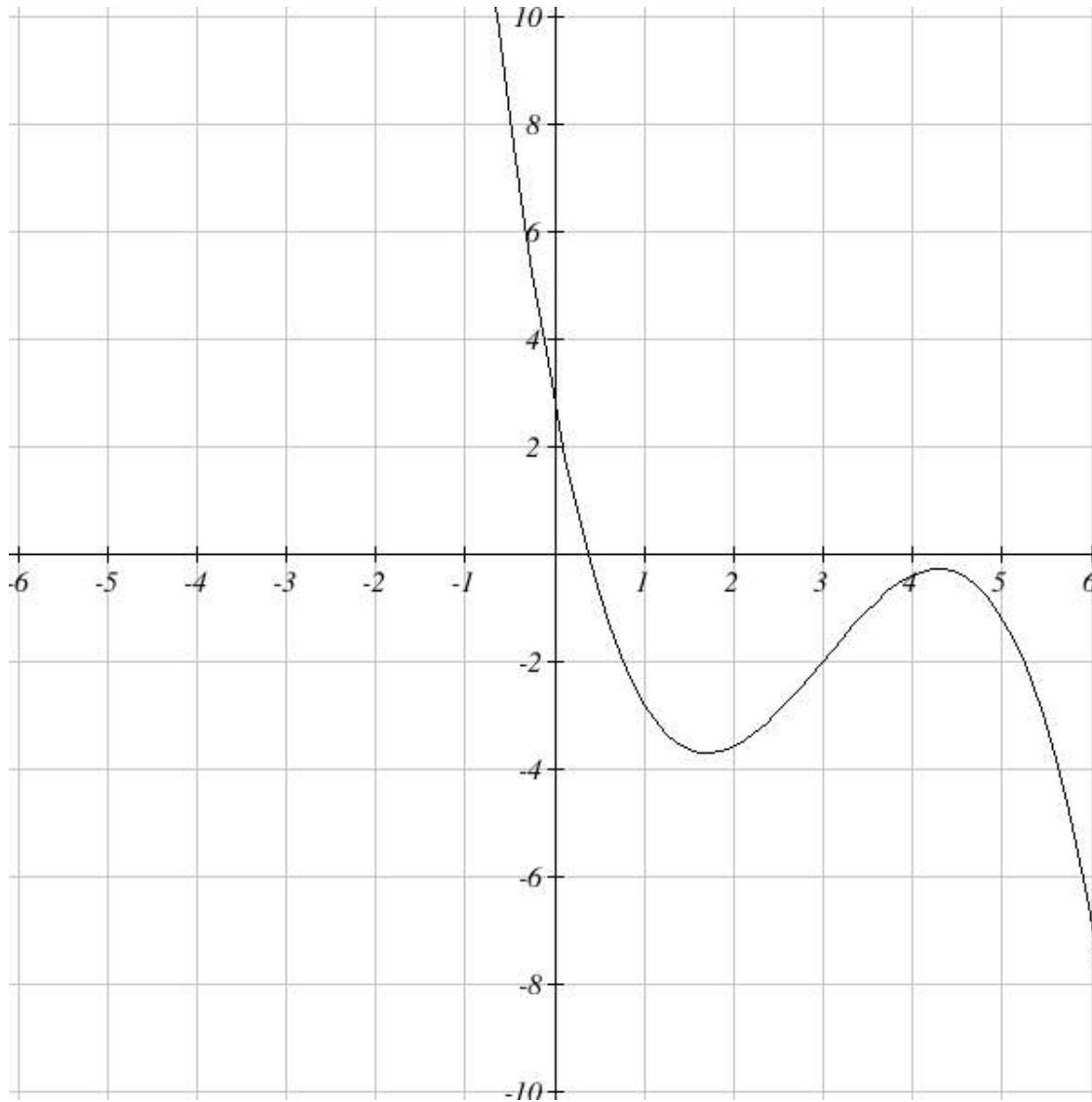
For the graph below, determine if it represents a function that is increasing or decreasing, and whether the function is concave up or concave down.



- Increasing
- Decreasing

- Concave up
- Concave down

21.



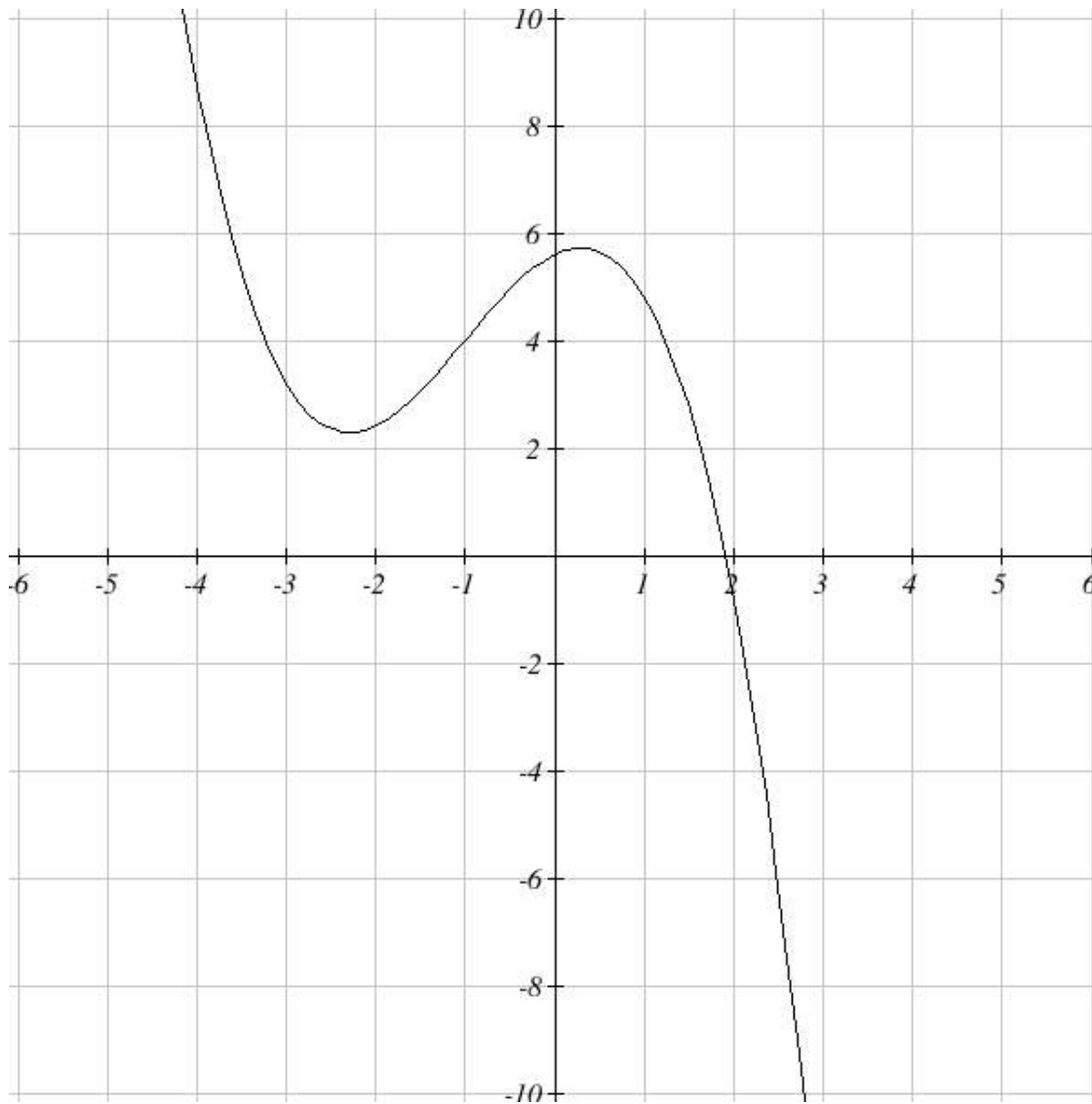
The function graphed above is:

Concave up on the interval(s) _____

Concave down on the interval(s) _____

There is an inflection point at: _____

22.



The function graphed above is:

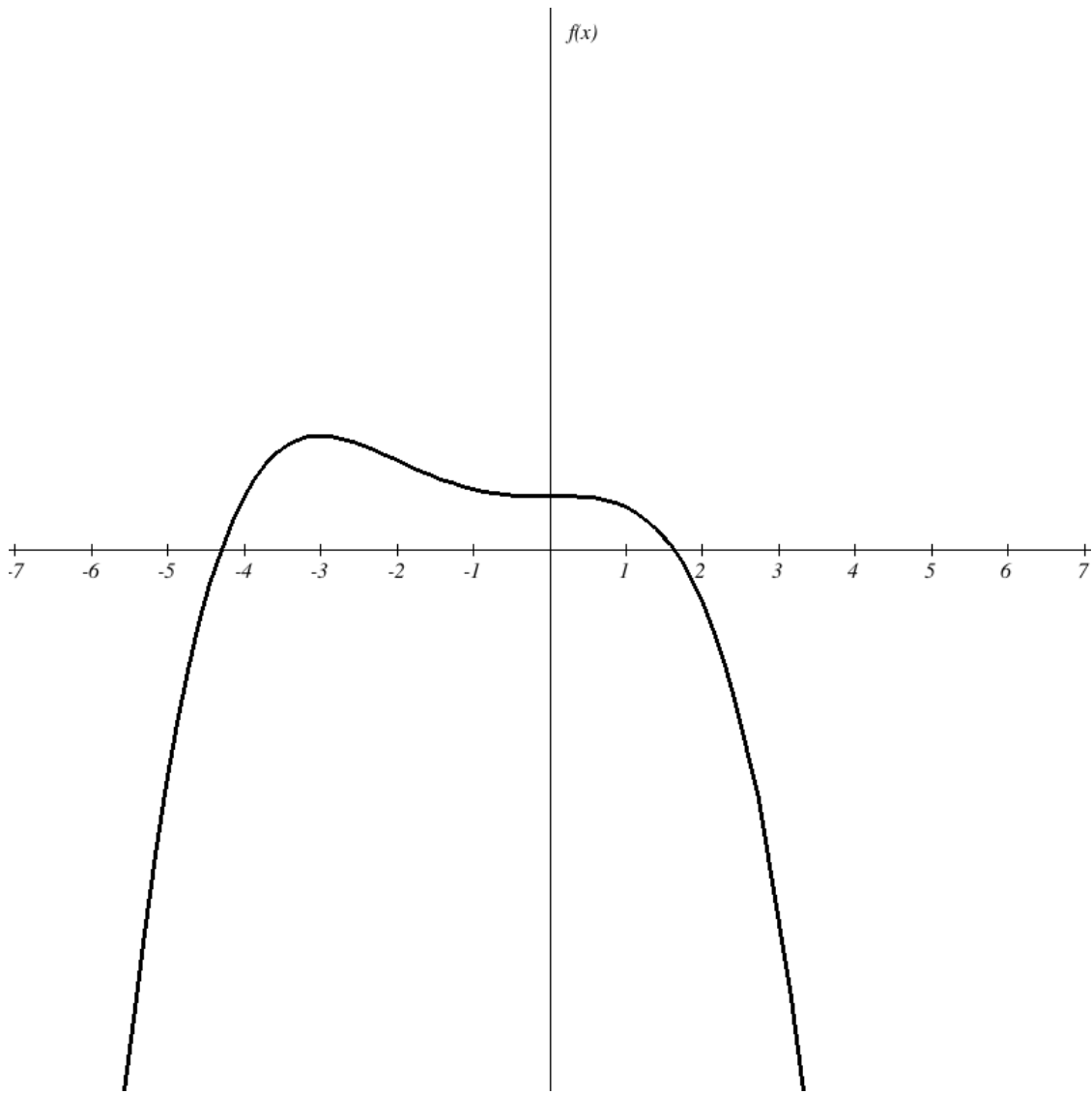
Concave up on the interval(s) _____

Concave down on the interval(s) _____

There is an inflection point at: _____

23.

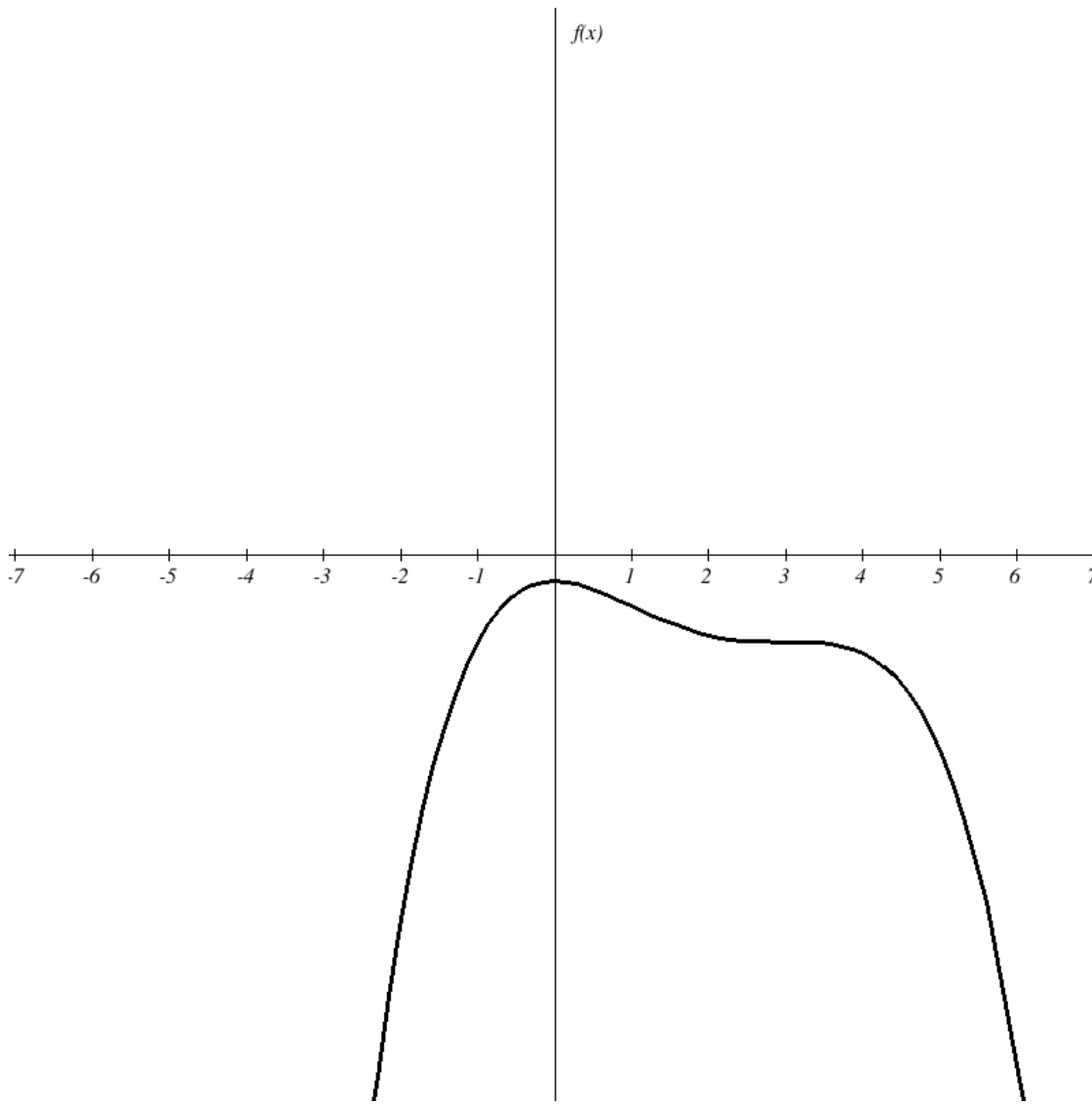
At which values of x does the function graphed below have inflection points?



The graph has inflection points at $x =$ _____

24.

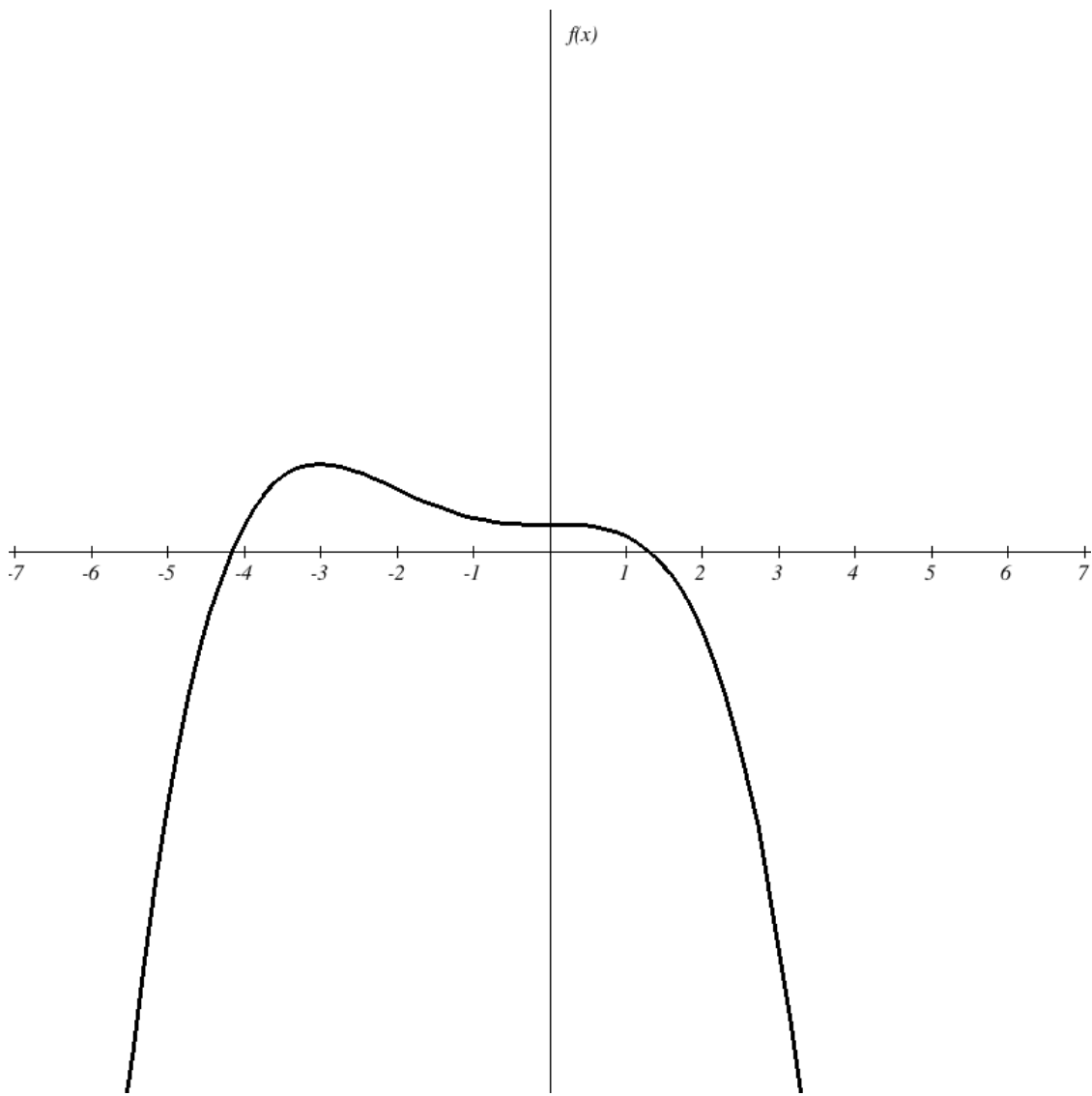
At which values of x does the function graphed below have inflection points?



The graph has inflection points at $x =$ _____

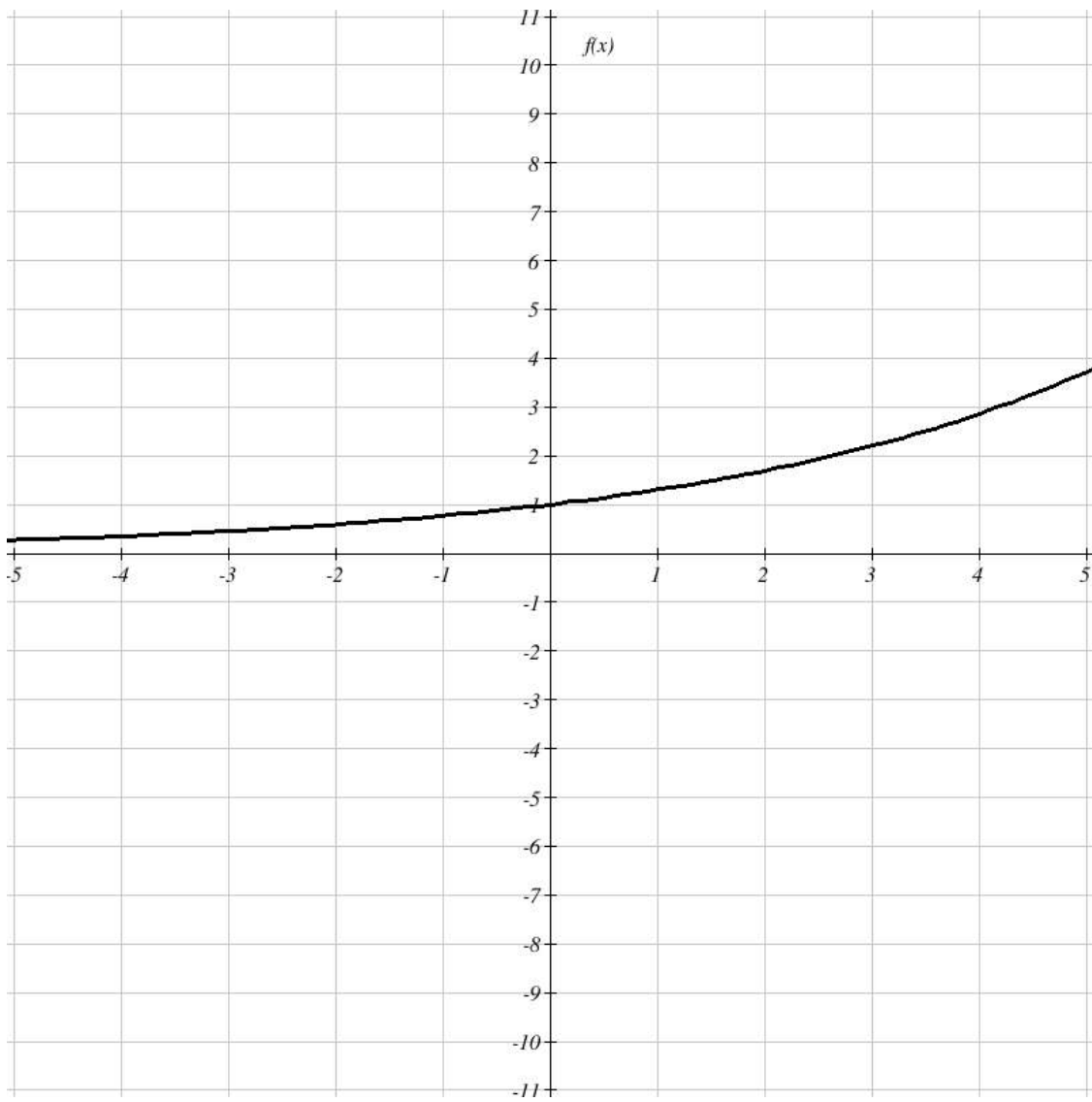
25.

At which values of x does the function graphed below have inflection points?



The graph has inflection points at $x =$ _____

26.
 For the graph of $f(x)$ below, identify the zeros of $f(x)$ and the intervals of each specified behavior.
 Answer DNE if there is no such interval/value.



The zeros of $f(x)$ are _____

$f(x)$ is positive on the interval(s) _____

$f(x)$ is negative on the interval(s) _____

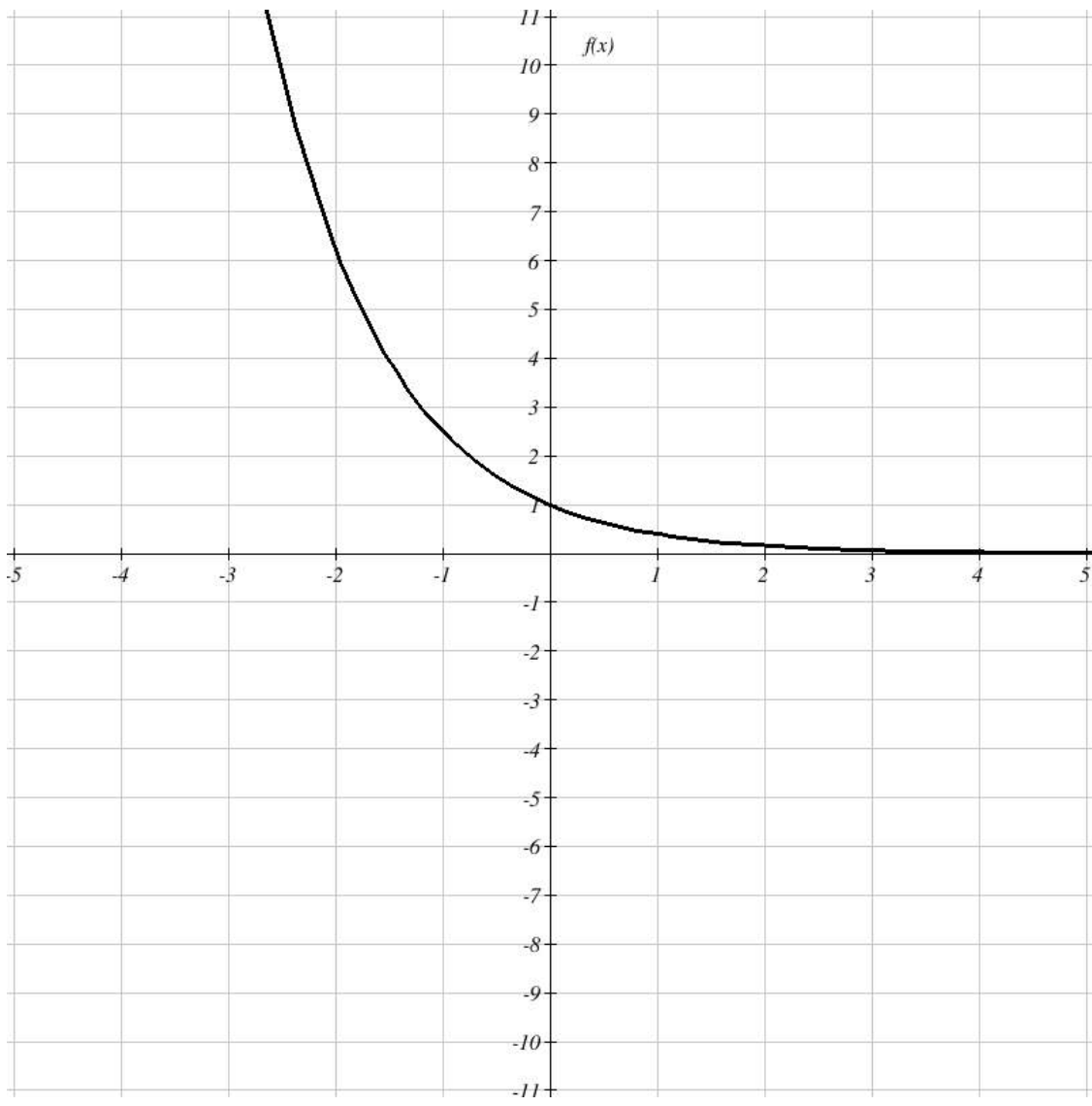
$f(x)$ is increasing on the interval(s) _____

$f(x)$ is decreasing on the interval(s) _____

$f(x)$ is concave up on the interval(s) _____

$f(x)$ is concave down on the interval(s) _____

27.
 For the graph of $f(x)$ below, identify the zeros of $f(x)$ and the intervals of each specified behavior.
 Answer DNE if there is no such interval/value.



The zeros of $f(x)$ are _____

$f(x)$ is positive on the interval(s) _____

$f(x)$ is negative on the interval(s) _____

$f(x)$ is increasing on the interval(s) _____

$f(x)$ is decreasing on the interval(s) _____

$f(x)$ is concave up on the interval(s) _____

$f(x)$ is concave down on the interval(s) _____

28.

Sketch the graph of a function that is increasing and concave up.

29.

Sketch the graph of a function that is increasing and concave down.

30.

Sketch the graph of a function that is decreasing and concave up.

31.

Sketch the graph of a function that is decreasing and concave down.

32.

Sketch the graph of a function that is decreasing on the interval $(-\infty, 2)$, constant on the interval $(2, 5)$ and increasing on the interval $(5, \infty)$.

33.

Sketch the graph of a function that is decreasing on the interval $(-\infty, -1)$, increasing on the interval $(-1, 3)$ and constant on the interval $(3, \infty)$.

34.

Sketch the graph of a function that is constant on the interval $(-\infty, 2)$, increasing on the interval $(2, 7)$ and decreasing on the interval $(7, \infty)$.

Lecture 7

Arithmetic combinations with domain analysis, composition with domain analysis, Decomposing functions

1.

Given the following functions, find each of the values:

$$f(x) = x^2 + 5x - 6$$

$$g(x) = x - 1$$

$$(f + g)(1) = \underline{\hspace{2cm}}$$

$$(f - g)(4) = \underline{\hspace{2cm}}$$

$$(f \cdot g)(3) = \underline{\hspace{2cm}}$$

$$\left(\frac{f}{g}\right)(-4) = \underline{\hspace{2cm}}$$

2.

Given the following functions, evaluate each of the following:

$$f(x) = x^2 - 4x - 5$$

$$g(x) = x + 1$$

$$(f + g)(-5) = \underline{\hspace{2cm}}$$

$$(f - g)(-5) = \underline{\hspace{2cm}}$$

$$(f \cdot g)(-5) = \underline{\hspace{2cm}}$$

$$\left(\frac{f}{g}\right)(-5) = \underline{\hspace{2cm}}$$

3.

Complete the table below.

x	-1	1	2	3	4
$f(x)$	4	-8		-5	
$g(x)$	10	3	1		
$(f - g)(x)$		-11		-4	-10
$(f + g)(x)$	14		-3		8

4.

Answer the following True or False:

If $f(x) = ax^2 + c$, then $f(x + 9) - f(x) = a(x + 9)^2 + c - ax^2 + c$.

- True
- False

5.

Let $f(x) = 2x + 4$ and $g(x) = 2x^2 + 2x$.

After simplifying,

$(f + g)(x) = \underline{\hspace{2cm}}$

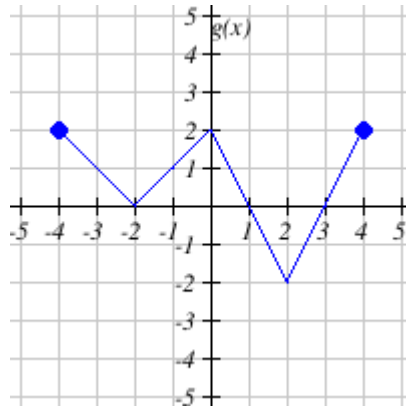
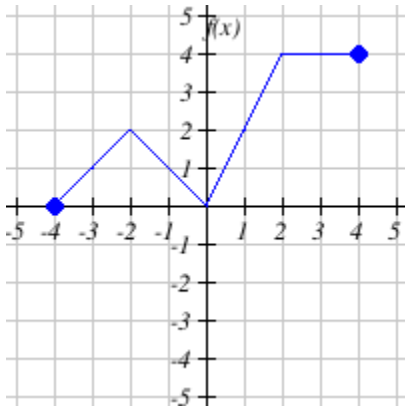
6.

Let $f(x) = 2x + 2$ and $g(x) = 3x^2 + 3x$. After simplifying,

$(fg)(x) = \underline{\hspace{2cm}}$

7.

The functions $f(x)$ and $g(x)$ are graphed below.



Determine $f(-2) = \underline{\hspace{2cm}}$

Determine $g(-2) = \underline{\hspace{2cm}}$

Determine $(f + g)(-2) = \underline{\hspace{2cm}}$

Determine $f(2) = \underline{\hspace{2cm}}$

Determine $g(2) = \underline{\hspace{2cm}}$

Determine $(f - g)(2) = \underline{\hspace{2cm}}$

Determine $f(4) = \underline{\hspace{2cm}}$

Determine $g(4) = \underline{\hspace{2cm}}$

Determine $(g - f)(4) = \underline{\hspace{2cm}}$

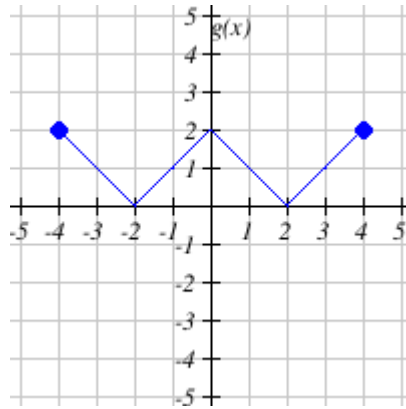
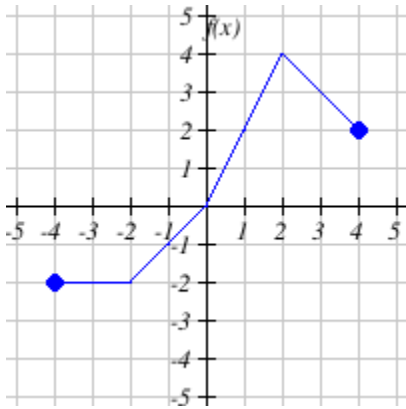
Determine $f(0) = \underline{\hspace{2cm}}$

Determine $g(0) = \underline{\hspace{2cm}}$

Determine $(fg)(0) = \underline{\hspace{2cm}}$

8.

The functions $f(x)$ and $g(x)$ are graphed below.



Determine $(f + g)(4) = \underline{\hspace{2cm}}$

Determine $(f - g)(2) = \underline{\hspace{2cm}}$

Determine $(fg)(3) = \underline{\hspace{2cm}}$

Determine $\left(\frac{f}{g}\right)(-3) = \underline{\hspace{2cm}}$

9.

Given the following functions, find each:

$$f(x) = x^2 + 2x - 24$$

$$g(x) = x - 4$$

$$(f + g)(x) = \underline{\hspace{2cm}}$$

$$(f - g)(x) = \underline{\hspace{2cm}}$$

$$(f \cdot g)(x) = \underline{\hspace{2cm}}$$

$$\left(\frac{f}{g}\right)(x) = \underline{\hspace{2cm}}$$

10.

Answer the following True or False:

Let f and g be functions. Then the domain of $\frac{f}{g}$ is the intersection of the domain of f and the domain of g

- True
- False

11.

Answer the following True or False:

Let f and g be functions. The domain of $f \cdot g$ is the intersection of the domain of f and the domain of g

- True
- False

12.

$$f(x) = x^2 + x - 20$$

$$g(x) = x - 4$$

Find $\left(\frac{f}{g}\right)(x)$

$$\left(\frac{f}{g}\right)(x) = \underline{\hspace{2cm}}$$

The domain of $\left(\frac{f}{g}\right)(x)$ is $x \neq \underline{\hspace{2cm}}$

13.

Suppose that

$$f(x) = 7x + 35$$

$$g(x) = x^2 - 3x - 40$$

The domain of $\left(\frac{f}{g}\right)(x)$ is $x \neq \underline{\hspace{2cm}}$

14.

Use the table of values to evaluate the expressions below.

x	$f(x)$	$g(x)$
0	7	0
1	0	9
2	8	6
3	5	1
4	1	7
5	2	2
6	6	5
7	3	8
8	4	4
9	9	3

$$f(g(1)) = \underline{\hspace{2cm}}$$

$$g(f(8)) = \underline{\hspace{2cm}}$$

$$f(f(5)) = \underline{\hspace{2cm}}$$

$$g(g(9)) = \underline{\hspace{2cm}}$$

15.

Use the table below to evaluate.

x	-10	-5	-3	2	3	6	8	12
$f(x)$	3	-3	-5	2	-10	8	12	6
$g(x)$	-10	3	6	-5	12	2	8	-3

$$(f \circ g)(-5) = \underline{\hspace{2cm}}$$

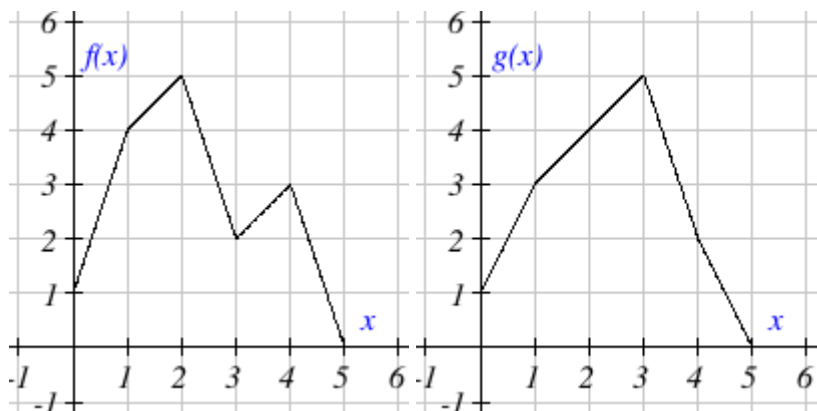
$$(g \circ f)(-10) = \underline{\hspace{2cm}}$$

$$(f \circ f)(-3) = \underline{\hspace{2cm}}$$

$$(g \circ g)(12) = \underline{\hspace{2cm}}$$

16.

Use the graphs to evaluate the expressions below.



$$f(g(3)) = \underline{\hspace{2cm}}$$

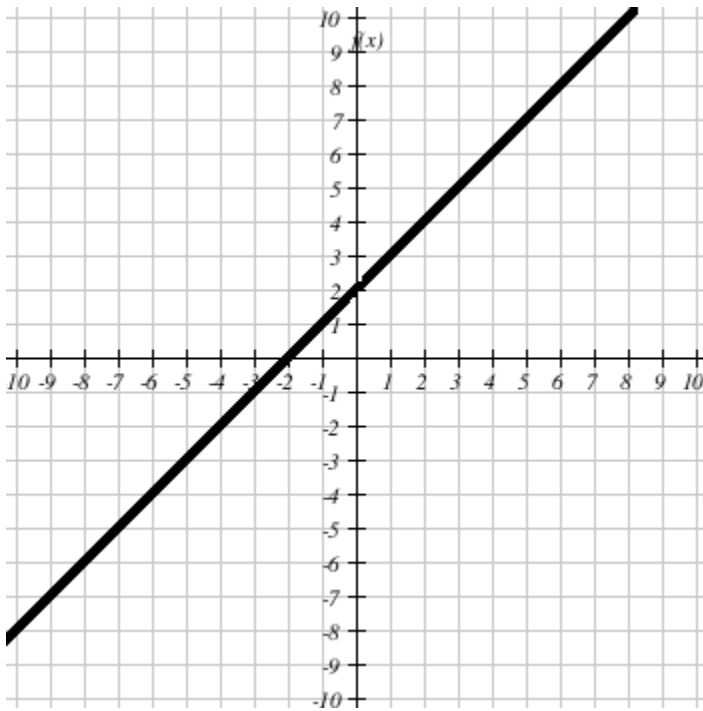
$$g(f(5)) = \underline{\hspace{2cm}}$$

$$f(f(1)) = \underline{\hspace{2cm}}$$

$$g(g(2)) = \underline{\hspace{2cm}}$$

17.

The graph of $f(x)$ and a table of values for $g(x)$ are given below. Use them to evaluate the given statements.



x	0	1	2	3	4	5	6	7	8	9
$g(x)$	7	8	5	6	4	2	3	0	1	9

$$(f + g)(4) = \underline{\hspace{2cm}}$$

$$(fg)(3) = \underline{\hspace{2cm}}$$

$$(g \circ f)(-1) = \underline{\hspace{2cm}}$$

$$(g \circ g)(7) = \underline{\hspace{2cm}}$$

18.

The function $D(p)$ gives the number of items that will be demanded when the price is p . The production cost, $C(x)$, is the cost of producing x items.

To determine the cost of production when the price is \$7, you would:

- Evaluate $D(C(7))$
- Solve $C(D(p)) = 7$
- Solve $D(C(x)) = 7$
- Evaluate $C(D(7))$

19.

Given that $f(x) = 2x - 8$ and $g(x) = 9 - x^2$, calculate

(a) $f(g(0)) =$ _____

(b) $g(f(0)) =$ _____

20.

Given that $f(x) = 8x - 3$ and $g(x) = -6x - 1$, determine each of the following. Make sure to fully simplify your answer.

(a) $(f \circ g)(x) =$ _____

(b) $(g \circ f)(x) =$ _____

(c) $(f \circ f)(x) =$ _____

(d) $(g \circ g)(x) =$ _____

21.

Given that $f(x) = -2x^2 - 5x$ and $g(x) = -5x + 6$, determine each of the following. Make sure to fully simplify your answer.

(a) $(f \circ g)(x) =$ _____

(b) $(g \circ f)(x) =$ _____

(c) $(f \circ f)(x) =$ _____

(d) $(g \circ g)(x) =$ _____

22.

Given that $f(x) = -2x^2 - 5x + 9$ and $g(x) = 4x + 6$, determine each of the following. Make sure to fully simplify your answer.

(a) $(f \circ g)(x) =$ _____

(b) $(g \circ f)(x) =$ _____

23.

Let $f(x) = 3x + 3$ and $g(x) = 3x^2 + 2x$.

After simplifying,

$(f \circ g)(x) =$ _____

24.

$f(x) = x^2 + 5x$

$(f \circ f)(x) =$ _____

25.

Given functions $m(x) = \frac{1}{\sqrt{x}}$ and $g(x) = x^2 - 4$, state the domains of the following functions using interval notation.

Domain of $\frac{m(x)}{g(x)}$: _____

Domain of $m(g(x))$: _____

Domain of $g(m(x))$: _____

26.

If $f(x) = x^2 + 4x + 5$, simplify each of the following.

$f(x + h) =$ _____

$f(x + h) - f(x) =$ _____

27.

Let $f(x) = 4x + 3$. Determine $(f \circ f \circ f)(x)$.

$(f \circ f \circ f)(x) =$ _____

28.

If $f(x) = x^4 + 2$, $g(x) = x - 2$ and $h(x) = \sqrt{x}$, then

$f(g(h(x))) =$ _____

29.

If $f(x) = x^4 + 8$, $g(x) = x - 3$, $h(x) = \sqrt{x}$, then

$f \circ g \circ h(x) =$ _____

30.

The function $h(x) = (x + 8)^7$ can be expressed in the form $f(g(x))$, where $f(x) = x^7$, and $g(x)$ is defined below:

$g(x) =$ _____

31.

The function $h(x) = \frac{1}{x+2}$ can be expressed in the form $f(g(x))$, where $g(x) = (x + 2)$, and $f(x)$ is defined as:

$$f(x) = \underline{\hspace{2cm}}$$

32.

Suppose that $h(x) = \frac{1}{9x^3+8}$.

Creat two different decompositions for $h(x)$.

In other words, construct four functions $f(x), g(x), m(x), n(x)$ so that:

$$h(x) = f(g(x))$$

$$h(x) = m(n(x))$$

$$f(x) \neq m(x)$$

$$g(x) \neq n(x)$$

Lecture 8

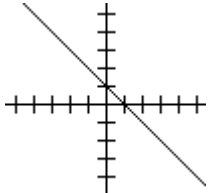
Library of basic functions (linear, square, cube, square root, reciprocal, abs value, piecewise)

Rigid Transformations (Translations, Reflections)

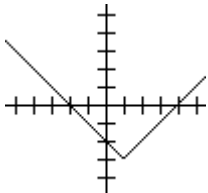
Nonrigid Transformations (Stretch/compression)

1.

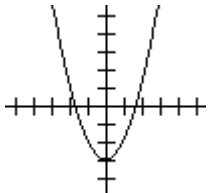
Match each graph with the corresponding function type.



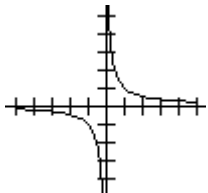
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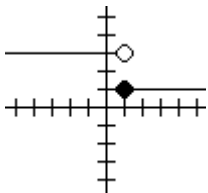
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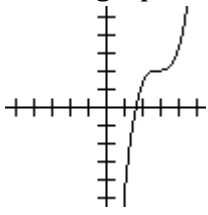


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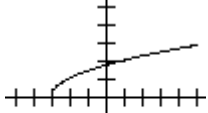
- a) Reciprocal
- b) Piecewise
- c) Linear
- d) Absolute Value
- e) Quadratic

2.

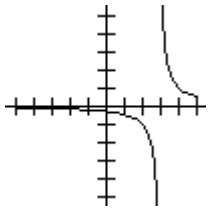
Match each graph with the corresponding function type.



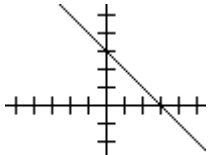
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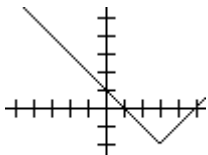
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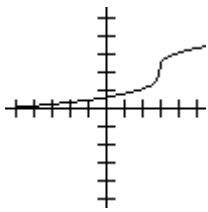
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- a) Absolute Value
- b) Rational
- c) Square Root
- d) Linear
- e) Cube Root
- f) Cubic

3.

If $f(5) = -5$, write an ordered pair that must be on the graph of $y = f(x - 1) - 3$

(_____ , _____)

4.

The graph of the function $y = f(x + 98)$ can be obtained from the graph of $y = f(x)$ by one of the following actions:

- shifting the graph of $f(x)$ to the left 98 units
- shifting the graph of $f(x)$ upwards 98 units
- shifting the graph of $f(x)$ to the right 98 units
- shifting the graph of $f(x)$ downwards 98 units

5.

Let $f(x) = 4\sqrt{x}$.

If $g(x)$ is the graph of $f(x)$ shifted up 6 units and right 3 units, write a formula for $g(x)$.

$g(x) =$ _____

6.

A table for $f(x)$ is shown below:

x	-2	-1	0	1	2
f(x)	2	1	-4	0	-1

A table for $g(x)$ is shown below:

x	-1	0	1	2	3
g(x)	2	1	-4	0	-1

Based on the table, $g(x) =$

- $f(x + 1)$
- $f(x - 1)$
- $f(x) - 1$
- $f(x) + 1$

A table for $h(x)$ is shown below:

x	-2	-1	0	1	2
h(x)	1	0	-5	-1	-2

Based on the table, $h(x) =$

- $f(x + 1)$
- $f(x) + 1$
- $f(x - 1)$
- $f(x) - 1$

7.

A table for $f(x)$ is shown below:

x	-2	-1	0	1	2
f(x)	-3	4	-1	-4	2

A table for $h(x)$ is shown below:

x	-2	-1	0	1	2
h(x)	-4	3	-2	-5	1

Based on the table, $h(x) =$

- $f(x) + 1$
- $f(x + 1)$
- $f(x - 1)$
- $f(x) - 1$

8.

Let $f(x) = x^3$

If $g(x)$ is the graph of $f(x)$ shifted up 4 units, write a formula for $g(x)$

$g(x) =$ _____

9.

Let $f(x) = x^4$

If $g(x)$ is the graph of $f(x)$ shifted left 2 units, write a formula for $g(x)$

$g(x) =$ _____

10.

Let $f(x) = 3\sqrt{x}$

If $g(x)$ is the graph of $f(x)$ shifted up 6 units and right 5 units, write a formula for $g(x)$

$g(x) =$ _____

11.

Given $f(x) = x^2$, after performing the following transformations: shift upward 32 units and shift 95 units to the right, the new function $g(x) =$ _____

12.

Suppose the graph of $y = 9x^2 + 7x - 1$ is stretched horizontally by a factor of 3.

The equation of the new graph will be $y =$ _____

13.

Suppose the graph of $y = 5x^2 - x - 10$ is stretched vertically by a factor of 3.

The equation of the new graph will be $y =$ _____

14.

Suppose the graph of $y = -4x^2 - 2x + 1$ is reflected across the x -axis.

The equation of the new graph will be $y =$ _____

15.

Suppose the graph of $y = -2x^2 - 4x + 4$ is reflected across the y -axis.

The equation of the new graph will be $y =$ _____

16.

Starting with the graph of $f(x) = 4^x$, write the formula for the function that results from

(a) shifting $f(x)$ 6 units upward. $y =$ _____

(b) shifting $f(x)$ 9 units to the right. $y =$ _____

(c) reflecting $f(x)$ about the y-axis. $y =$ _____

17.

The graph of the function $f\left(\frac{1}{8}x\right)$ can be obtained from the graph of $y = f(x)$

by one of the following actions:

- horizontally stretching the graph of $f(x)$ by a factor 8
- horizontally compressing the graph of $f(x)$ by a factor 8
- vertically stretching the graph of $f(x)$ by a factor 8
- vertically compressing the graph of $f(x)$ by a factor 8

18.

Describe a function $g(x)$ in terms of $f(x)$ if the graph of g is obtained by vertically stretching f by a factor of 9, then shifting the graph of f to the right 2 units and upward 2 units.

$g(x) = Af(x + B) + C$, where

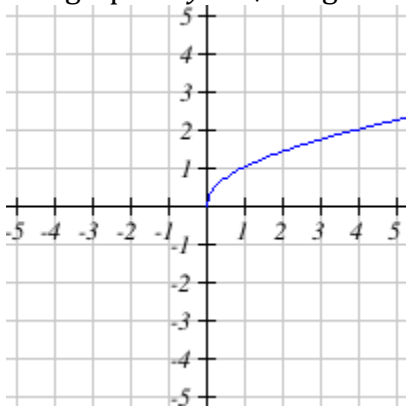
$A =$ _____

$B =$ _____

$C =$ _____

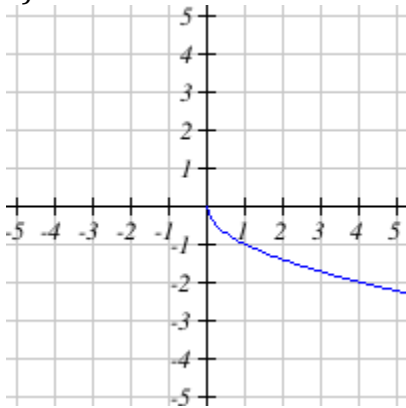
19.

The graph of $y = \sqrt{x}$ is given below:



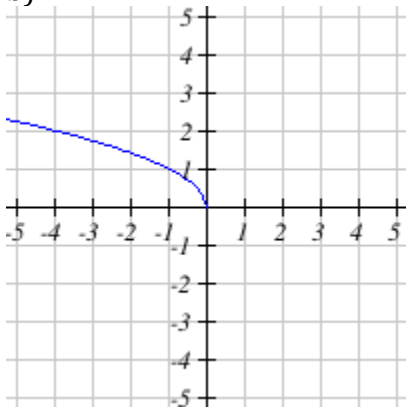
Find a formula for each of the transformations whose graphs are given below.
Recall that square root is entered as sqrt.

a)



$y =$ _____

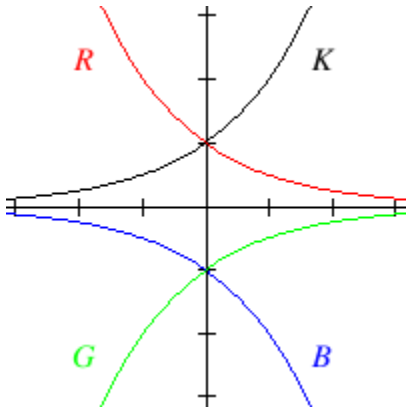
b)



$y =$ _____

20.

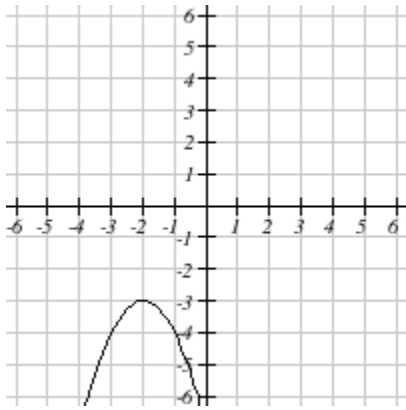
The graph of $f(x) = 2^x$ is shown in black (K). Match each transformation of this function with a graph below.



- 2^{-x}
- -2^{-x}
- -2^x

- a) red (R)
- b) blue (B)
- c) green (G)

21.

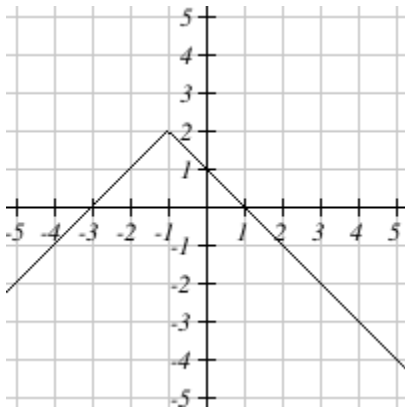


The graph above is a transformation of the function x^2 .

Give the function in the graph above.

$g(x) =$ _____

22.

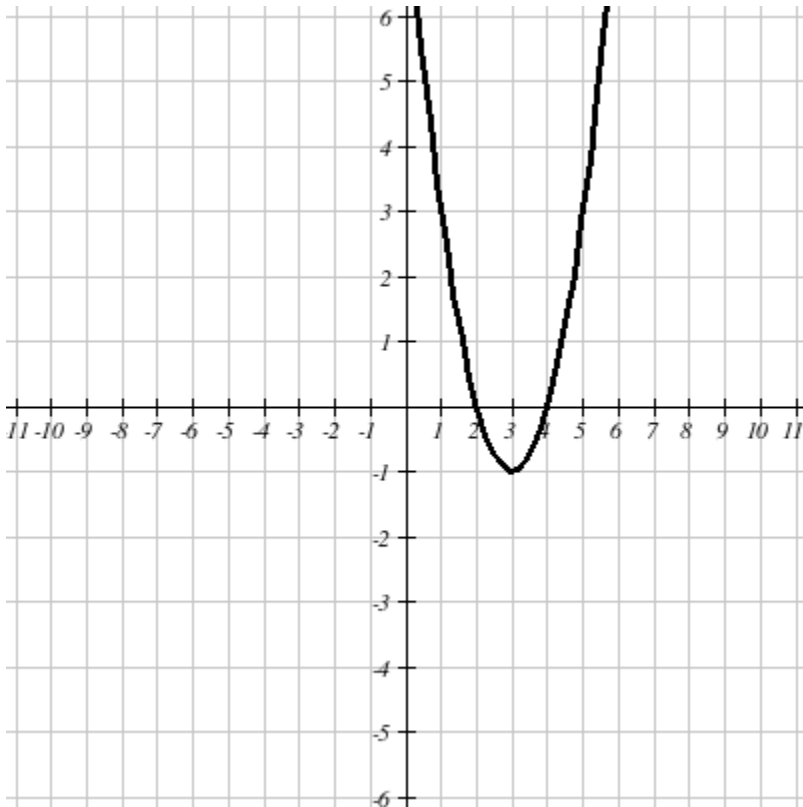


Write an equation for the function graphed above

$y =$ _____

23.

Consider the function $f(x) = (x - 3)^2 - 1$ graphed below: Graph the function $h(x)$, if $h(x)$ is the translation of f 3 units right and 2 units down.



Give the coordinates of the local minimum of $h(x)$ are: _____

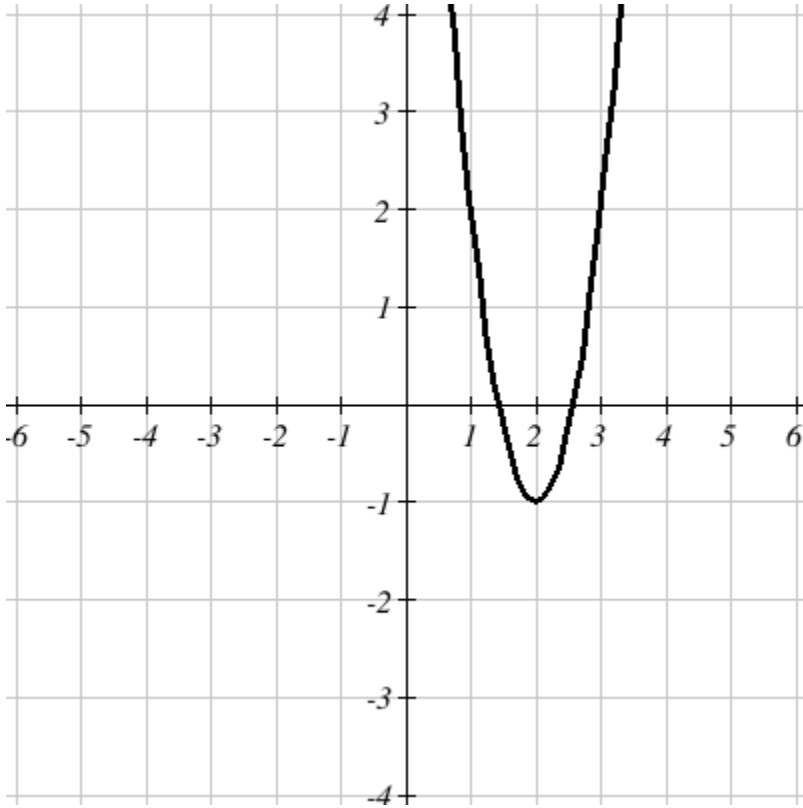
What is the formula for $h(x)$?

$h(x) =$ _____

24.

Consider the function $f(x) = 3(x - 2)^2 - 1$ passing through the point $(1,2)$.

Graph the function $h(x)$, if $h(x)$ is the reflection of f across the x-axis and y-axis

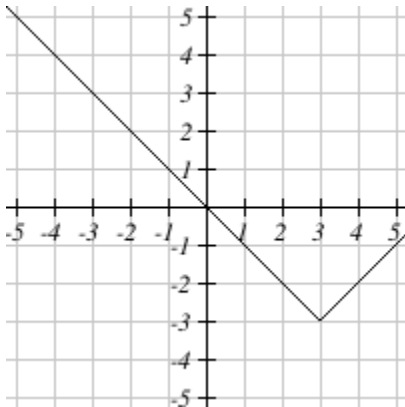


What is the coordinates of the local minimum of $h(x)$? _____

What is the formula of $h(x)$?

$h(x) =$ _____

25.

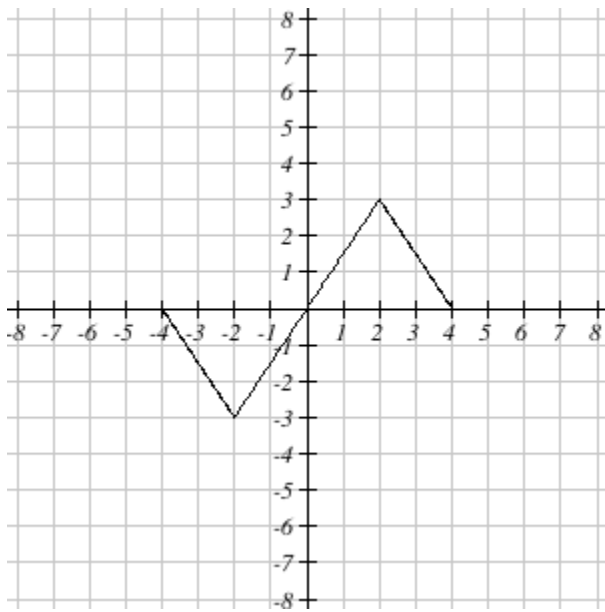


Complete an equation for the function graphed above

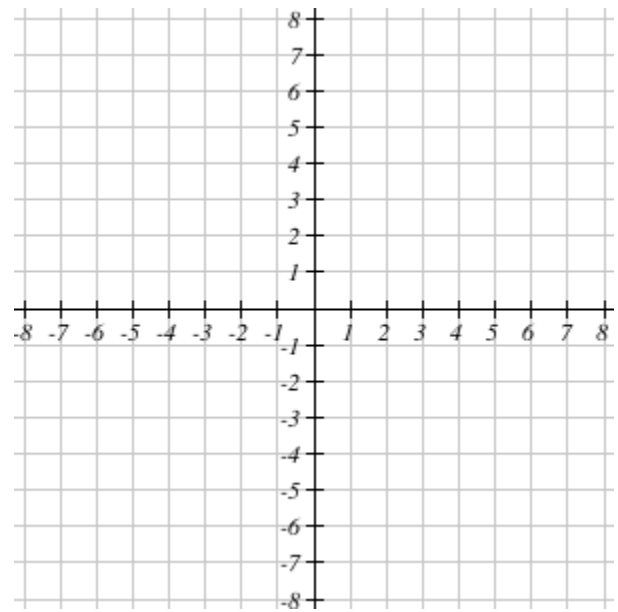
$y =$ _____

26.

The graph of $y = f(x)$ is shown below.



Draw the graph of $g(x) = f(x + 2)$ below.



27.

Determine the parent function from which the graph of the function shown below can be obtained. Next, identify each transformation that can be applied to the parent function in order to obtain the graph of the function shown below.

$$f(x) = 5\sqrt[3]{x + 7}$$

a) Choose the correct parent function.

- $y = x^2$
- $y = x^3$
- $y = |x|$
- $y = \sqrt{x}$
- $y = \sqrt[3]{x}$

b) Identify any reflections needed

c) Identify any stretch/compressions needed

d) Identify any vertical shift needed

e) Identify any horizontal shift needed

28.

Determine the parent function from which the graph of the function shown below can be obtained. Next, identify each transformation that can be applied to the parent function in order to obtain the graph of the function shown below.

$$g(x) = \frac{3}{4}\sqrt[3]{x+8} - 6$$

a) Choose the correct parent function.

- $y = x^2$
- $y = x^3$
- $y = |x|$
- $y = \sqrt{x}$
- $y = \sqrt[3]{x}$

b) Identify any reflections needed

c) Identify any stretch/compressions needed

d) Identify any vertical shift needed

e) Identify any horizontal shift needed

29.

Determine the parent function from which the graph of the function shown below can be obtained. Next, identify each transformation that can be applied to the parent function in order to obtain the graph of the function shown below.

$$g(x) = 8\sqrt[3]{-x} - 2$$

a) Choose the correct parent function.

- $y = x^2$
- $y = x^3$
- $y = |x|$
- $y = \sqrt{x}$
- $y = \sqrt[3]{x}$

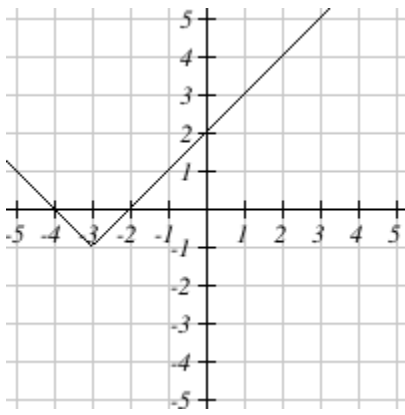
b) Identify any reflections needed

c) Identify any stretch/compressions needed

d) Identify any vertical shift needed

e) Identify any horizontal shift needed

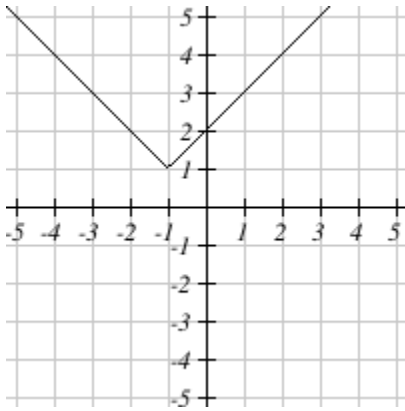
30.



The graph above is the graph of:

- $y = |x + 3| - 1$
- $y = |x - 1| - 3$
- $y = |x - 3| - 1$

31.

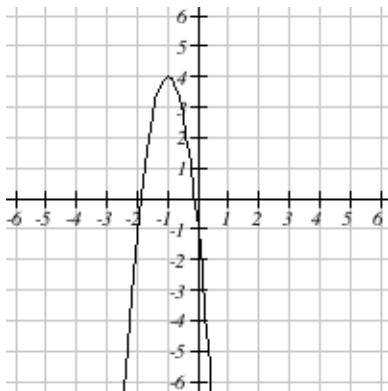
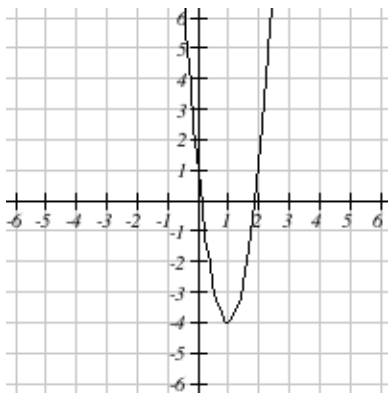
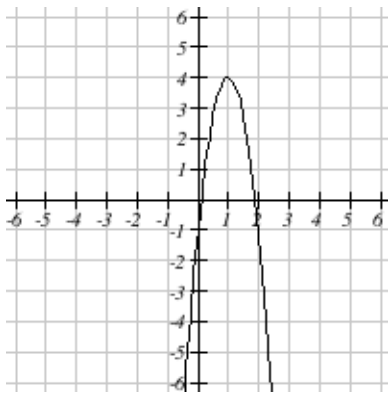
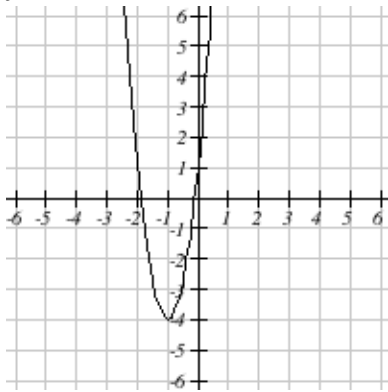


Write an expression for the function graphed above: _____

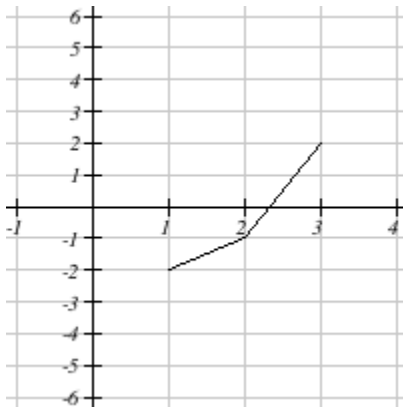
32.

Match the function with its graph.

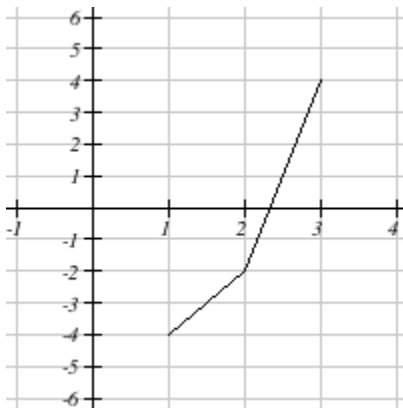
$$y = 5(x + 1)^2 - 4$$



33.



The graph **above** shows the function $f(x)$. The graph **below** shows $g(x)$.



$g(x)$ is a transformation of $f(x)$.

$g(x) = Af(Bx)$, where:

A = _____

B = _____

Lecture 9

Inverse of a relation, One-to-one functions, Horizontal Line test, Inverse functions, properties of inverses, Finding inverses graphically/algebraically, Restricting domain to get inverse

1.

Which of the following are one-to-one functions?

- $G = \{(-5,0), (-1,1), (-3,0), (9,1), (-4, -2), (3, -1)\}$
- $R = \{(-3,0), (1,3), (4,6), (9,7), (11,13), (15,16)\}$
- $F = \{(-3, -2), (0,0), (1,3), (6,5), (9,7), (16,13)\}$
- $S = \{(-5,0), (2, -6), (9,1), (-2, -7), (-3, -8), (8,4)\}$
- $M = \{(0, -2), (2,3), (2,5), (6,9), (7, -6), (16,10)\}$

2.

Select all of the following tables which represent y as a function of x **and** are one-to-one.

x	5	6	6
y	2	8	11

x	5	6	13
y	2	8	11

x	5	6	13
y	2	8	8

3.

Answer the following True or False:

If the graph of $y = f(x)$ passes the horizontal line test then $f(x)$ has an inverse.

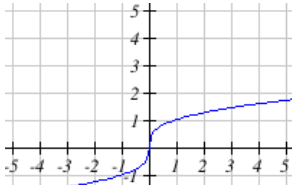
- True
- False

4.

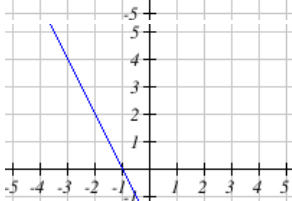
Why does the horizontal line test tell us whether the graph of a function is one-to-one?

- When a horizontal line intersects the graph of a relation more than once, it indicates that for that input there is more than one output, which means the relation is not one-to-one.
- When a horizontal line intersects the graph of a function more than once, it indicates that for that output there is more than one input, which means the function is not one-to-one.
- When a horizontal line intersects the graph of a relation only once, it indicates that for that output there is more than one input, which means the relation is not one-to-one.
- When a horizontal line intersects with the graph of a function more than once, it indicates that for that input there is more than one output, which means the function is not one to one.

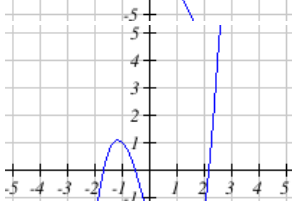
5. Select all of the following graphs which represent y as a function of x , and is **one-to-one**.



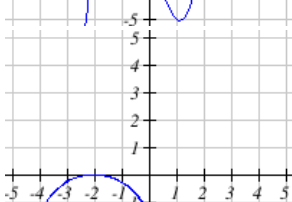
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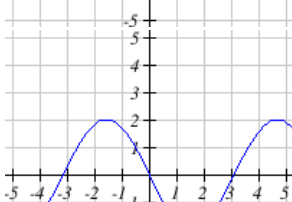
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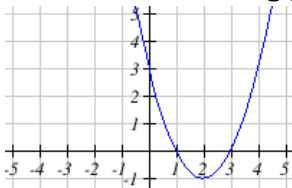
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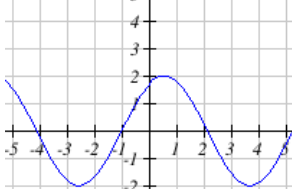
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6.

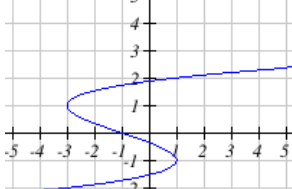
Select all of the following graphs which are **one-to-one** functions.



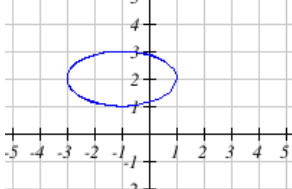
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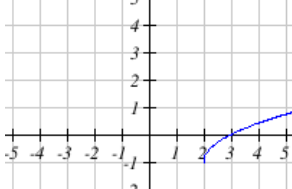
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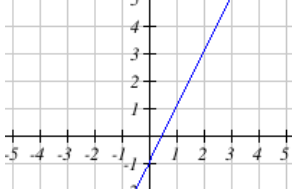
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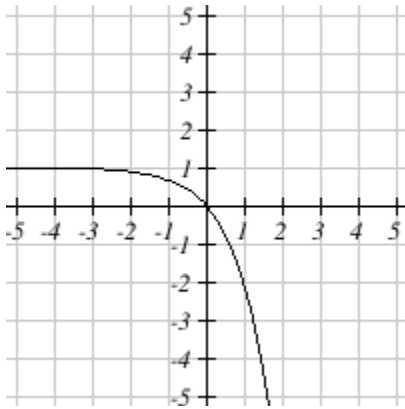
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7.

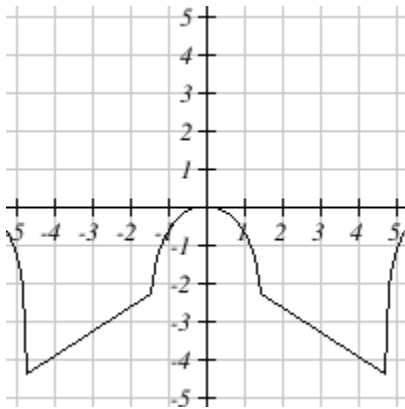
Determine if the given graph is a one-to-one function.



- This relation is not a one-to-one function
- This relation is a one-to-one function

8.

Determine if the given graph is a one-to-one function.



- This relation is a one-to-one function
- This relation is not a one-to-one function

9.

Answer the following True or False:

If the domain of $f(x)$ is $(3,11)$ then the domain of $f^{-1}(x)$ is also $(3,11)$.

- True
- False

10.

Answer the following True or False:

If $f(x) = \frac{1}{6x^2+2x-4}$ then the inverse of $f(x)$ is $f^{-1}(x) = 6x^2 + 2x - 4$.

- True
- False

11.

If f is one-to-one and $f(-5) = 6$, then

$$f^{-1}(6) = \underline{\hspace{2cm}} \text{ and } (f(-5))^{-1} = \underline{\hspace{2cm}}.$$

12.

Assume that the function f is a one-to-one function.

(a) If $f(6) = 4$, then $f^{-1}(4) = \underline{\hspace{2cm}}$.

(b) If $f^{-1}(-4) = -3$, then $f(-3) = \underline{\hspace{2cm}}$.

13.

Use the table below to fill in the missing values.

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	4	9	0	3	8	6	7	2	5	1

$$f(5) = \underline{\hspace{2cm}}$$

$$\text{if } f(x) = 8, \text{ then } x = \underline{\hspace{2cm}}$$

$$f^{-1}(6) = \underline{\hspace{2cm}}$$

$$\text{if } f^{-1}(x) = 4, \text{ then } x = \underline{\hspace{2cm}}$$

14.

Below is the table for the function $f(x)$.

x	1	4	9	11	16
y	3	5	8	12	15

Choose the one table below which is the inverse function $f^{-1}(x)$.

x	1/3	1/5	1/8	1/12	1/15
y	3	5	8	12	15

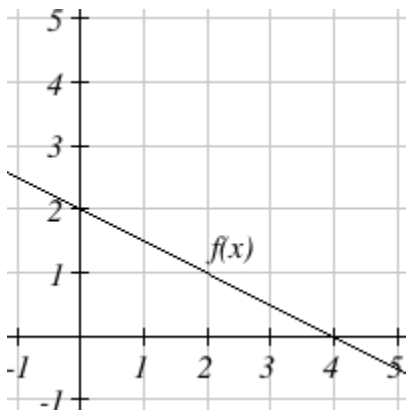
x	3	5	8	12	15
y	1	4	9	11	16

x	1	4	9	11	16
y	1/3	1/5	1/8	1/12	1/15

x	16	11	9	4	1
y	15	12	8	5	3

15.

Use the graph below to fill in the missing values.



$f(0) = \underline{\hspace{2cm}}$

If $f(x) = 0$, $x = \underline{\hspace{2cm}}$

$f^{-1}(0) = \underline{\hspace{2cm}}$

If $f^{-1}(x) = 0$, $x = \underline{\hspace{2cm}}$

16.

If $f(x) = x + 3$ and $g(x) = x - 3$,

(a) $f(g(x)) = \underline{\hspace{2cm}}$

(b) $g(f(x)) = \underline{\hspace{2cm}}$

(c) Thus $g(x)$ is called an $\underline{\hspace{2cm}}$ function of $f(x)$

17.

Are the following functions inverses?

$$f(x) = 4x + 5$$

$$g(x) = \frac{x}{4} - 5$$

- Yes, they are inverses
- No, they are not inverses

18.

Are the following functions inverses?

$$f(x) = \sqrt[3]{x - 1}$$

$$g(x) = (x + 1)^3$$

- Yes, they are inverse
- No, they are not inverses

19.

Are the following functions inverses?

$$f(x) = \sqrt[3]{x + 6}$$

$$g(x) = x^3 - 6$$

- No, they are not inverses
- Yes, they are inverse

20.

Let $f(x) = 9 - x$

$$f^{-1}(x) = \underline{\hspace{2cm}}$$

21.

Let $f(x) = 3x + 2$

$f^{-1}(x) = \underline{\hspace{2cm}}$

22.

Find the inverse function of $f(x) = 4 + \sqrt[3]{x}$.

$f^{-1}(x) = \underline{\hspace{2cm}}$

23.

Find the inverse function of $f(x) = 6 + \sqrt[3]{x}$.

$f^{-1}(x) = \underline{\hspace{2cm}}$

24.

(a) Find the inverse function of $f(x) = 9x - 6$.

$f^{-1}(x) = \underline{\hspace{2cm}}$

(b) The graphs of f and f^{-1} are symmetric with respect to the line defined by $y = \underline{\hspace{2cm}}$

25.

Use algebra to find the inverse of the function

$f(x) = x^8, x \geq 0, y \geq 0$

The inverse function is $f^{-1}(x) = \underline{\hspace{2cm}}$ where $x \geq 0, y \geq 0$

26.

Use algebra to find the inverse of the function

$f(x) = x^3$

The inverse function is $f^{-1}(x) = \underline{\hspace{2cm}}$

27.

Use algebra to find the inverse of the function

$f(x) = \frac{1}{4}x$

The inverse function is $f^{-1}(x) = \underline{\hspace{2cm}}$

28.

Use algebra to find the inverse of the function $f(x) = 5x^9 - 2$

The inverse function is $f^{-1}(x) =$ _____

29.

Let $f(x) = (x - 8)^2$

Find a domain on which f is one-to-one and non-decreasing: _____

Find the inverse of f restricted to this domain.

$f^{-1}(x) =$ _____

30.

Find the inverse function of the function $f(x) = -5x^2$ if $x \geq 0$.

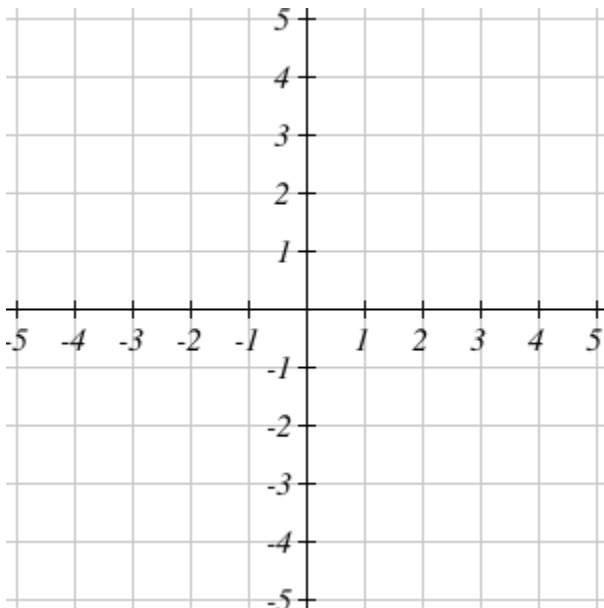
$f^{-1}(x) =$ _____ where _____ (give the domain of the inverse function in this second answer box)

31.

Find the inverse of the function $f(x) = 4x - 1$.

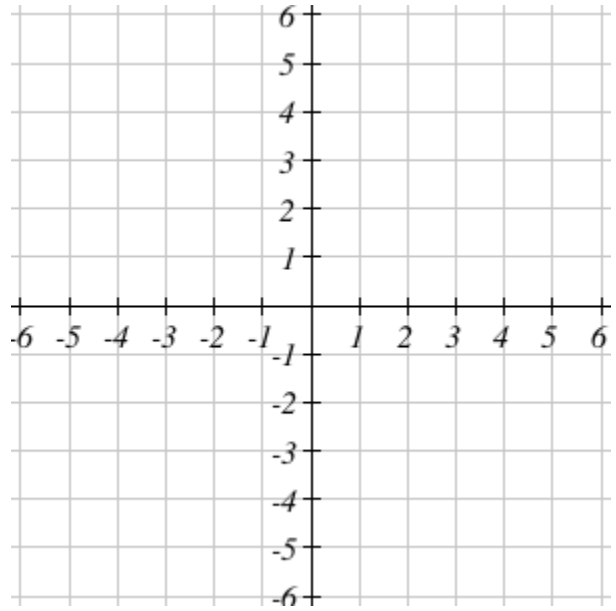
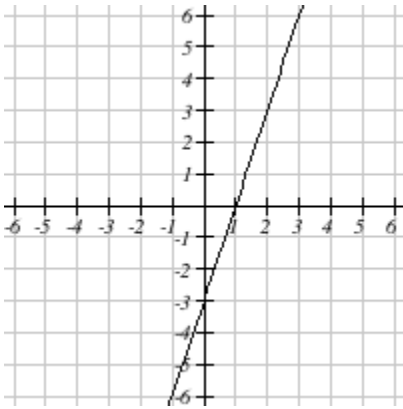
$f^{-1}(x) =$ _____

Draw the functions f and f^{-1}



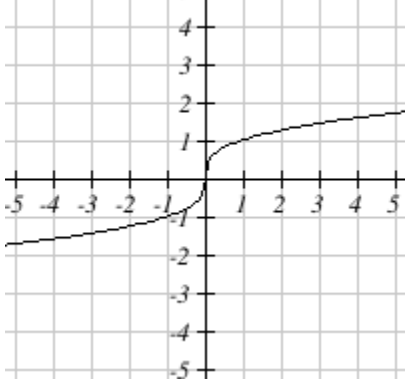
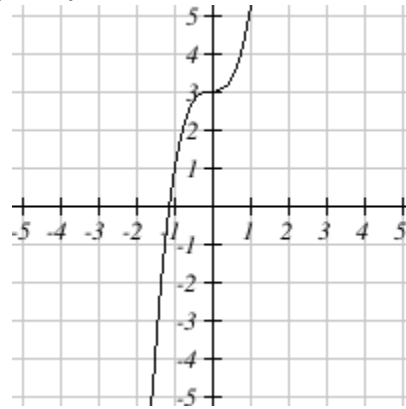
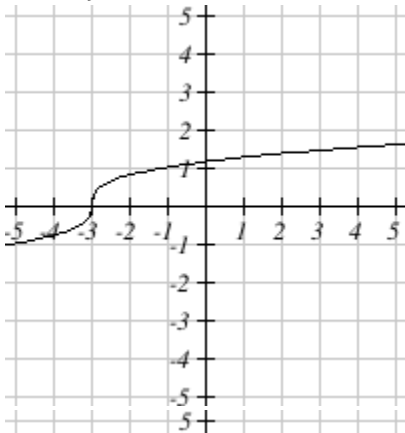
32.

The graph of $y = f(x)$ is shown. Sketch the graph of $y = f^{-1}(x)$.



33.

Given that $f(x) = 2x^3 - 3$, which of the following is the graph of $f^{-1}(x)$?



•

•

•

Lecture 10

Slope, Rate of change, Horiz/Vert Lines, Slope-Int, Pt-Slope, Parallel/Perpendicular

1.

Find the slope between the points $(-4, -3)$ and $(1, 3)$. Express your answer as an integer or reduced fraction.

2.

Find the slope between the points $(\frac{2}{11}, \frac{10}{7})$ and $(\frac{13}{11}, \frac{13}{7})$

3.

Find the slope between the points $(6, 1)$ and $(9, 1)$. Answer DNE if the slope between the points is undefined.

4.

Find the slope between the points $(-10, 5)$ and $(-10, 8)$. Answer DNE if the slope between the points is undefined.

5.

Give two points with integer coordinates that have a slope of $\frac{3}{5}$ between them.

First point: (_____ , _____)

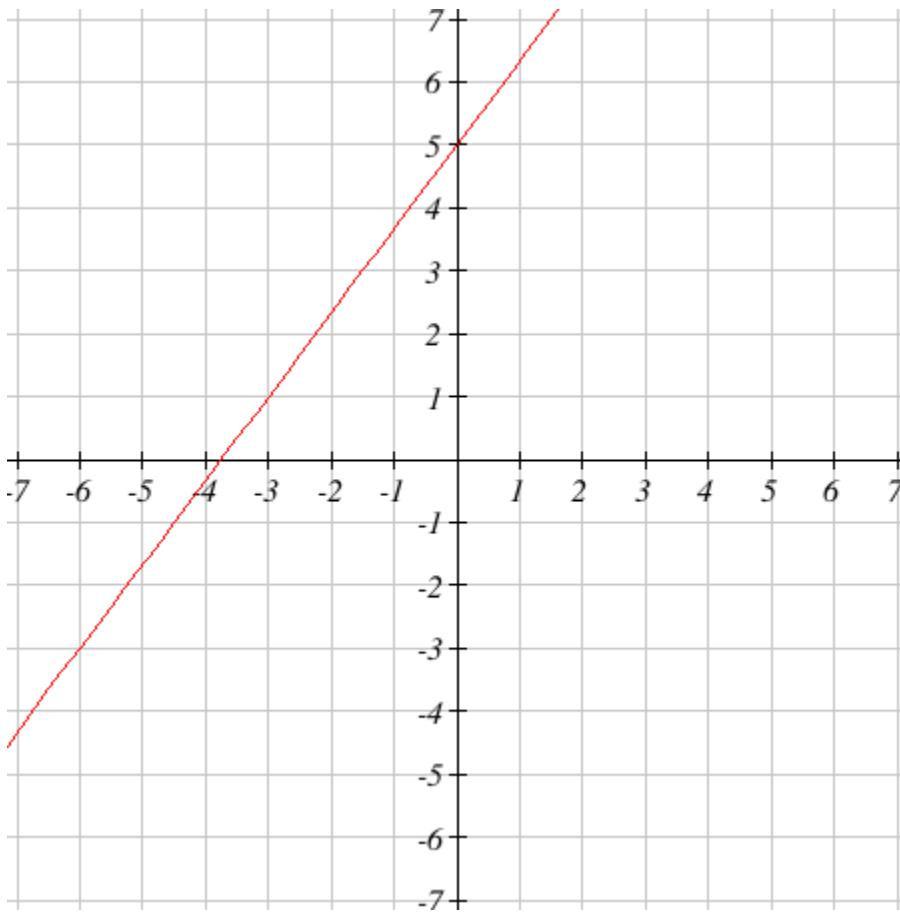
Second point: (_____ , _____)

6.

Is the point $(18, -77)$ on the graph of the function $f(x) = -4x - 5$?

- Yes
- No
- It is impossible to determine

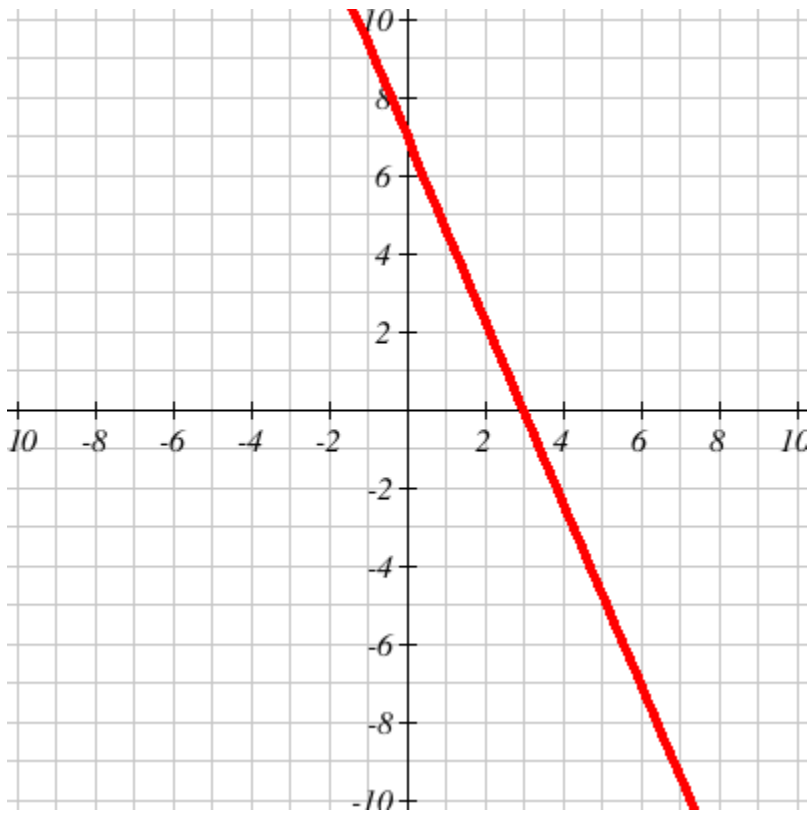
7.
Find the slope of the line graphed below. Give your answer as an integer or as a reduced fraction.



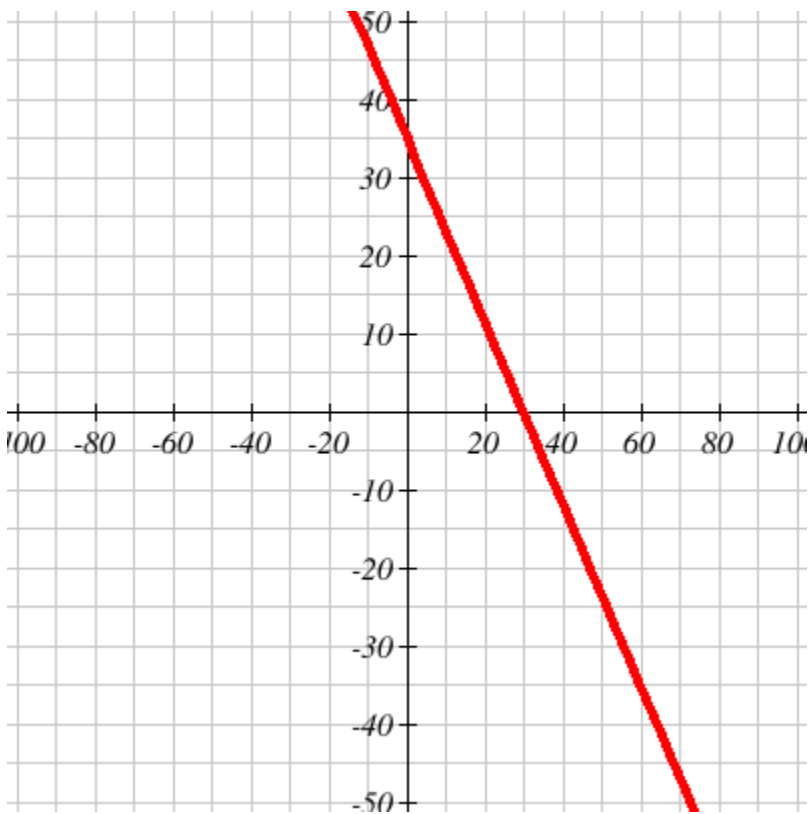
The slope of the line is _____

8.

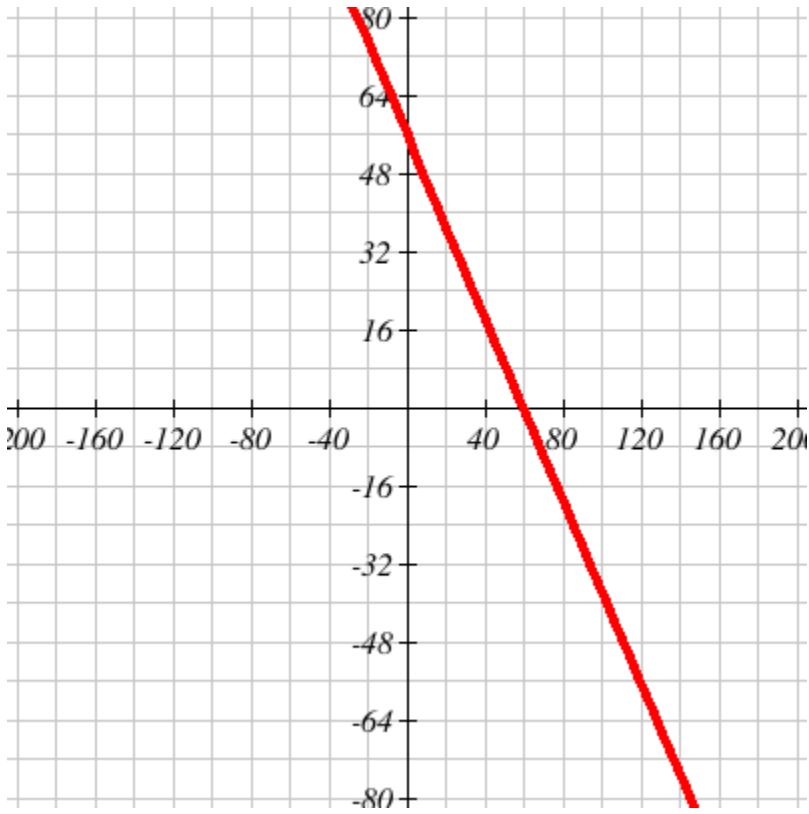
Find the slope of each line graphed below. Give your answers as reduced fractions.



The slope of this line is _____

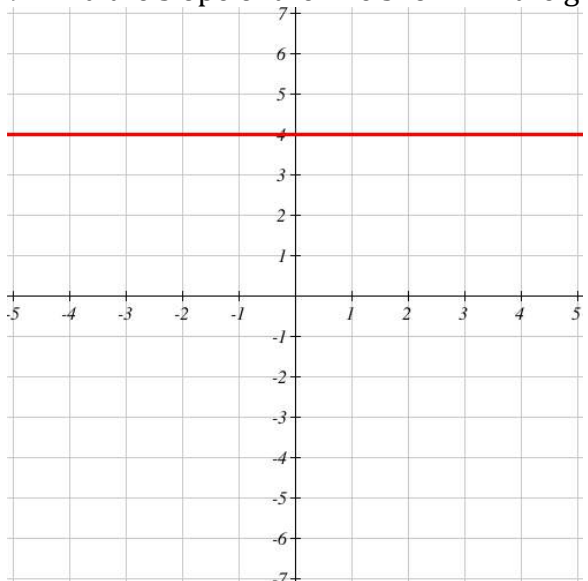


The slope of this line is _____



The slope of this line is _____

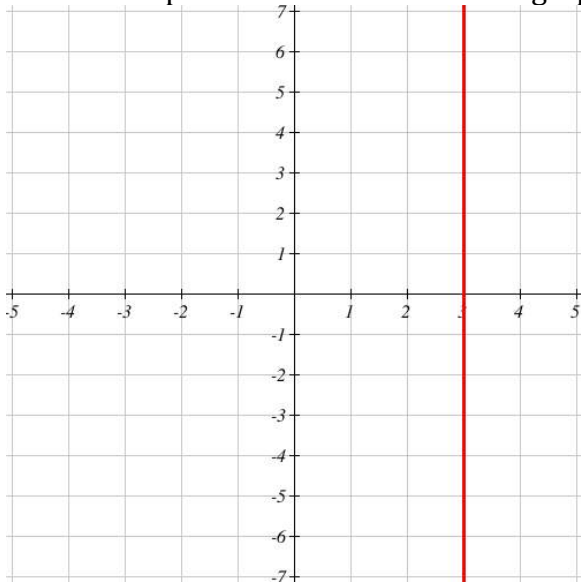
9. Find the slope of the line shown in the graph below. Answer DNE if the slope does not exist.



The slope is: _____

10.

Find the slope of the line shown in the graph below. Answer DNE if the slope does not exist



The slope is: _____

11.

Suppose you are driving. You notice that after driving for 3 hours, you are 135 miles from Seattle. You continue driving, and calculate that after driving 5 hours you are 215 miles from Seattle.

What was your rate of travel between these two observations?

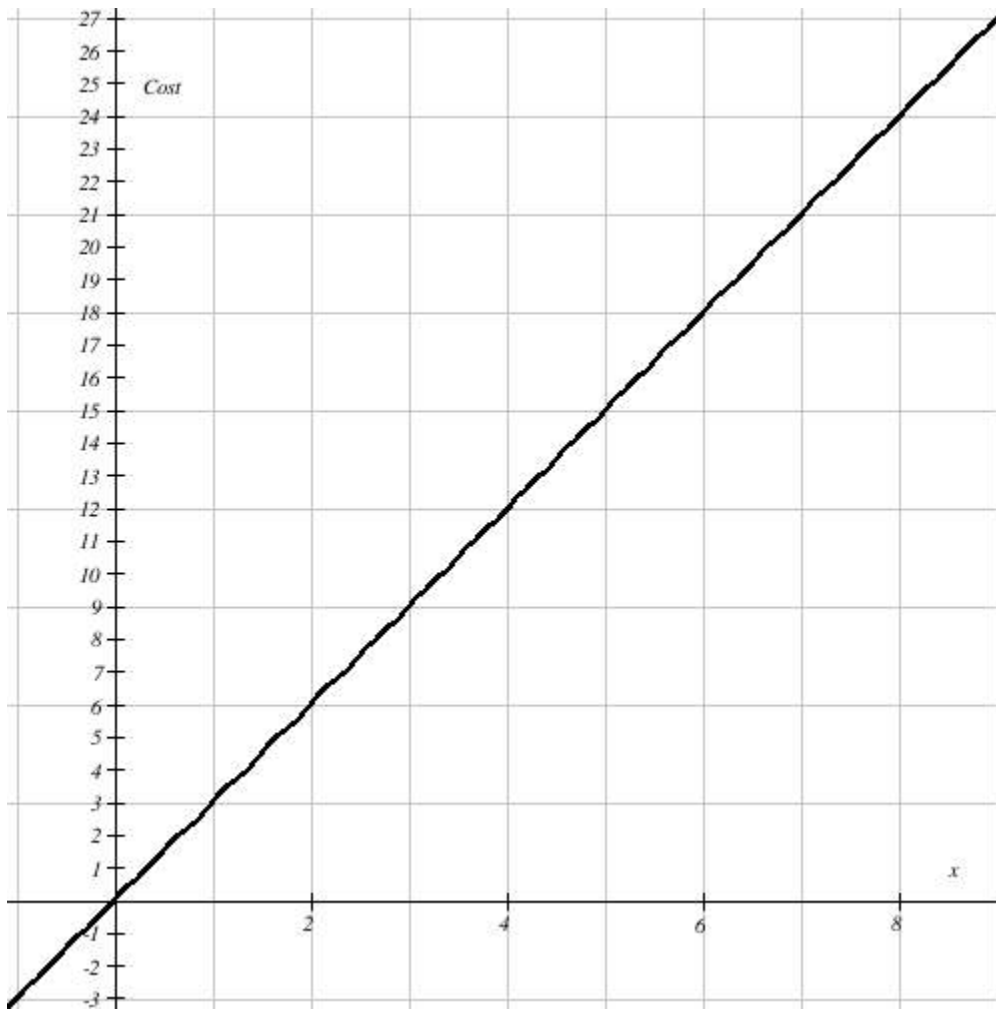
_____ miles per hour

12.

Which of the following rates are equivalent to the rate 11 pounds per 2 months?

- 66 pounds per year
- $\frac{11}{2}$ pounds per month
- 5.5 pounds per month
- 55 pounds every 10 months
- one pound per $\frac{2}{11}$ months

13.
The graph below shows the cost of buying x square yards of carpet. Find the slope of the graph and interpret the result.



The slope of the graph is

What does the slope represent?

- The cost per square yard of carpet
- The size of the carpet
- The cost per hour of installation
- The cost of installation
- The total cost of the carpet

14.

Is the function $f(x)$ shown in the table below a linear function?

x	-2	-1	0	1	2	3
f(x)	8	5	2	-1	-4	-7

- No, the ratio of change in input to change in output is not constant
- Yes, the ratio of change in input to change in output is constant
- Yes, each input has exactly one output
- No, there are multiple outputs for at least one input
- Triceratops

15.

Is the function $f(x)$ shown in the table below a linear function?

X	-1	0	1	2	3	4
f(x)	-3	1	-3	-15	-35	-63

- No, there are multiple outputs for at least one input
- Yes, the ratio of change in input to change in output is constant
- Yes, each input has exactly one output
- No, the ratio of change in input to change in output is not constant
- Wombat

16.

Give three different points that lie on the line $y = 4x + 2$ by choosing different values for x .
Note that you must answer all three questions in order for your responses to be scored correctly.

If x is _____, then the point on the line is _____.

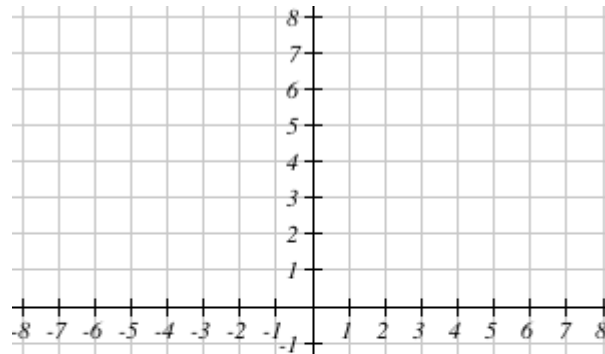
If x is _____, then the point on the line is _____.

If x is _____, then the point on the line is _____.

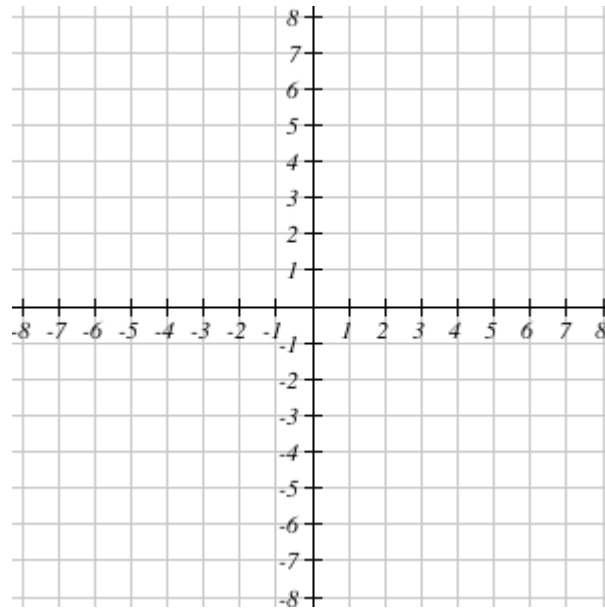
17.

Draw the graph of each of the 5 equations below. Make sure to scroll down to see all the graphs.

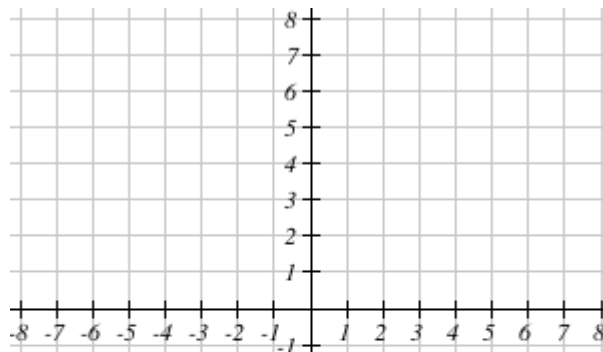
$$h(x) = -\frac{5}{3}x + 2$$



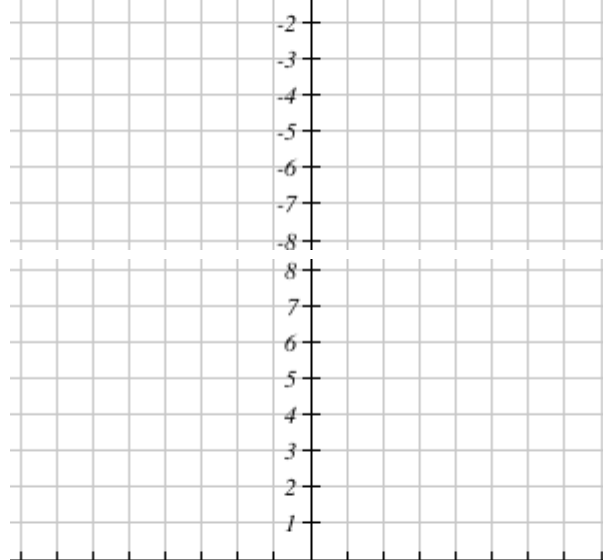
$$g(x) = 0$$



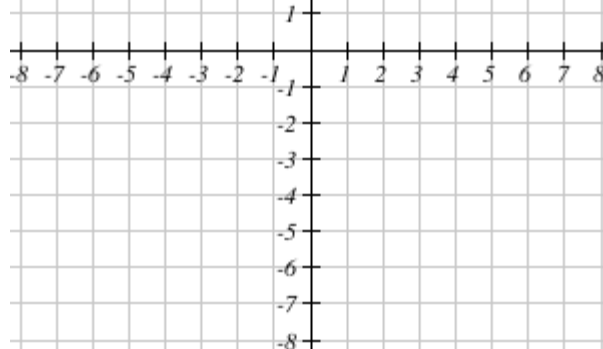
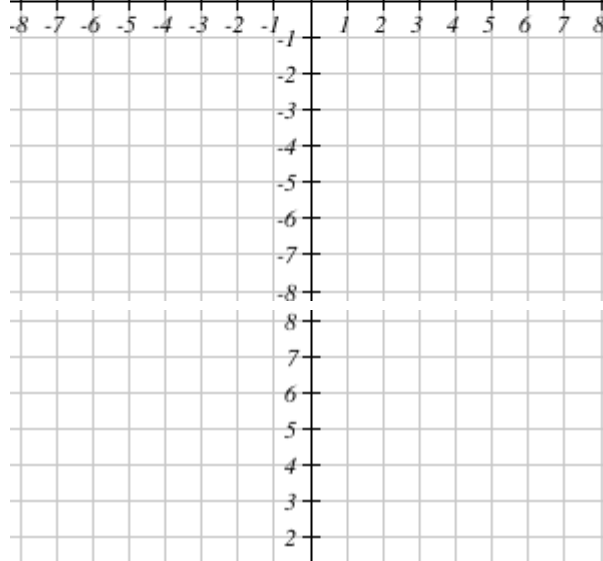
$$x = 5$$



$$k(x) = -\frac{3}{5}x + 5$$

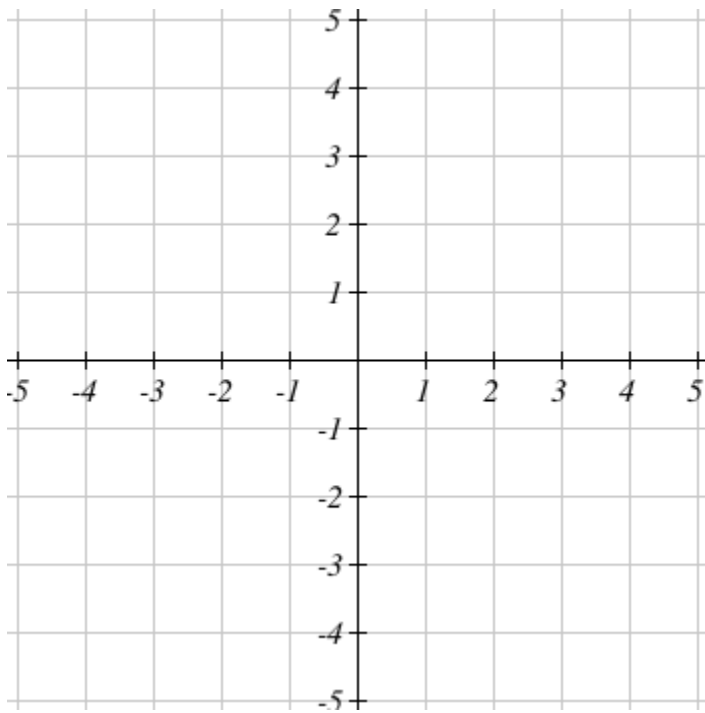


$$q(x) = -\frac{2}{3}x - 5$$



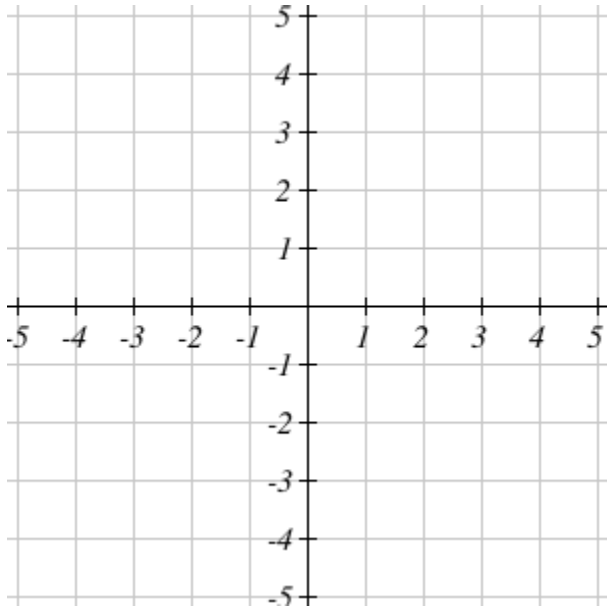
18.

Graph the line that has an x -intercept of $(2,0)$ and y -intercept of $(0,3)$.



19.

Sketch a graph of $f(x) = -\frac{1}{2}x + 2$.

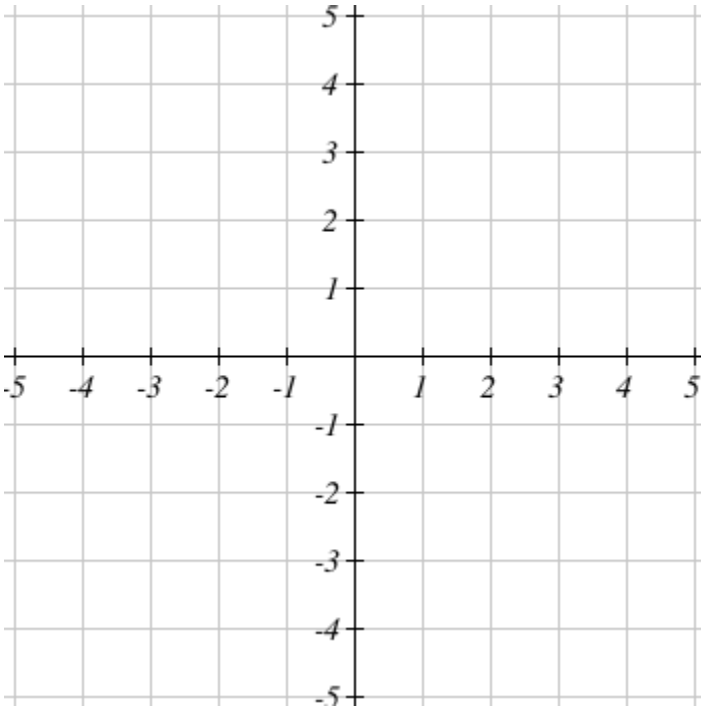


20.

Find the x -intercept and y -intercept of the graph of $4x + 2y = 8$. Then graph.

x -intercept = (_____, 0)

y -intercept = (0, _____)



21.

Given the equation $y = -2x - 3$

The x intercept is: _____

The y intercept is: _____

22.

Find the x and y intercepts of the equation: $-2x - 5y = 30$

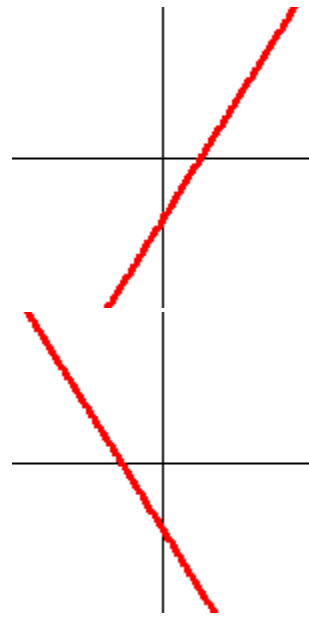
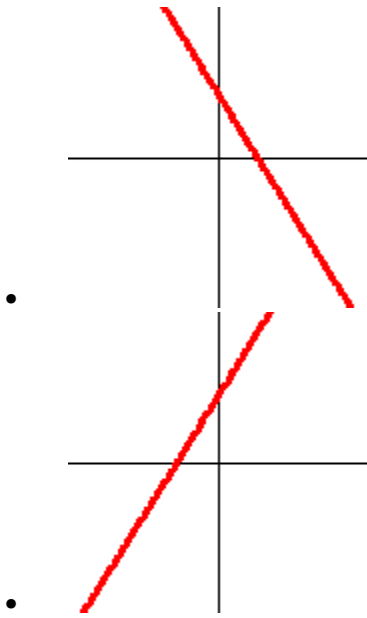
The intercepts are: _____

x intercept = (_____, 0)

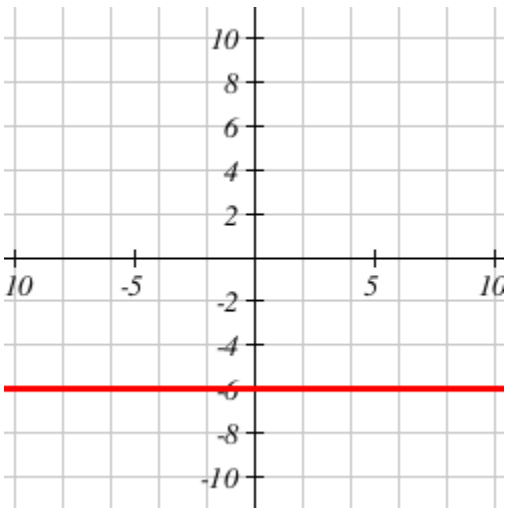
y intercept = (0, _____)

23.

The graphs shown below correspond to functions of the form $f(x) = mx + b$. Which of the graphs has $m > 0$ and $b < 0$?

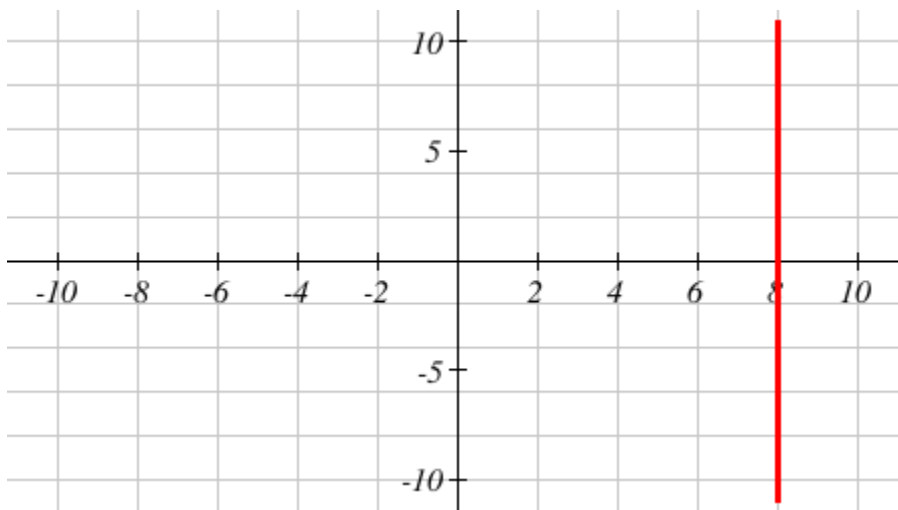


24.



Find the equation of the line shown.

25.



Find the equation of the line shown.

26.

Construct a linear function that passes through the point $(3,15)$. Give your answer in slope-intercept form and use rational numbers or integers.

$f(x) =$ _____

27.

Put the following equation in slope-intercept form: $y - 3 = -\frac{1}{6}(x - 1)$

$y =$ _____

28.

Find the equation for the line that passes through the points $(-5, -8)$ and $(-8, -9)$. Give your answer in the point-slope form $y = m(x - x_1) + y_1$.

$y =$ _____

29.

Find the equation of the line with slope = 5 and passing through $(7,-3)$. Write the equation in point-slope form AND slope-intercept forms. Include the full equation in your answers.

point-slope form: _____

slope-intercept form: _____

30.

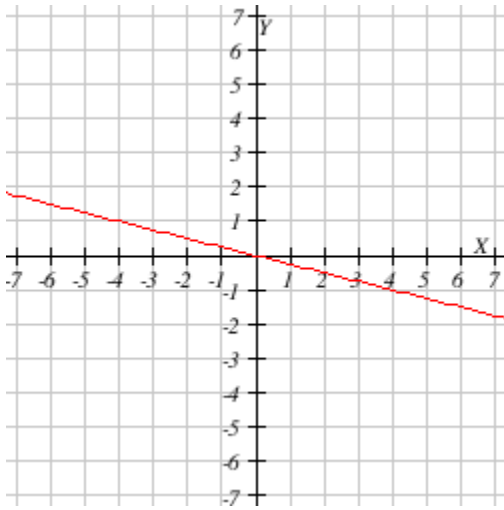
A line with equation $y = mx + b$ passes through the origin and the point $(7, -8)$. Find the values of m and b .

$m =$ _____

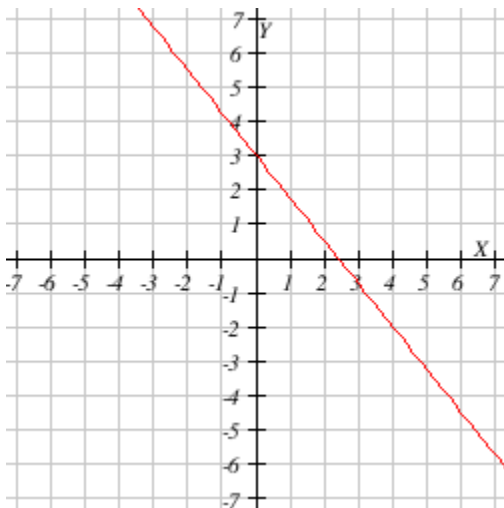
$b =$ _____

31.

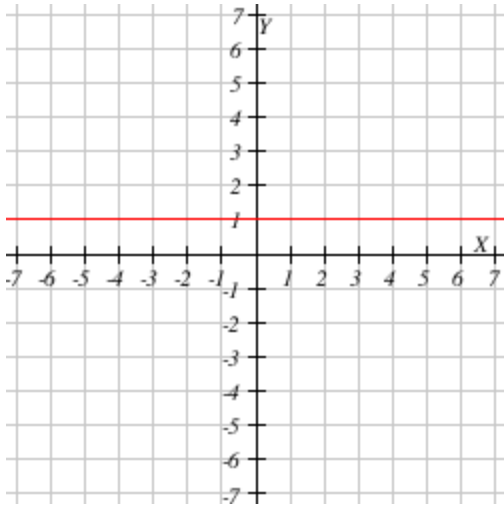
Give the equation of each line in the 5 graphs below. Make sure to scroll down to see all the graphs. Give your answers in slope-intercept form unless the line is vertical.



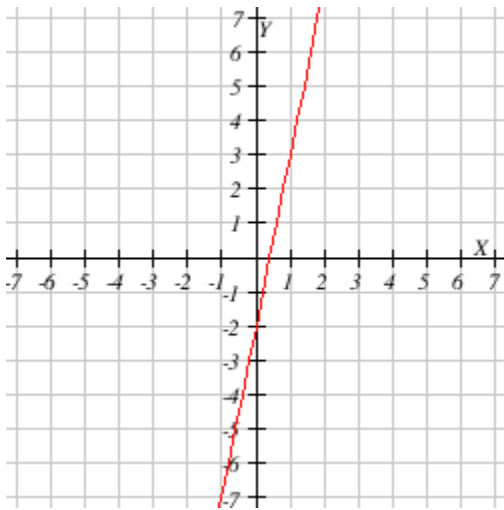
The equation of the line is:



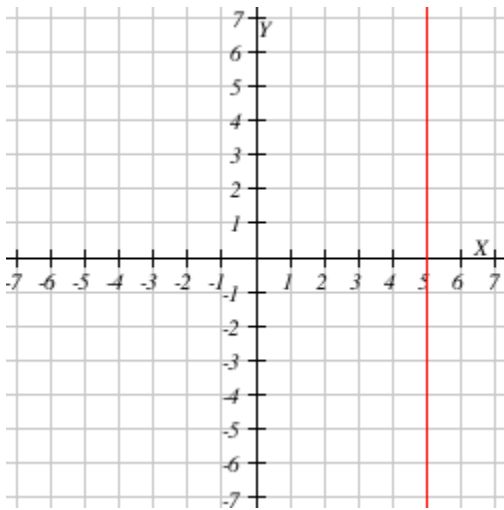
The equation of the line is:



The equation of the line is:



The equation of the line is:



The equation of the line is:

32.

Write an equation for a line parallel to $y = 4x - 2$ and passing through the point $(1,1)$.

$$y = \underline{\hspace{2cm}}$$

33.

Construct two different linear functions $f(x)$ and $g(x)$ that are parallel to each other. Write them below in slope-intercept form.

$$f(x) = \underline{\hspace{2cm}}$$

$$g(x) = \underline{\hspace{2cm}}$$

34.

Two lines are perpendicular if..

- they have the same y-intercept
- they are not parallel
- they intersect at a right angle
- they never intersect
- they both own cats. Oh wait, that's purrrpendicular

35.

Write an equation for a line perpendicular to $y = -5x + 3$ and passing through the point $(-15,1)$.

$$y = \underline{\hspace{2cm}}$$

36.

Find an equation of the line that is perpendicular to $8x - 10y = -4$ and has a y-intercept of 1. Write your answer in slope-intercept form.

$$y = \underline{\hspace{1cm}}x + \underline{\hspace{1cm}}$$

37.

Construct two different linear functions $f(x)$ and $g(x)$ that are perpendicular to each other and have nonzero y-intercepts. Write them below in slope-intercept form.

$$f(x) = \underline{\hspace{2cm}}$$

$$g(x) = \underline{\hspace{2cm}}$$

38.

John is on vacation and needs to arrange transportation. A rental car costs 70 dollars per day plus a one-time cost of 20 dollars for insurance. Construct a function $C(x)$ that gives the total cost of renting a car for x days.

$$C(x) = \underline{\hspace{2cm}}$$

If John has budgeted 370 dollars for the rental, how many days can he afford?

 days

39.

In 1991, the moose population in a park was measured to be 4040. By 1999, the population was measured again to be 4680. If the population continues to change linearly:

A.) Find a formula for the moose population, P , in terms of t , the years since 1990.

$$P(t) = \underline{\hspace{2cm}}$$

B.) What does your model predict the moose population to be in 2002?

40.

Depreciation is the decrease or loss in value of an item due to age, wear, or market conditions. We usually consider depreciation on expensive items like cars. Businesses use depreciation as a loss when calculating their income and taxes.

One company buys a new bulldozer for \$131500. The company depreciates the bulldozer linearly over its useful life of 25 years. Its salvage value at the end of 25 years is \$19000.

A) Express the value of the bulldozer, V , as a function of how many years old it is, t . Make sure to use function notation.

B) The value of the bulldozer after 2 years is \$.

Lecture 11

Quadratic function general form, graphs(parabolas), intercepts, vertex, standard form (and converting general/standard), domain/range, axis of symmetry, applications

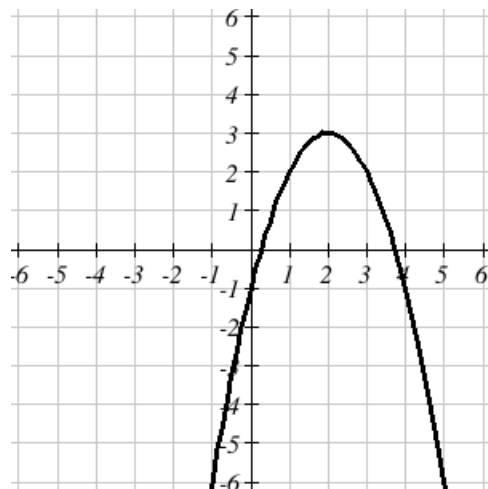
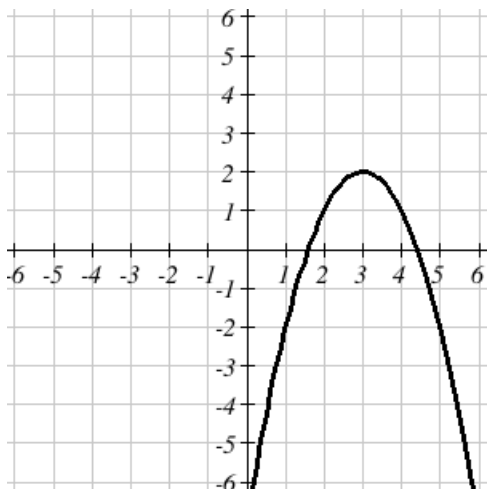
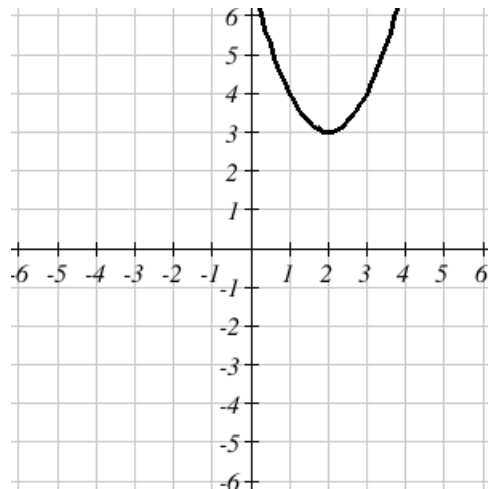
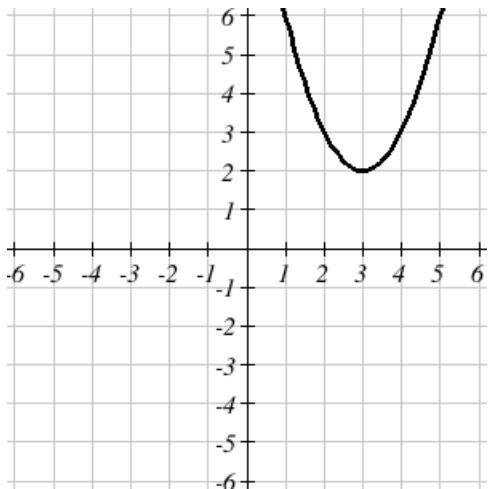
1.

The vertex of a parabola is _____

- the highest or lowest point on a parabola
- the point where the parabola intersects its axis of symmetry
- the point where the parabola changes from increasing to decreasing
- all of the above
- none of the above

2.

Which of these parabolas opens downwards and has its vertex at the point (2,3)?



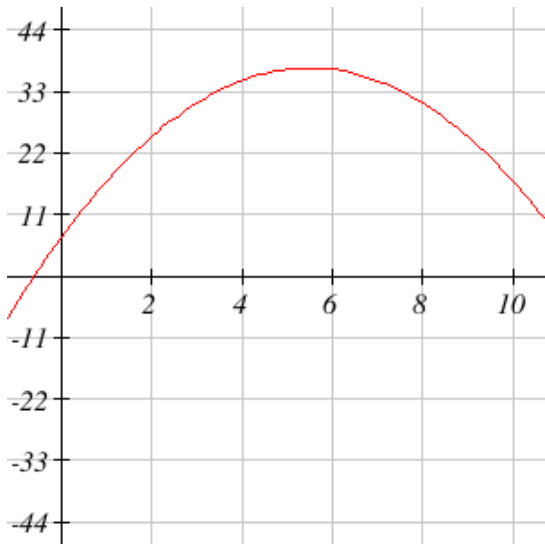
3.

Describe how the graph of $g(x) = -1.6x^2$ compares to the graph of $f(x) = x^2$.

The graph of $g(x)$ is (wider/narrower) than the graph of $f(x)$.

The graph of $g(x)$ opens in the (opposite/same) direction as the graph of $f(x)$.

4.

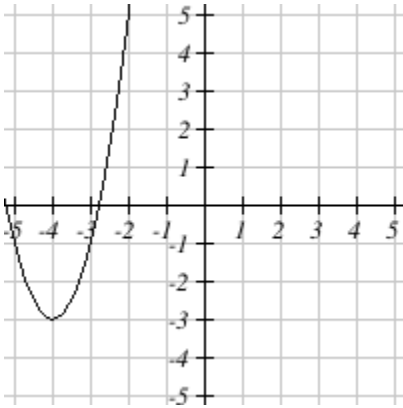


Use the y-intercept, vertex, and shape of the graph to determine which of the choices below is the equation for the parabola.

- $y = x^2 + 11x + 7$
- $y = x^2 - 11x - 7$
- $y = -x^2 + 11x - 7$
- $y = -x^2 - 11x + 7$
- $y = x^2 - 11x - 7$
- $y = -x^2 + 11x + 7$

5.

Consider the function graphed below.



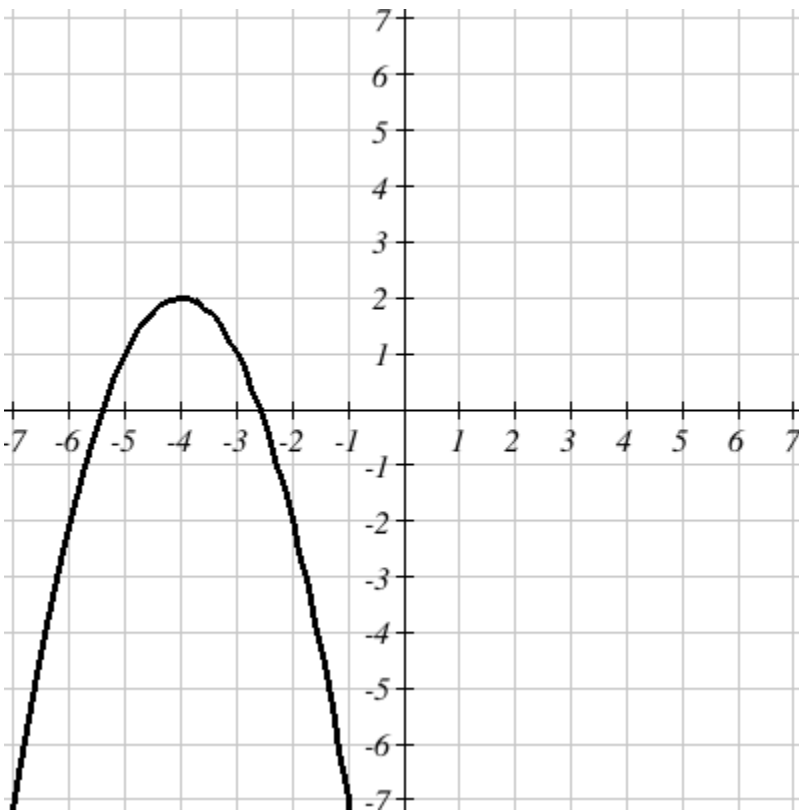
The function has a Select an answer minimum maximum value of at $x =$ _____

The function is increasing on the interval(s): _____

The function is decreasing on the interval(s): _____

6.

For the parabola graphed below, identify its vertex, state the intervals where it is increasing and decreasing.



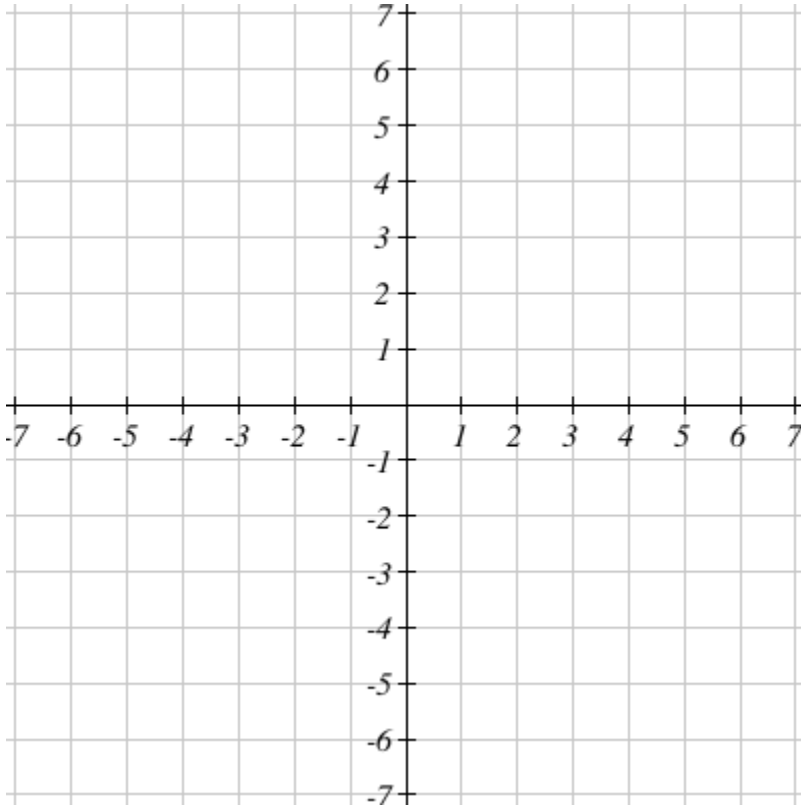
The vertex of the parabola is _____

The parabola is increasing on the interval _____

The parabola is decreasing on the interval _____

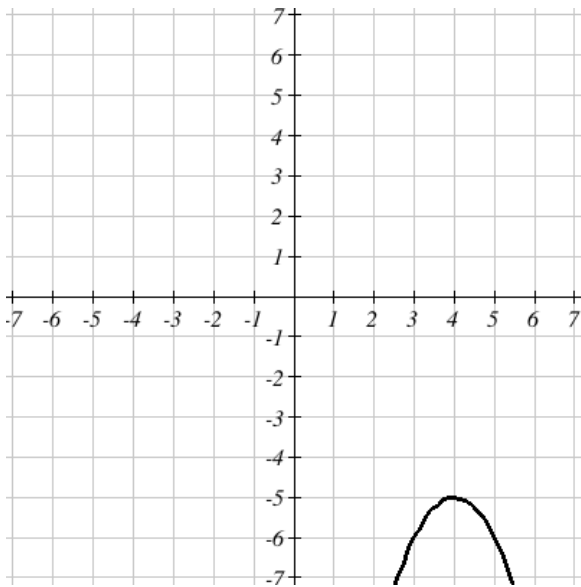
7.

Graph $f(x) = x^2 - 8x + 15$ below.



8.

For the parabola graphed below, identify its vertex, axis of symmetry, and state if it opens upwards or downwards.



The vertex of the parabola is _____

The axis of symmetry is _____

The parabola opens Select an answer downwards upwards _____

9.

Give three different points that lie on the parabola $y = x^2 + 5x + 4$.

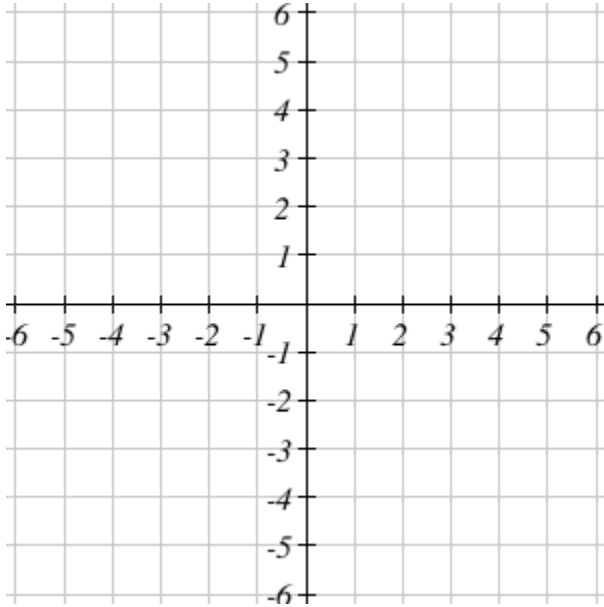
If x is _____, then the point on the parabola is _____.

If x is _____, then the point on the parabola is _____.

If x is _____, then the point on the parabola is _____.

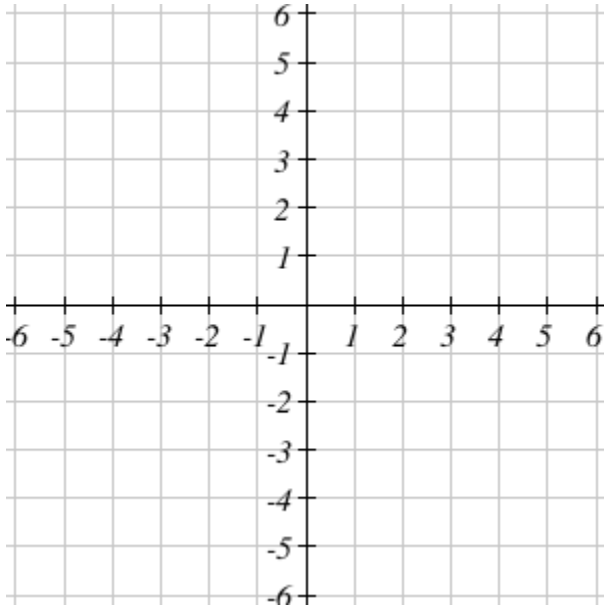
10.

Sketch a graph of $f(x) = -x^2 + 1$



11.

Sketch a graph of $f(x) = -2x^2 - 2$

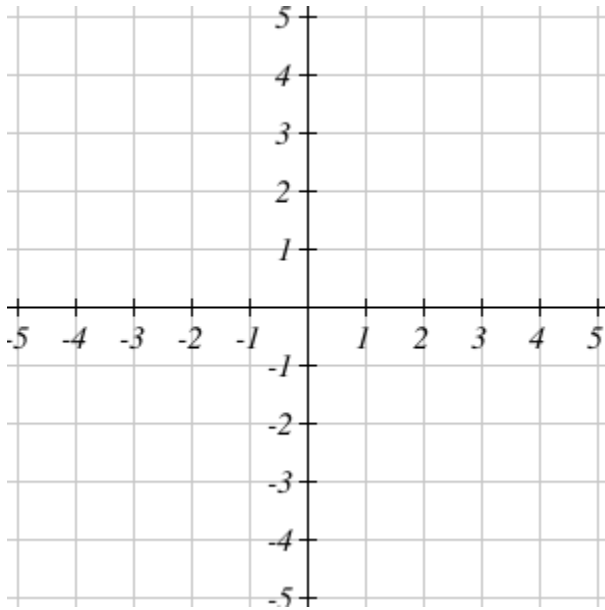


12.

Identify the vertex, and the y -intercept then graph $f(x) = x^2 + 2x + 2$.

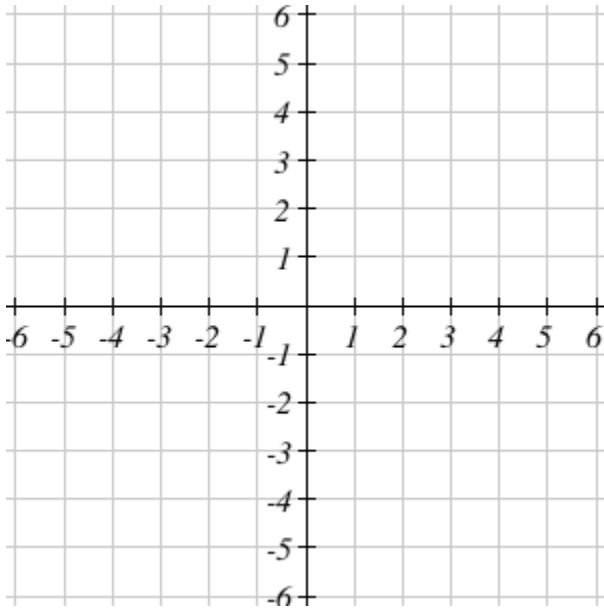
The vertex is (_____, _____)

The y -intercept is (0, _____)



13.

Graph the equation $y = x^2 - 4x + 1$ below and then state its domain and range.



The domain of the equation is:

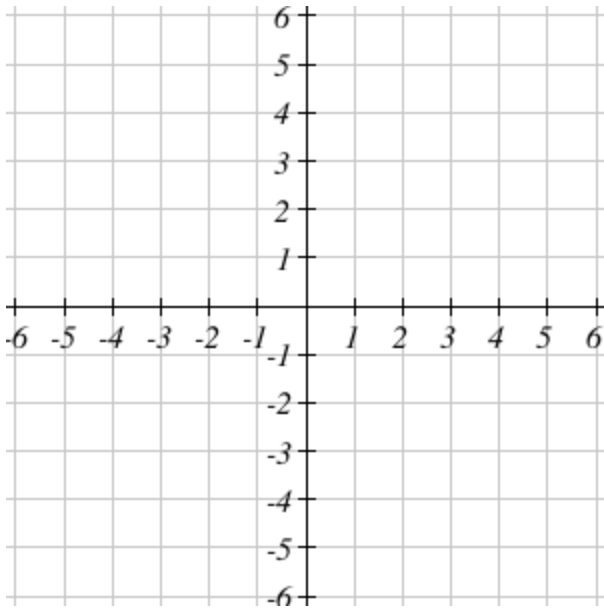
- $x \leq 2$
- All real numbers
- $x \leq -3$
- $x \geq -3$
- $x \geq 2$

The range of the equation is:

- All real numbers
- $y \geq -3$
- $y \leq 2$
- $y \leq -3$
- $y \geq 2$

14.

Graph the equation $y = -x^2 + 8x - 13$ below and then state its domain and range.



The domain of the equation is:

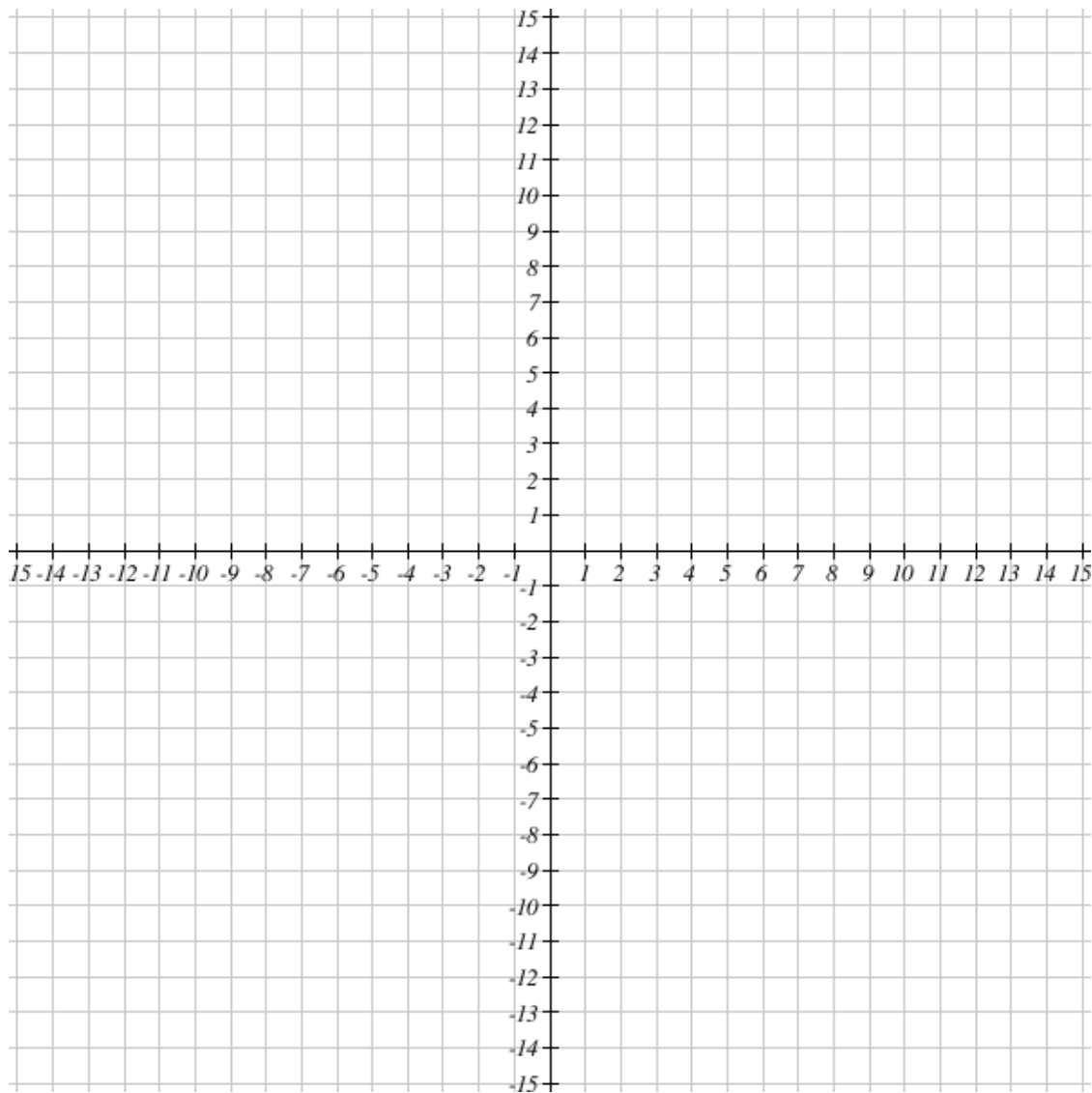
- $x \leq 4$
- $x \leq 3$
- $x \geq 4$
- All real numbers
- $x \geq 3$

The range of the equation is:

- $y \leq 3$
- $y \geq 3$
- All real numbers
- $y \geq 4$
- $y \leq 4$

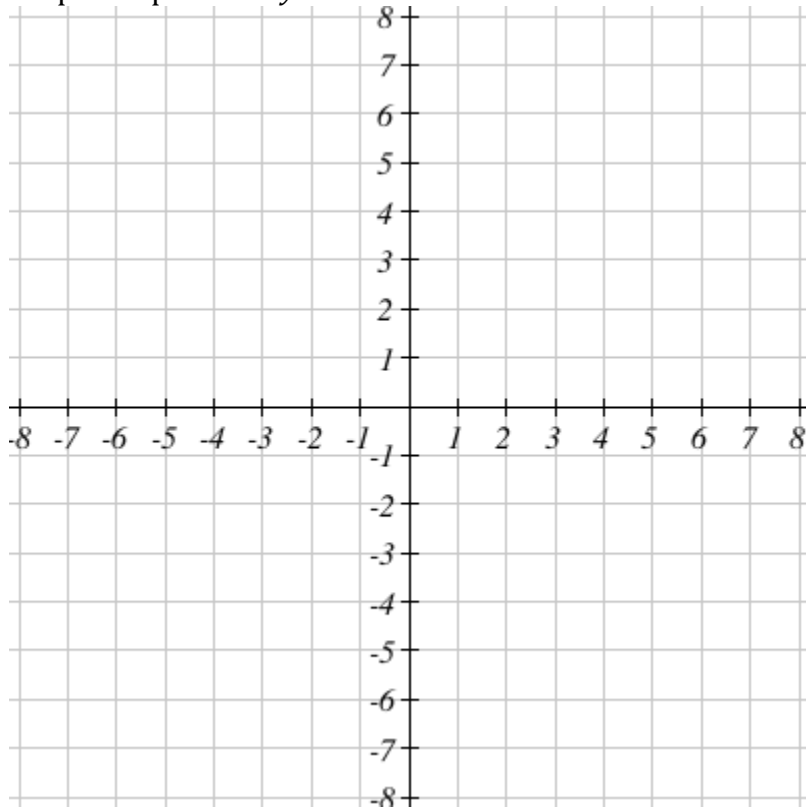
15.

Graph the function $g(x) = -0.5x^2 - 4x - 7$ on the axes below



16.

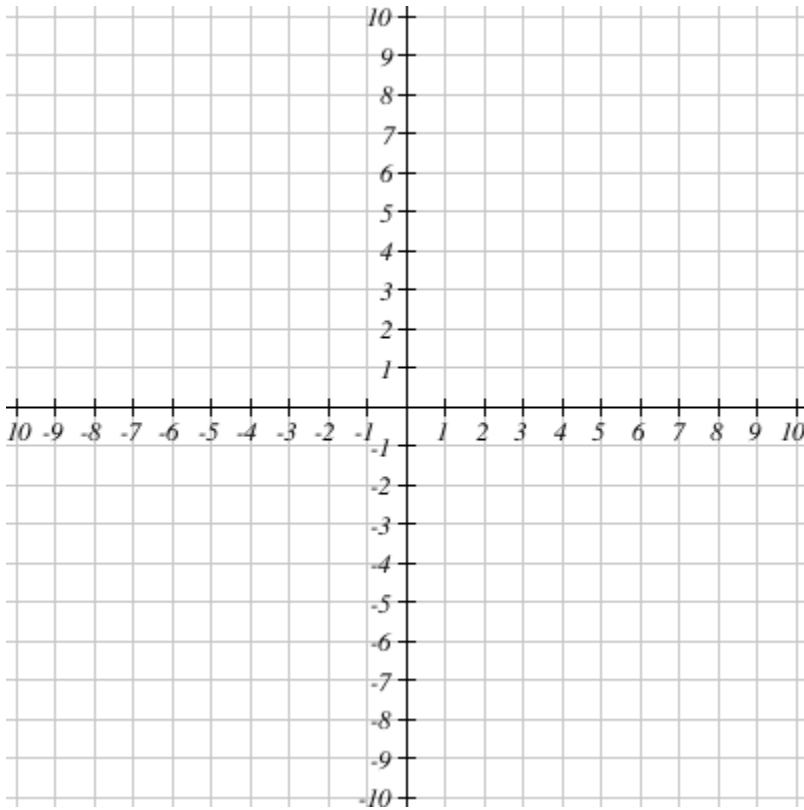
Graph the parabola $y = x^2 - 4x + 1$ below.



17.

Graph the parabola then fill in the blanks about the vertex, any intercepts, the domain, and range. Enter intercepts as ordered pairs, aka points.

$$y = x^2 - 8x + 12$$



Vertex = _____

y -intercept = _____

x -intercepts = _____

Domain: _____

Range: _____

Equation of the Axis of Symmetry: _____

18.

Consider the quadratic function $f(x) = -x^2 + 4x + 5$.

Determine the following:

The left x -intercept is $x =$ _____

The right x -intercept is $x =$ _____

The y -intercept is $y =$ _____

The vertex is (_____ , _____)

The line of symmetry has the equation _____

19.

Consider the parabola given by the equation: $f(x) = -4x^2 + 24x - 24$

Find the following for this parabola:

A) The vertex: _____

B) The vertical intercept is the point _____

C) Find the coordinates of the two x -intercepts of the parabola and write them as ordered pairs. First, give the exact values, and then give decimals rounded to two decimal places.

Exact coordinates: _____

Decimal coordinates: _____

20.

Let $f(x) = 3x^2 - 18x + 19$. Find the x -intercepts of the graph of $y = f(x)$.

The x -intercepts are located at

- One or more solutions: _____
- No solution

(Give the coordinates of ALL the x -intercepts, or if no x -intercepts exist, select that choice)

21.

Find b and c so that $y = -6x^2 + bx + c$ has vertex $(5, -9)$.

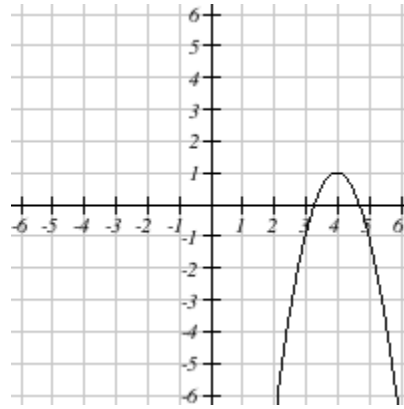
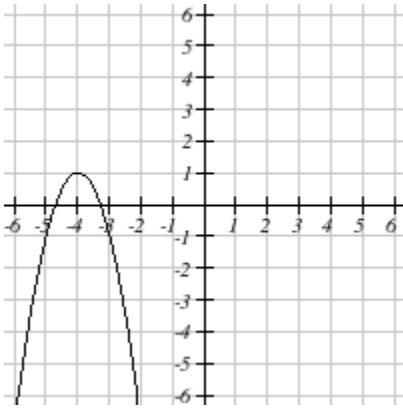
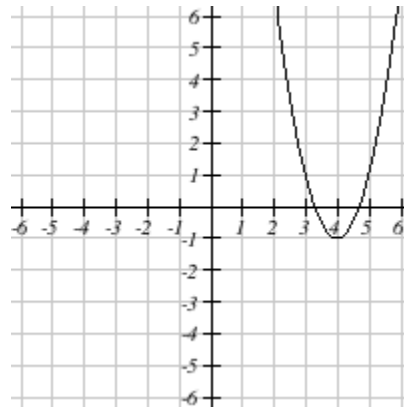
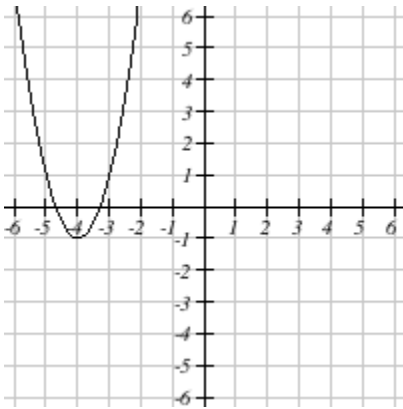
$b =$ _____.

$c =$ _____.

22.

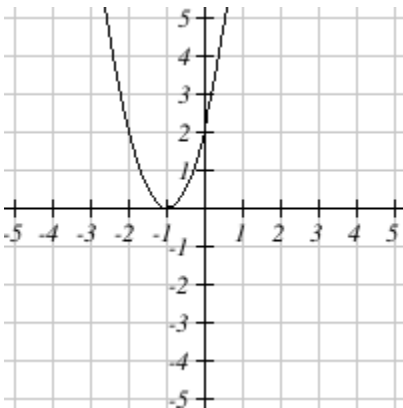
Match the function with its graph.

$$y = 2(x + 4)^2 - 1$$



23.

Write an equation (any form) for the quadratic graphed below:



y = _____

24.
Determine the equation of the quadratic function with vertex (1,4) and passing through the point (-1,16)

$y = \underline{\hspace{2cm}}$

25.
Given the following quadratic function: $y = x^2 - 10x + 16$

a) Complete the square to write the quadratic in standard form: $y = \underline{\hspace{2cm}}$

b) Identify the vertex of the parabola: $\underline{\hspace{2cm}}$

26.
For each quadratic expression below, complete the square to write it in standard form.

a) $x^2 + 4x + 5$

Standard form: $\underline{\hspace{2cm}}$

b) $x^2 - 16x + 56$

Standard form: $\underline{\hspace{2cm}}$

27.
Find the formula, in standard form $y = ax^2 + bx + c$, for a quadratic that has roots at $x = \sqrt{17}$ and $x = -\sqrt{17}$, and has leading coefficient of 1.

$y = \underline{\hspace{2cm}}$

28.
Find the formula, in standard form $y = ax^2 + bx + c$, for a quadratic that has roots at $x = -3 + 3\sqrt{7}$ and $x = -3 - 3\sqrt{7}$, and has leading coefficient of 1.

$y = \underline{\hspace{2cm}}$

29.

On Utapau, while riding a boga, General Kenobi dropped his lightsaber 440 feet down onto the platform where Commander Cody was. $h(s) = -14s^2 + 440$, gives the height after s seconds.

a) What type of function would best model this situation?

- Rational or Inverse
- Linear
- Quadratic
- Exponential

b) Evaluate $h(5) =$ _____

30.

This question is not about solving the stated problem, but about understanding it.

A rocket is launched, and its height above sea level t seconds after launch is given by the equation $h(t) = -4.9t^2 + 1400t + 220$.

a) From what height was the rocket launched?

To answer this question, we'd find:

- The t intercept
- The h intercept
- The t coordinate of the vertex
- The h coordinate of the vertex

b) What is the maximum height the rocket reaches?

To answer this question, we'd find:

- The t intercept
- The h intercept
- The t coordinate of the vertex
- The h coordinate of the vertex

c) If the rocket will splash down in the ocean, when will it splash down?

To answer this question, we'd find:

- The t intercept
- The h intercept
- The t coordinate of the vertex
- The h coordinate of the vertex

31.

A rocket is fired upward from some initial distance above the ground. Its height (in feet), h , above the ground t seconds after it is fired is given by $h(t) = -16t^2 + 80t + 1056$.

What is the rocket's maximum height?

_____feet

How long does it take for the rocket to reach its maximum height?

_____seconds

After it is fired, the rocket reaches the ground at $t =$ _____ seconds

32.

A person standing close to the edge on top of a 56-foot building throws a ball vertically upward. The quadratic function $h = -16t^2 + 104t + 56$ models the ball's height above the ground, h , in feet, t seconds after it was thrown.

a) What is the maximum height of the ball?

_____feet

b) How many seconds does it take until the ball hits the ground?

_____seconds

33.

Suppose that you have 760 feet of rope and want to use it to make a rectangle. What dimensions should you make your rectangle if you want to enclose the maximum possible area?

The length should be _____ feet

The width should be _____ feet

The total area enclosed is _____ square feet.

34.

Josh wants to build a rectangular enclosure for his animals. One side of the pen will be against the barn, so he needs no fence on that side. The other three sides will be enclosed with wire fencing. If Josh has 450 feet of fencing, you can find the dimensions that maximize the area of the enclosure.

a) Let w be the width of the enclosure (perpendicular to the barn) and let l be the length of the enclosure (parallel to the barn). Write an function for the area A of the enclosure in terms of w . (HINT first write two equations with w and l and A . Solve for l in one equation and substitute for l in the other).

$A(w) =$ _____

b) What width w would maximize the area?

$w =$ _____ft

c) What is the maximum area?

$A =$ _____square feet

Lecture 12

Power functions, Terminology, smoothness, continuity, end behavior, zeros, multiplicity, graph sketching.
Division algorithm, synthetic division

1.

When a polynomial $f(x)$ is evaluated at a particular value of x , is it possible for more than one value to result?

- Yes, a polynomial will have more than one output for every input.
- No, a polynomial can only have more than one output for some inputs.
- No, a polynomial is a function so it can only have one output for each input.
- Yes, a polynomial can have more than one output for some inputs.

2.

Let $f(x) = 3 \cdot x^4$.

$$f\left(-\frac{1}{3}\right) = \underline{\hspace{2cm}}$$

3.

Find $f(-2)$ for $f(x) = 2x^3 + x^2 - 3x + 1$

$$f(-2) = \underline{\hspace{2cm}}$$

4.

Add the polynomials:

$$(11x^5 - 8x^4 + 4x^3 - 10x^2) + (-x^5 + 11x^4 + 5x^3 + 6) = \underline{\hspace{2cm}}$$

5.

Perform the indicated operation and simplify:

$$\left(\frac{1}{7}x + \frac{1}{2}\right)^2 = \underline{\hspace{2cm}}$$

6.

Describe the end behavior (long run behavior) of $f(x) = x^2$

As $x \rightarrow -\infty$, $f(x) \rightarrow$

- ∞
- $-\infty$
- 0

As $x \rightarrow \infty$, $f(x) \rightarrow$

- ∞
- $-\infty$
- 0

7.

Describe the long run behavior of $f(x) = x^9$

As $x \rightarrow -\infty$, $f(x) \rightarrow$

- ∞
- $-\infty$
- 0

As $x \rightarrow \infty$, $f(x) \rightarrow$

- ∞
- $-\infty$
- 0

8.

Describe the long run behavior of $f(r) = -2r^9 - 5r^7 - r^3 + 5$

As $r \rightarrow -\infty$, $f(r) \rightarrow$

- ∞
- $-\infty$
- 0

As $r \rightarrow \infty$, $f(r) \rightarrow$

- ∞
- $-\infty$
- 0

9.

Describe the long run behavior of $f(n) = 2(n + 4)^3(n - 3)^2(n - 1)^3$

As $n \rightarrow -\infty$, $f(n) \rightarrow$

- ∞
- $-\infty$
- 0

As $n \rightarrow \infty$, $f(n) \rightarrow$

- ∞
- $-\infty$
- 0

10.

Find the degree, leading coefficients, and the maximum number of real zeros of the polynomial.

$$f(x) = 6x^8 - 3 + 3x^5 + 5x^7$$

Degree = _____

Leading Coefficient = _____

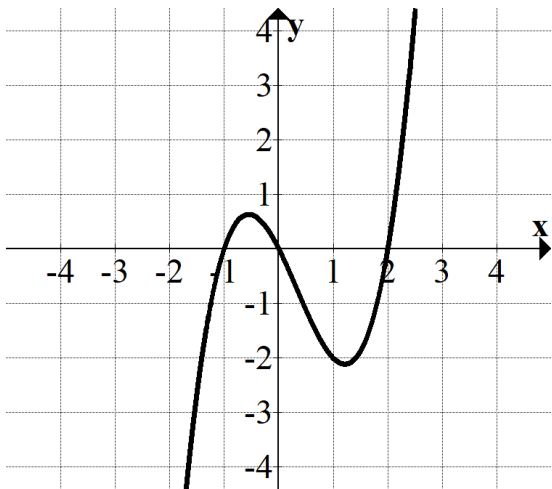
Maximum number of real zeros = _____

11.

Given the function $f(r) = 5r^{12} + 2r^2 + r^{11} + r$

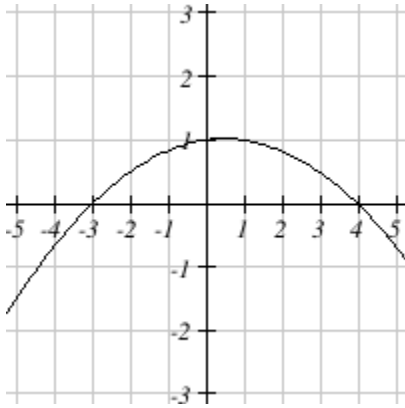
There are at most _____ x-intercepts, and at most _____ turning points.

12.



What is the least possible degree of the polynomial graphed above? _____

13.



What is the least possible degree of the polynomial graphed above? _____

14.

Given the function $P(x) = (x - 6)(x + 5)(x - 3)$

its P -intercept is _____

its x -intercepts are _____

15.

Given the function $f(x) = 4x^4 - 8x^3 - 140x^2$:

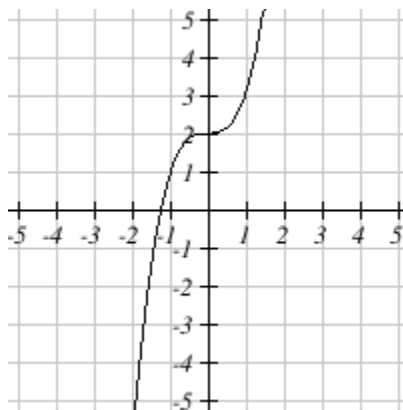
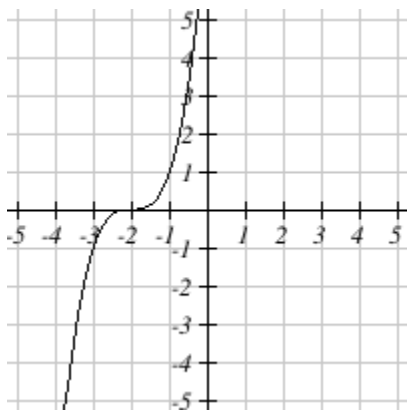
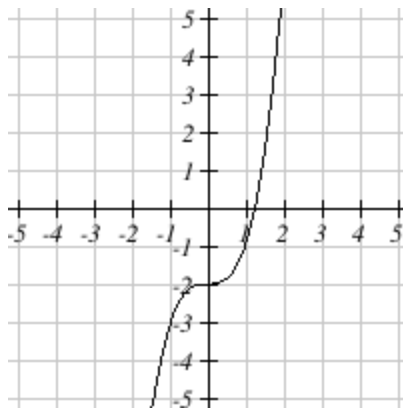
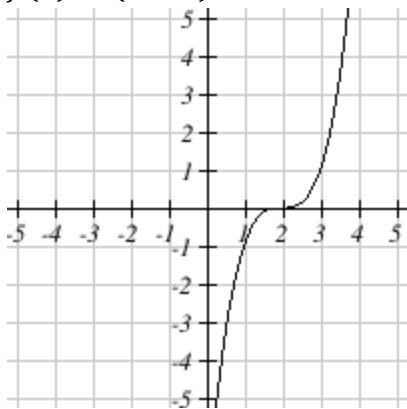
its f -intercept is _____

its x -intercepts are _____

16.

Pick the graph of the following function.

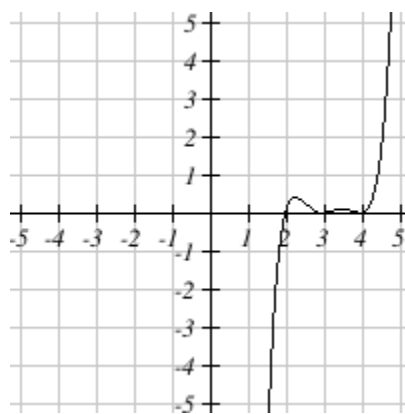
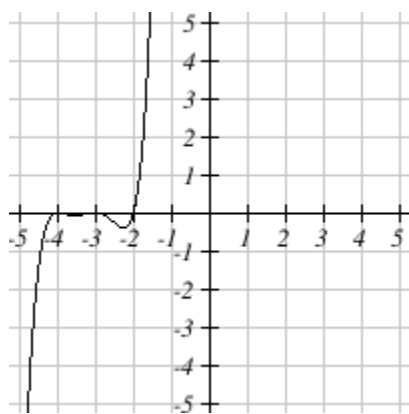
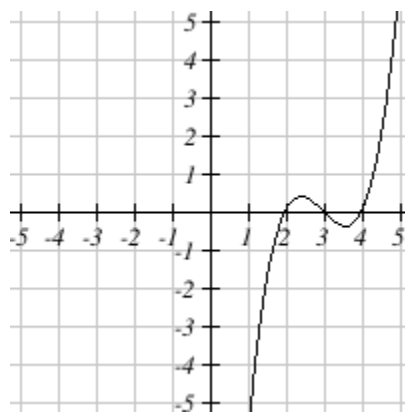
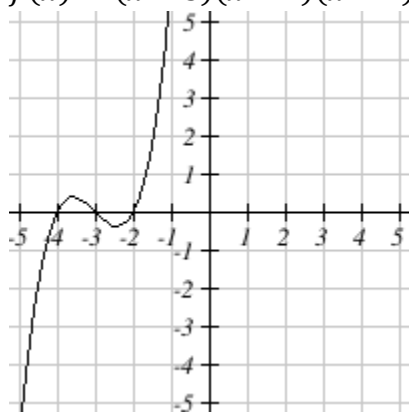
$$f(x) = (x + 2)^3$$



17.

Pick the graph of the following function.

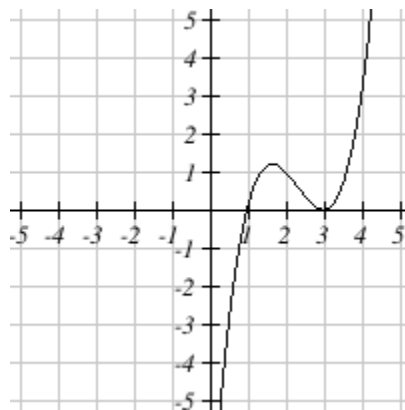
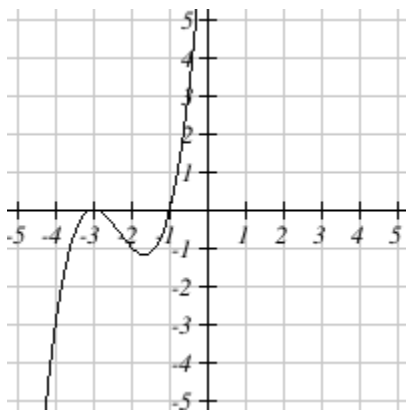
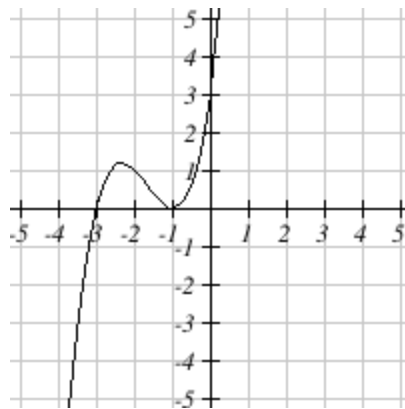
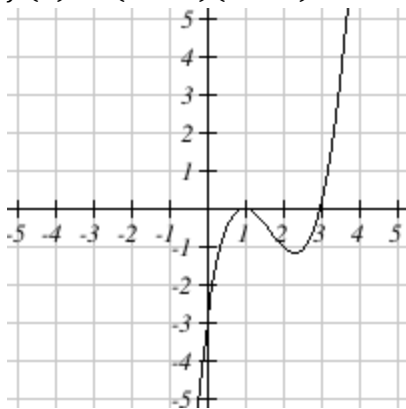
$$f(x) = (x - 3)(x - 4)(x - 2)$$



18.

Pick the graph of the following function.

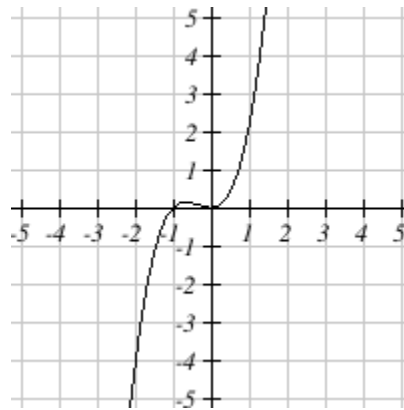
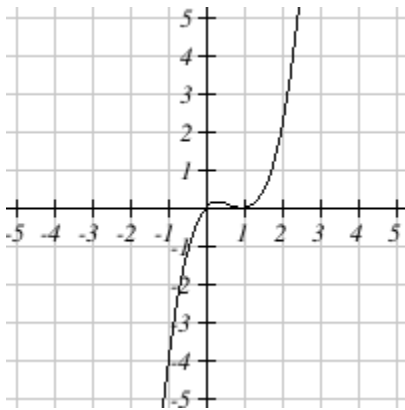
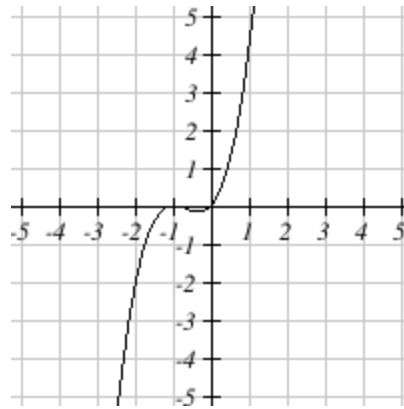
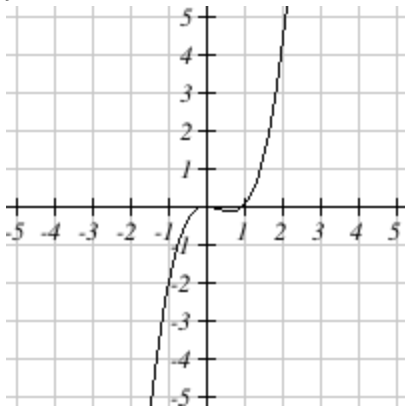
$$f(x) = (x - 1)(x - 3)^2$$



19.

Pick the graph of the following function.

$$f(x) = x^2(x - 1)$$



20.

The polynomial of degree 5, $P(x)$, has leading coefficient 1, has roots of multiplicity 2 at $x = 3$ and $x = 0$, and a root of multiplicity 1 at $x = -3$.

Find the formula for $P(x)$.

$$P(x) = \underline{\hspace{2cm}}$$

21.

The polynomial of degree 4, $P(x)$, has a root of multiplicity 2 at $x = 4$ and roots of multiplicity 1 at $x = 0$ and $x = -4$. It goes through the point $(5,9)$.

Find a formula for $P(x)$.

$$P(x) = \underline{\hspace{2cm}}$$

22.

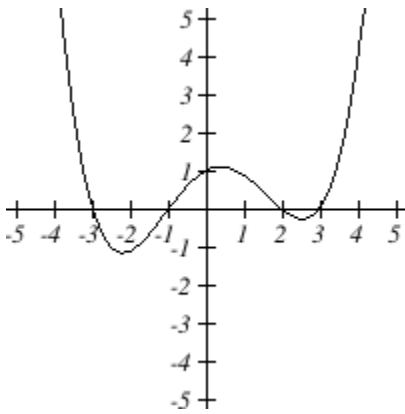
The polynomial of degree 3, $P(x)$, has a root of multiplicity 2 at $x = 5$ and a root of multiplicity 1 at $x = -4$. The y -intercept is $y = -10$.

Find a formula for $P(x)$.

$P(x) =$ _____

23.

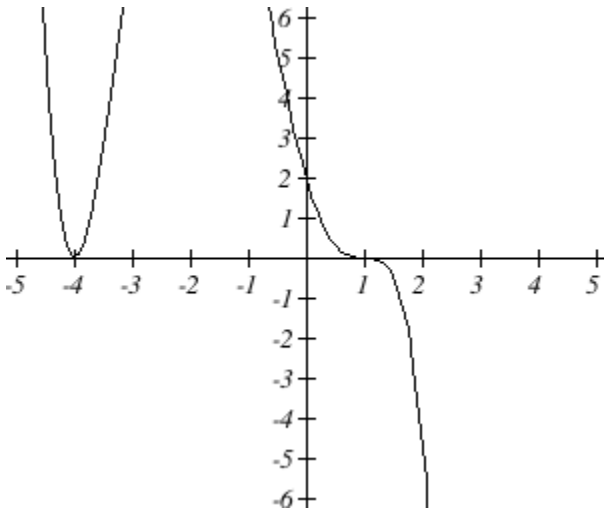
Write an expression in factored form for the polynomial of least possible degree graphed below.



$y(x) =$ _____

24.

Write an expression in factored form for the polynomial of least possible degree graphed below.



$y(x) =$ _____

25.

Find the quotient and remainder using long division for: $\frac{2x^3-14x^2+7x-30}{2x^2+5}$

The quotient is _____

The remainder is _____

26.

Find the quotient and remainder using long division for: $\frac{x^2+6x+15}{x+3}$.

The quotient is _____

The remainder is _____

27.

Divide. If there is a remainder, express it in the form $\frac{r}{x+2}$.

$$(x^3 - 5x - 2) \div (x + 2) = \underline{\hspace{2cm}}$$

28.

Divide. If there is a remainder, express it in the form $\frac{r}{x-4}$.

$$(x^3 + 3x^2 - 33x + 27) \div (x - 4) = \underline{\hspace{2cm}}$$

29.

Find the quotient and remainder using synthetic division for: $\frac{x^3+8x^2+17x+14}{x+2}$

The quotient is _____

The remainder is _____

30.

Find the quotient and remainder using synthetic division: $\frac{x^3+10}{x+2}$

The quotient is _____

The remainder is _____

31.

Find the quotient and remainder using synthetic division: $\frac{x^4 - 5x^3 - 31x - 25}{x - 6}$

The quotient is _____

The remainder is _____

32.

Find the quotient and remainder using synthetic division for

$$\frac{x^5 - x^4 + 5x^3 - 5x^2 + 7x - 10}{x - 1}$$

The quotient is _____

The remainder is _____

Lecture 13

Intro, Imaginary unit, roots of negative numbers, complex numbers, operations, conjugate, division/rationalizing, solutions to quadratic equations, powers of i .

1.

$$\sqrt{-1} =$$

- 1
- i
- $-i$
- -1

2.

For the complex number $-1 - 20i$...

The real part is _____

The imaginary part is _____

3.

Express $\sqrt{-100}$ as a complex number, in terms of i :

$$\sqrt{-100} = \underline{\hspace{2cm}}$$

4.

Express $7 - \sqrt{-81}$ as a complex number in the form $a + bi$.

$$7 - \sqrt{-81} = \underline{\hspace{2cm}}$$

5.

Express in terms of i :

$$\sqrt{-26} = \underline{\hspace{2cm}}$$

6.

Express in terms of i and simplify:

$$\sqrt{-18} = \underline{\hspace{2cm}}$$

7.

Simplify the expression:

$$\sqrt{-80} = \underline{\hspace{2cm}}$$

8.

Perform the indicated operations & simplify.

Add: $(14 - 25i) + (5 + 17i)$

sum = _____

Subtract: $(14 - 25i) - (5 + 17i)$

difference = _____

9.

Perform the indicated operation & simplify. Express the answer in terms of i (as a complex number).

$$\sqrt{-7} \cdot \sqrt{-175} = \underline{\hspace{2cm}}$$

10.

Perform the indicated operation & simplify. Express the answer in terms of i (as a complex number).

$$(3i)(-12i) = \underline{\hspace{2cm}}$$

11.

Multiply and express your answer as a complex number.

$$(4 - 7i)(-6 + 8i) = \underline{\hspace{2cm}}$$

12.

Perform the indicated operation & simplify. Express the answer as a complex number.

$$(6 - 11i)(-7 + 5i) = \underline{\hspace{2cm}}$$

13.

Multiply and express your answer as a complex number.

$$(6 + 5i)(6 + 5i) = \underline{\hspace{2cm}}$$

14.

Perform the indicated operation & simplify. Express the answer in terms of i (as a complex number).

$$2i(4 + 7i) = \underline{\hspace{2cm}}$$

15.

Perform the indicated operation & simplify. Express the answer in terms of i (as a complex number).

$$(2 + 2i)(3 + 5i)(4 + 4i) = \underline{\hspace{2cm}}$$

16.

What is the complex conjugate of the number $10 + 4i$? $\underline{\hspace{2cm}}$

17.

In this problem you are going to investigate what happens when you multiply a complex number by its conjugate using the number $3 - 3i$.

First, multiply it by something that is not its conjugate:

$$(3 - 3i)(-8 - 2i) = \underline{\hspace{2cm}}$$

Now, multiply it by its conjugate:

$$(3 - 3i)(3 + 3i) = \underline{\hspace{2cm}}$$

You should notice a difference in those two results. To test it, try another one:

$$(-10 - 4i)(-10 + 4i) = \underline{\hspace{2cm}}$$

When you multiply a complex number by its conjugate the result will be a(n)

- real
- hyperreal
- surreal
- imaginary

number.

18.

Simplify:

$$\frac{3+i}{4i} = \underline{\hspace{2cm}}$$

19.

Divide and simplify:

$$(192 + 89i) \div (-12 + 11i) = \underline{\hspace{2cm}}$$

20.

Simplify:

$$\frac{-2i}{2+5i} = \underline{\hspace{2cm}}$$

21.

Simplify each of the following completely:

$$i^{13} = \underline{\hspace{2cm}}$$

$$i^{40} = \underline{\hspace{2cm}}$$

$$i^{48} = \underline{\hspace{2cm}}$$

$$i^{38} = \underline{\hspace{2cm}}$$

$$i^{14} = \underline{\hspace{2cm}}$$

$$i^{31} = \underline{\hspace{2cm}}$$

$$i^{24} = \underline{\hspace{2cm}}$$

$$i^{77} = \underline{\hspace{2cm}}$$

22.

Write in the form $a + bi$: $-4 + i - 6i^2 + 5i^3 - 5i^4 = \underline{\hspace{2cm}}$

23.

Solve $m^2 - 3 = 0$ using complex numbers if necessary.

$$m = \underline{\hspace{2cm}}$$

24.

Find all complex-number solutions. Write solutions in terms of i

$$x^2 = -25$$

$$x = \underline{\hspace{2cm}}$$

25.

Solve $m^2 + 11 = 0$ using complex numbers if necessary.

$m =$ _____

26.

Solve the equation $x^2 + 6x = -10$, using complex numbers if necessary.

$x =$ _____

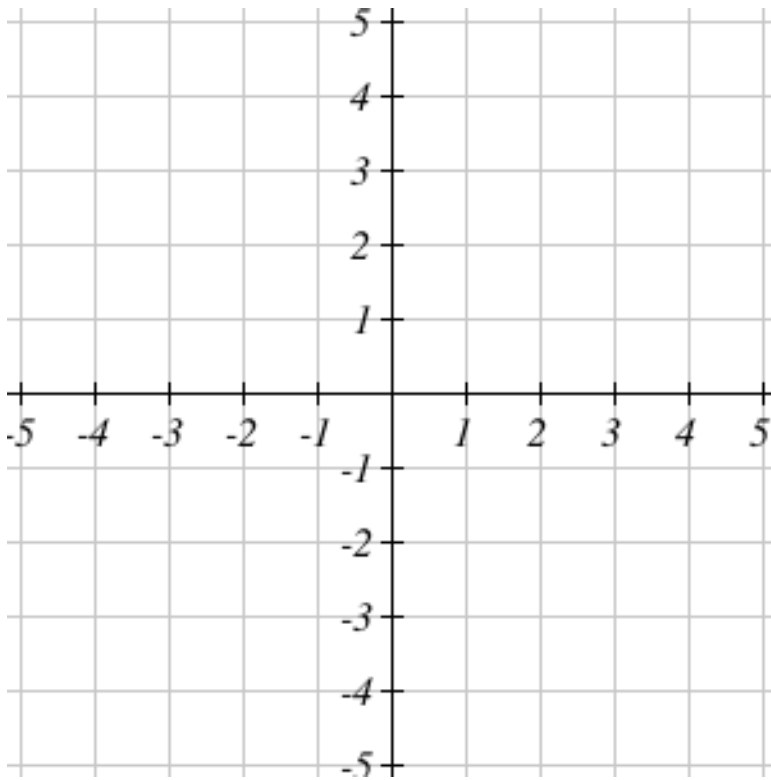
27.

Solve the equation: $x^2 - 8x - 3 = -73$.

Fully simplify all answers, including non-real solutions. $x =$ _____

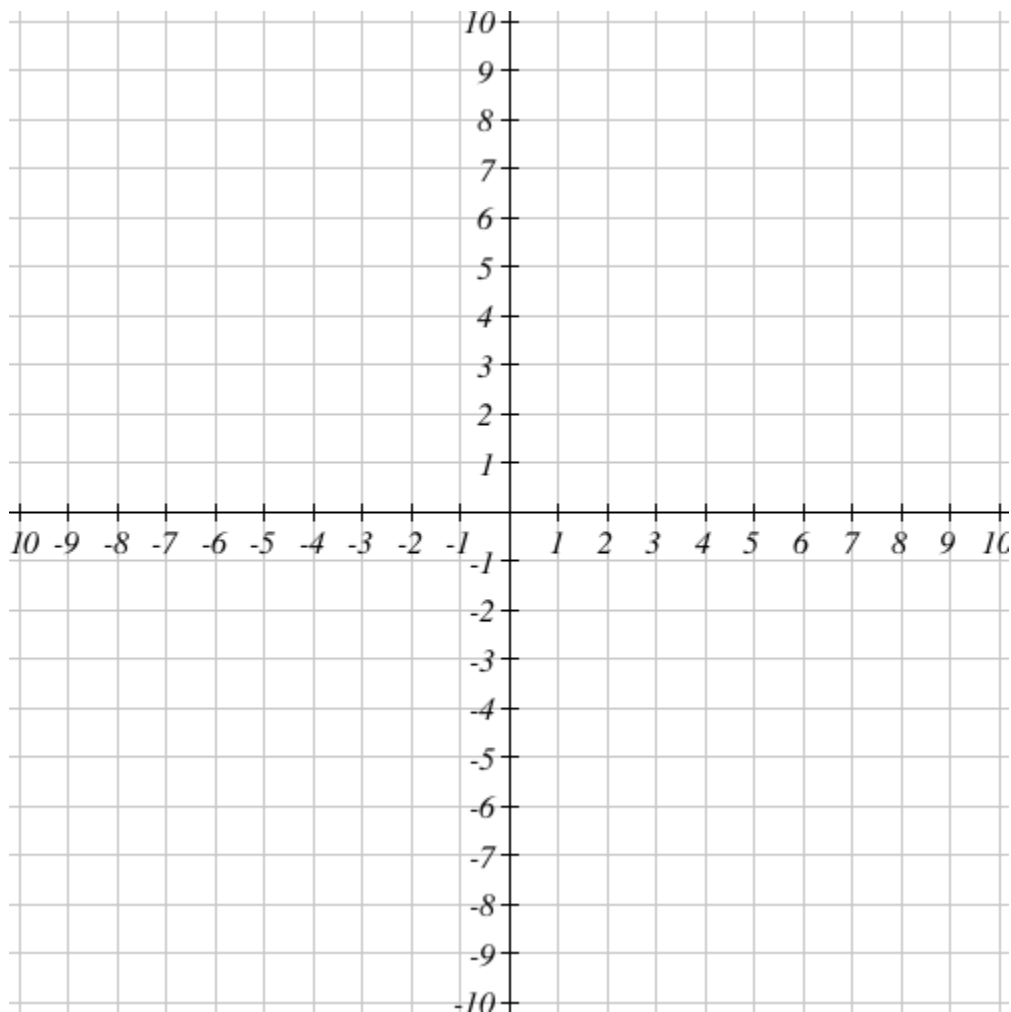
28.

Plot the number $-1 - 2i$.



29.

Visualize the addition of $(7 + 5i) + (-4 + i)$ by plotting the initial point, and the result.



Lecture 14

Remainder/factor theorem, Rational Zero theorem, Fundamental Theorem of Algebra, complex conjugate theorem

1.

Is $(x - 5)$ a factor of $-4x^3 + 25x^2 - 19x - 27$?

- No, it is a not factor.
- Yes, it is a factor.

2.

Find all zeros of the function $f(x) = 9x^3 - 6x^2 - 29x + 10$. Give the zeros separated by commas.

The zeros are $x =$ _____

3.

Find all zeros of $f(x) = x^3 - 2x^2 - 6x + 4$. Give the zeros separated by commas. Give exact value, not decimal approximations.

The zeros are $x =$ _____

4.

Find all zeros of $f(x) = x^3 - x^2 - 18x - 10$. Give the zeros separated by commas. Give exact value, not decimal approximations.

The zeros are $x =$ _____

5.

Find all zeros of $f(x) = x^3 - 5x^2 + x - 5$.

The zeros are $x =$ _____

6.

Find all zeros of $f(x) = 9x^3 + 27x^2 + 30x + 12$. Give the zeros separated by commas. Give exact value, not decimal approximations.

The zeros are $x =$ _____

7.

Given $P(x) = x^3 + x^2 + 4x + 4$. Write P in factored form (as a product of linear factors). Be sure to write the full equation, including $P(x) =$ _____.

8.

Given $P(x) = 3x^5 - 7x^4 + 5x^3 - 25x^2 - 28x + 12$, and that $2i$ is a zero, write P in factored form (as a product of linear factors). Be sure to write the full equation, including $P(x) = \underline{\hspace{2cm}}$.

9.

The value of $x^3 + 9x^2 - 11x - 34$ when $x = 2$ is $\underline{\hspace{2cm}}$.

10.

For the cubic polynomial $f(x) = 6x^3 - 22x^2 - 80x - 24$, use the Rational Zeros Theorem to list all possible rational zeros. Then find all zeros of the polynomial.

Possible rational zeros: +/-:

- 1, 2, 3, 6, 1/2, 3/2, 1/3, 2/3, 1/4, 3/4, 1/6, 1/8, 3/8, 1/12, 1/24
- 1, 2, 3, 4, 6, 8, 12, 24, 1/2, 3/2, 1/3, 2/3, 4/3, 8/3, 1/4, 3/4, 1/6, 1/8, 3/8, 1/12, 1/24
- 1, 2, 3, 4, 6, 8, 12, 24, 1/2, 3/2, 1/3, 2/3, 1/4, 3/4, 1/6, 1/8, 3/8, 1/12, 1/24
- 1, 2, 3, 4, 6, 8, 12, 24, 1/2, 3/2, 1/3, 2/3, 4/3, 8/3, 1/6

Actual zeros of the polynomial: $\underline{\hspace{2cm}}$

11.

For the polynomial $g(x) = 2x^4 - 5x^3 - 39x^2 + 90x + 54$, use the Rational Zeros Theorem to list all possible rational zeros of the polynomial. Then identify all roots of the polynomial.

Possible rational roots: +/- :

- 1, 2, 3, 6, 9, 18, 27, 54, 1/2, 1/3, 2/3, 1/6, 1/9, 2/9, 1/18, 1/27, 2/27, 1/54
- 1, 2, 3, 6, 9, 18, 27, 54, 1/2, 3/2, 9/2, 27/2
- 1, 2, 1/2, 1/3, 2/3, 1/6, 1/9, 2/9, 1/18, 1/27, 2/27, 1/54
- 1, 2, 3, 6, 9, 18, 27, 54, 1/2, 3/2, 9/2, 27/2, 1/3, 2/3, 1/6, 1/9, 2/9, 1/18, 1/27, 2/27, 1/54

List all the zeros (real or complex) of the polynomial: $\underline{\hspace{2cm}}$

12.

For the cubic polynomial $f(x) = 6x^3 - 22x^2 - 80x - 24$, use the Rational Zeros Theorem to list all possible rational zeros. Then find all zeros of the polynomial.

Possible rational zeros: +/-

- 1, 2, 3, 4, 6, 8, 12, 24, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{12}$, $\frac{1}{24}$
- 1, 2, 3, 4, 6, 8, 12, 24, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{8}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{12}$, $\frac{1}{24}$
- 1, 2, 3, 6, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{1}{12}$, $\frac{1}{24}$
- 1, 2, 3, 4, 6, 8, 12, 24, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{8}{3}$, $\frac{1}{6}$

Actual zeros of the polynomial: _____

13.

For the polynomial $g(x) = 2x^4 - 5x^3 - 39x^2 + 90x + 54$, use the Rational Zeros Theorem to list all possible rational zeros of the polynomial. Then identify all roots of the polynomial. Possible rational roots: \pm ...

- 1, 2, 3, 6, 9, 18, 27, 54, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{9}{2}$, $\frac{27}{2}$
- 1, 2, 3, 6, 9, 18, 27, 54, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{9}{2}$, $\frac{27}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{2}{9}$, $\frac{1}{18}$, $\frac{1}{27}$, $\frac{2}{27}$, $\frac{1}{54}$
- 1, 2, 3, 6, 9, 18, 27, 54, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{2}{9}$, $\frac{1}{18}$, $\frac{1}{27}$, $\frac{2}{27}$, $\frac{1}{54}$
- 1, 2, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{2}{9}$, $\frac{1}{18}$, $\frac{1}{27}$, $\frac{2}{27}$, $\frac{1}{54}$

List all the zeros (real or complex) of the polynomial: _____

Lecture 15

Domain, Simplifying/equivalence, operations, complex fractions, negative exponent examples, finding LCD, solving equations

1.

Simplify to lowest terms, if possible: $\frac{10z^4}{45z^6} = \underline{\hspace{2cm}}$

2.

Simplify the following expression completely.

$$\frac{3z-18}{6} = \underline{\hspace{2cm}}$$

3.

Simplify the following expression completely.

$$\frac{72}{8m+32} = \underline{\hspace{2cm}}$$

4.

Simplify the rational expression given below:

$$\frac{(x-12)(x-5)}{x-5}$$

The expression simplifies to $\underline{\hspace{2cm}}$ with the restriction that $x \neq \underline{\hspace{2cm}}$

5.

Simplify the following expression completely.

$$\frac{2n+14}{n^2+13n+42} = \underline{\hspace{2cm}} \text{ with the restriction that } x \neq \underline{\hspace{2cm}}$$

6.

Simplify

$$\frac{x^2+9x+18}{x^2-9} = \underline{\hspace{2cm}} \text{ with the restriction that } x \neq \underline{\hspace{2cm}}$$

7.

Simplify the rational expression given below:

$$\frac{11 + x}{x + 11}$$

The expression simplifies to _____ with the condition that $x \neq$ _____

8.

Simplify the rational expression given below:

$$\frac{15 - x}{x - 15}$$

The expression simplifies to _____ with the condition that $x \neq$ _____

9.

Divide and simplify:

$$\frac{700x^{10}}{16y^9} \div \frac{245x^5}{28y^6} = \underline{\hspace{2cm}}$$

10.

Simplify:

$$\frac{x^2+2x-15}{x^2-9} \cdot \frac{x^2-x-12}{x^2+7x+10} = \underline{\hspace{2cm}} \text{ with the restriction that } x \neq \underline{\hspace{2cm}}$$

11.

Simplify:

$$\frac{x^2-4x-5}{x^2-6x+5} \div \frac{x^2+5x+4}{x^2+x-2} = \underline{\hspace{2cm}} \text{ with the restriction } x \neq \underline{\hspace{2cm}}$$

12.

Find the least common denominator of the expressions $\frac{1}{x+10}$ and $\frac{1}{x-12}$.

The least common denominator is _____

13.

Find the least common denominator of the expressions $\frac{1}{(x-2)(x-10)}$ and $\frac{1}{(x-10)(x+5)}$

The least common denominator is _____

14.

Find the least common denominator of the expressions $\frac{1}{(x+10)^9(x+8)^2}$ and $\frac{1}{(x+10)^2(x+8)^7}$

The least common denominator is _____

15.

Add:

$$\frac{-1}{x+1} + \frac{-1}{x-3} = \underline{\hspace{2cm}}$$

16.

Combine into a single fraction:

$$\frac{8}{k-8} + \frac{5}{(k-8)^2} = \underline{\hspace{2cm}}$$

17.

Combine into a single fraction:

$$\frac{2x}{x^2-9} + \frac{x}{x-3} = \underline{\hspace{2cm}}$$

18.

Add and simplify: $\frac{x}{x+3} + \frac{2x+15}{x^2+9x+18} = \underline{\hspace{2cm}}$ with the restriction that $x \neq \underline{\hspace{2cm}}$

19.

What would be a good first step for solving the equation given below?

$$\frac{8}{x+7} = 7$$

- Add $x + 7$ to both sides
- Multiply both sides by 7
- Subtract 7 from both sides
- Multiply both sides by $x + 7$
- Keep your feet shoulder-width apart, and lift with your legs, not your back

20.

Which value would you have to eliminate if it appeared as a possible solution to the equation below?

$$\frac{2}{x-18} + \frac{x-10}{17} = 4$$

- 18
- 10
- 4
- 2
- 17

21.

Solve: $\frac{x+2}{3} = \frac{7}{21}$

$x =$ _____

22.

Solve the equation $\frac{9}{x} = \frac{6}{3x} + 1$.

$x =$ _____

23.

Solve the rational equation: $-\frac{7}{s} + \frac{7}{4} = -\frac{10}{s}$

Answer: $s =$ _____

24.

Solve the equation below:

$$\frac{x}{x+10} = \frac{1}{3}$$

$x =$ _____

25.

Solve the rational equation: $\frac{-10t}{t^2-36} - \frac{8}{t+6} = \frac{1}{t-6}$

Answer: $t =$ _____

26.

Solve for x.

$$\frac{x}{x+6} + \frac{7}{x-9} = \frac{-17x-12}{x^2-3x-54}$$

x = _____

27.

Solve for x. If there is no solution, answer DNE.

$$\frac{x}{x+7} - \frac{9}{x+3} = \frac{-16x-84}{x^2+10x+21}$$

x = _____

28.

Solve the equation $\frac{x+1}{x-1} = \frac{-12}{x+3} + \frac{8}{x^2+2x-3}$

- x = _____
- No solution

29.

If the same number is subtracted from both the numerator and the denominator of $\frac{11}{13}$, the result is $\frac{3}{4}$. Find the number.

The number is _____

30.

John can mow a lawn in 80 minutes. Rocky can mow the same lawn in 120 minutes. How long does it take for both John and Rocky to mow the lawn if they are working together? Express your answer as a reduced fraction.

_____minutes

31.

One inlet pipe can fill an empty pool in 6 hours, and a drain can empty the pool in 10 hours. How long will it take the pipe to fill the pool if the drain is left open?

_____hours

Lecture 16

Domain, Graphical features (asymptotes, holes, zeros, end behavior), curve sketching

1.

Determine the vertical asymptotes and holes of the rational function shown below.

$$f(x) = 8 \frac{(x - 19)(x + 29)}{(x - 19)(x - 48)(x + 75)}$$

(a) holes? _____

(b) vertical asymptotes? _____

Answer DNE if none.

2.

Determine the vertical asymptotes and holes of the rational function shown below.

$$f(x) = \frac{(x + 7)(x - 10)}{(x - 10)(x - 6)}$$

Holes

$x =$ _____

Vertical Asymptotes

$x =$ _____

3.

Determine the vertical asymptotes and holes of the rational function shown below.

$$f(x) = \frac{x^2 + 5x + 4}{x^2 - 5x - 6}$$

If there is not a hole or asymptote, record DNE as your answer.

Holes

$x =$ _____

Vertical Asymptotes

$x =$ _____

4.

$$\text{Let } f(x) = \frac{2x^2 - 9x + 9}{2x^2 - 1x - 1}.$$

This function has...

- 1) A y-intercept at the point _____
- 2) x-intercepts at the point(s) _____
- 3) Vertical asymptotes at $x =$ _____

5.

For each function, determine the long run behavior.

$$\frac{x^2+1}{x^3+2} \text{ has}$$

- No horizontal asymptote
- a Horizontal asymptote at $y=0$
- a Horizontal asymptote at $y=1$

$$\frac{x^2+1}{x^2+2} \text{ has}$$

- No horizontal asymptote
- a Horizontal asymptote at $y=0$
- a Horizontal asymptote at $y=1$

$$\frac{x^3+1}{x^2+2} \text{ has}$$

- No horizontal asymptote
- a Horizontal asymptote at $y=0$
- a Horizontal asymptote at $y=1$

6.

$$\text{Find the horizontal asymptote of } f(x) = \frac{2x - x^3 - 3}{3x^3 - 2x^2 - 5}.$$

$$y = \underline{\hspace{2cm}}$$

7.

$$\text{Let } f(x) = \frac{2x^2 - 9x + 4}{3x^2 + 11x + 6}.$$

This function has:

- 1) A y -intercept at the point _____
- 2) x -intercepts at the point(s) _____
- 3) Vertical asymptotes at $x =$ _____
- 4) Horizontal asymptote at $y =$ _____

8.

Write an equation for a rational function with the following properties. Assume the multiplicity of each factor is 1 or 2.

Vertical asymptotes at $x = -2$ and $x = -3$

x -intercepts at $x = 2$ and $x = -1$

y -intercept at 4

$y =$ _____

9.

Write an equation for a rational function with following properties. Assume the multiplicity of each factor is 1 or 2.

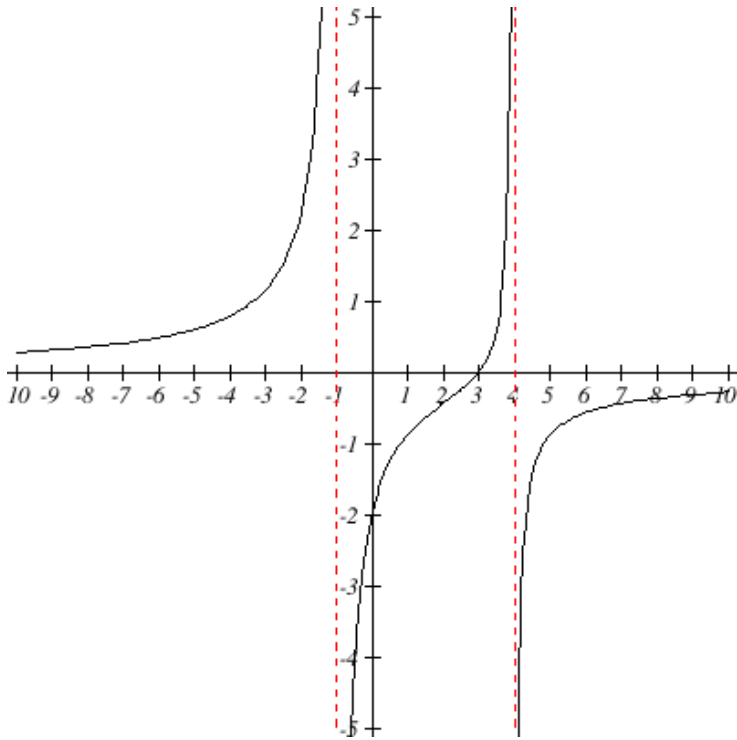
Vertical asymptotes at $x = -4$ and $x = -1$

x -intercepts at $x = 6$ and $x = 2$

Horizontal asymptote at $y = 7$

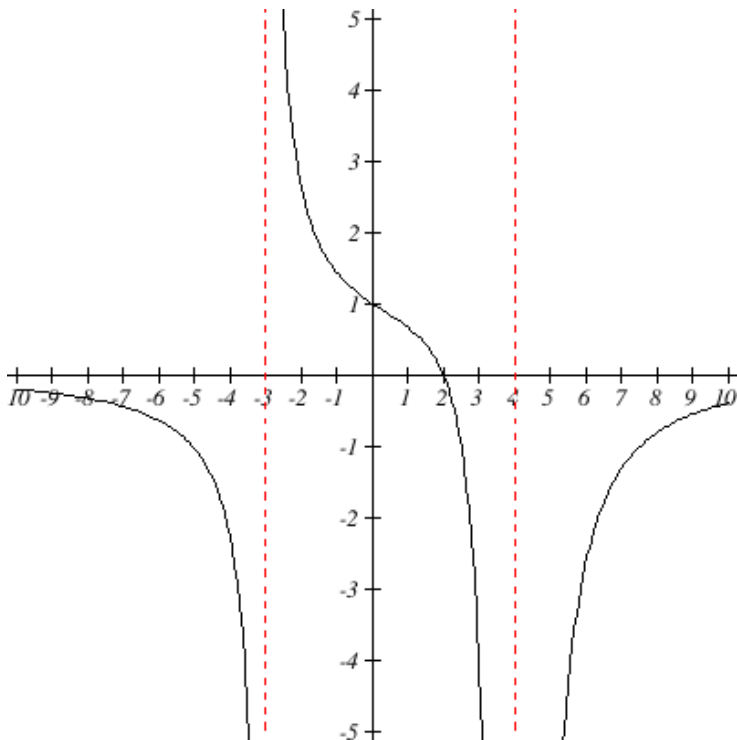
$y =$ _____

10.
Write an equation for the function graphed below. Assume the multiplicity of each factor is 1 or 2.



$y =$ _____

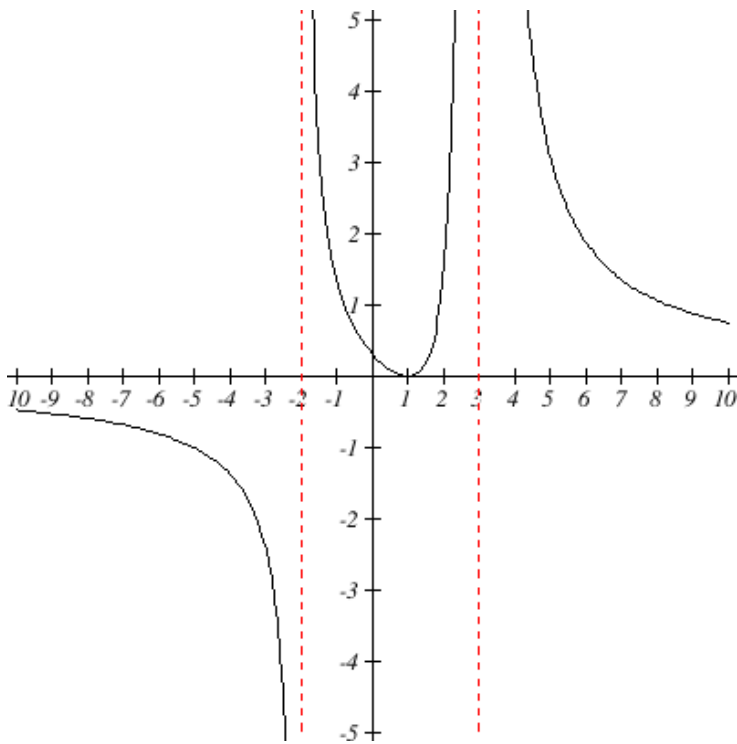
11.
Write an equation for the function graphed below. Assume the multiplicity of each factor is 1 or 2.



$y =$ _____

12.

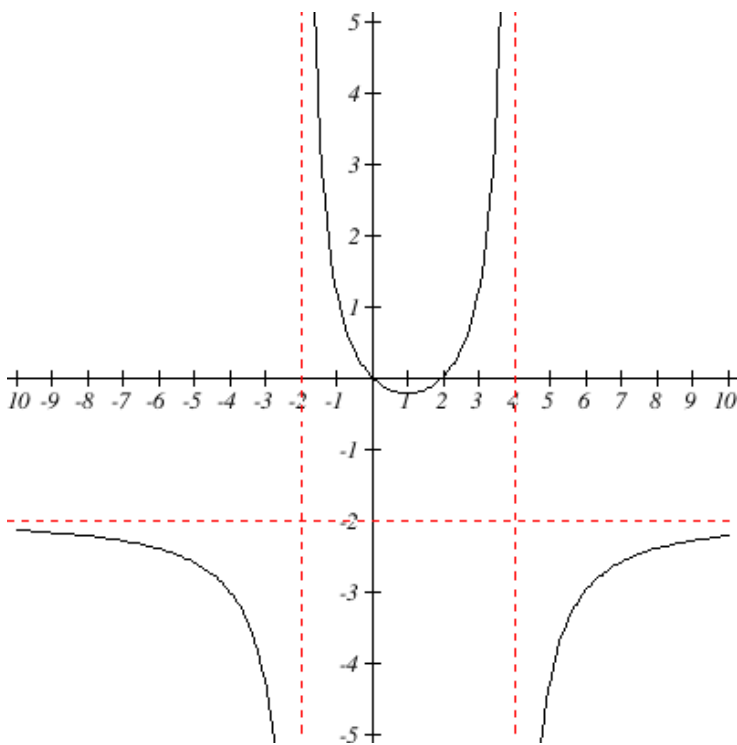
Write an equation for the function graphed below. Assume the multiplicity of each factor is 1 or 2. The y intercept is at (0,0.3)



$y =$ _____

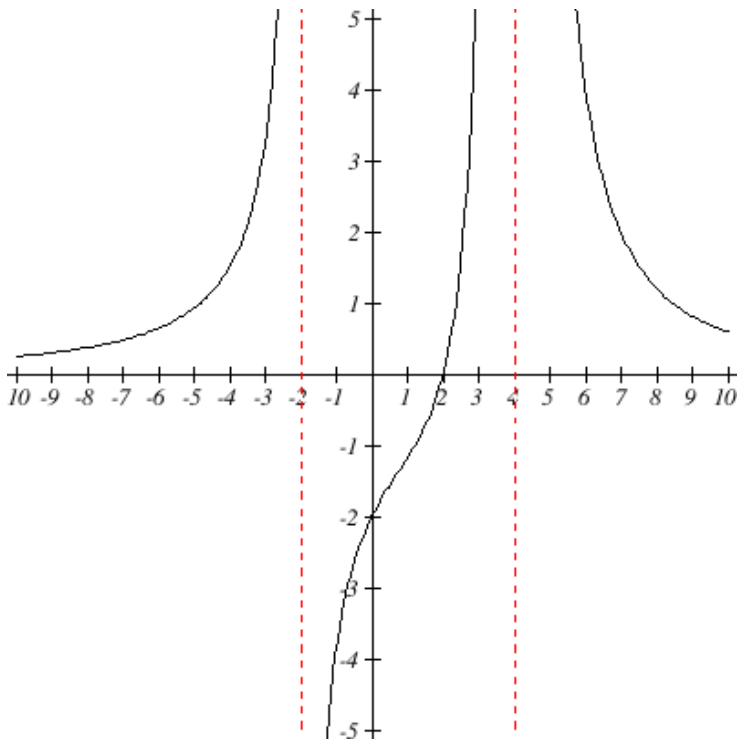
13.

Write an equation for the function graphed below. Assume the multiplicity of each factor is 1 or 2.



$y =$ _____

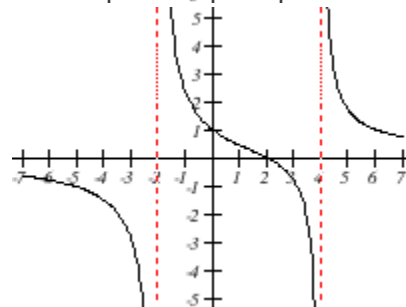
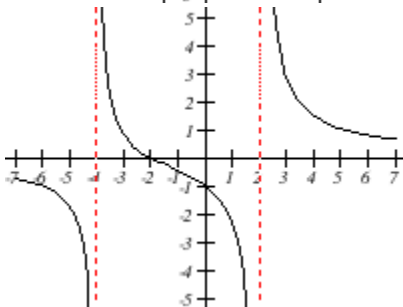
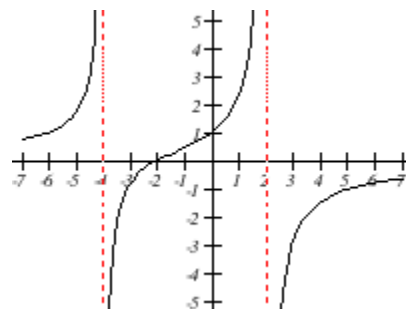
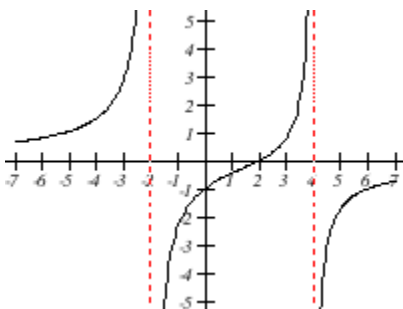
14. Write an equation for the function graphed below. Assume the multiplicity of each factor is 1 or 2.



$y = \underline{\hspace{2cm}}$

15. Select the choice that is a graph of the function

$$f(x) = -4 \frac{x - 2}{(x + 2)(x - 4)}$$



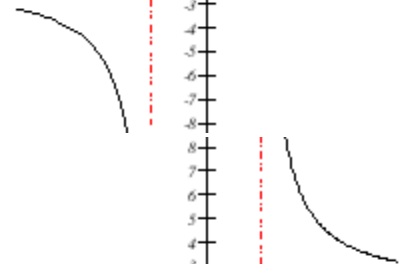
16.

Select the choice that is a graph of the function

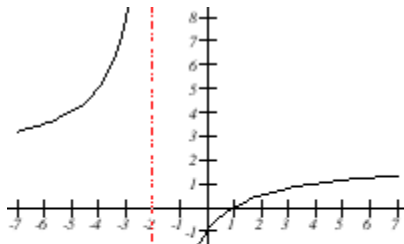
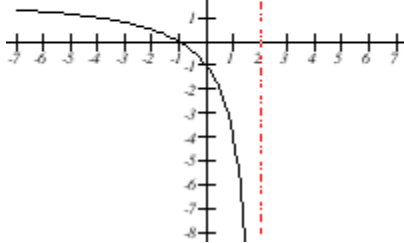
$$f(x) = 2 \cdot \frac{x + 1}{x - 2}$$



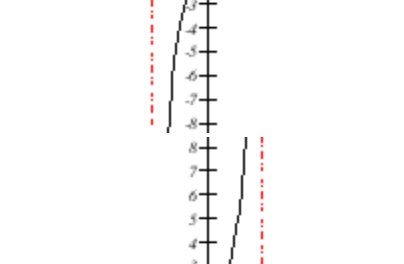
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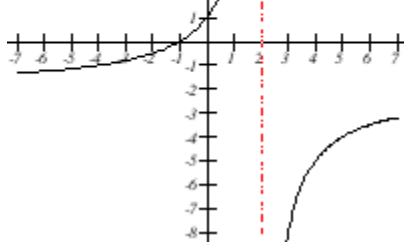
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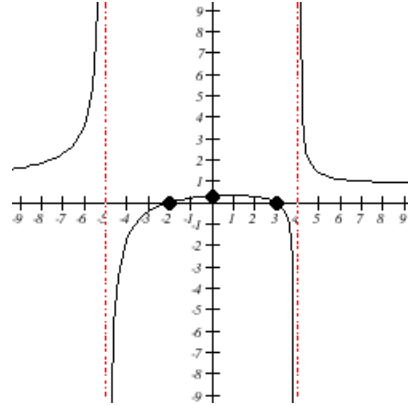
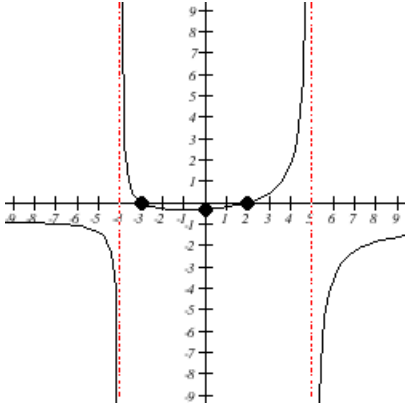
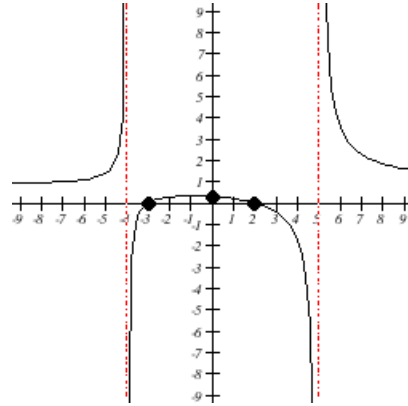
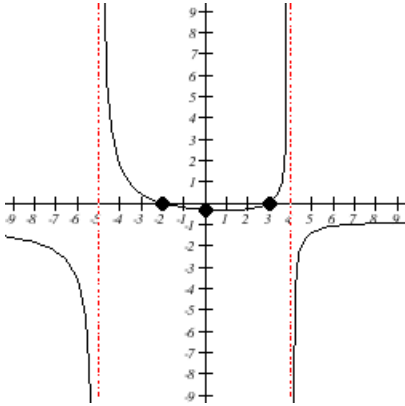


17.

Select the choice that is a graph of the function

$$f(x) = -\frac{(x+3)(x-2)}{(x+4)(x-5)}$$

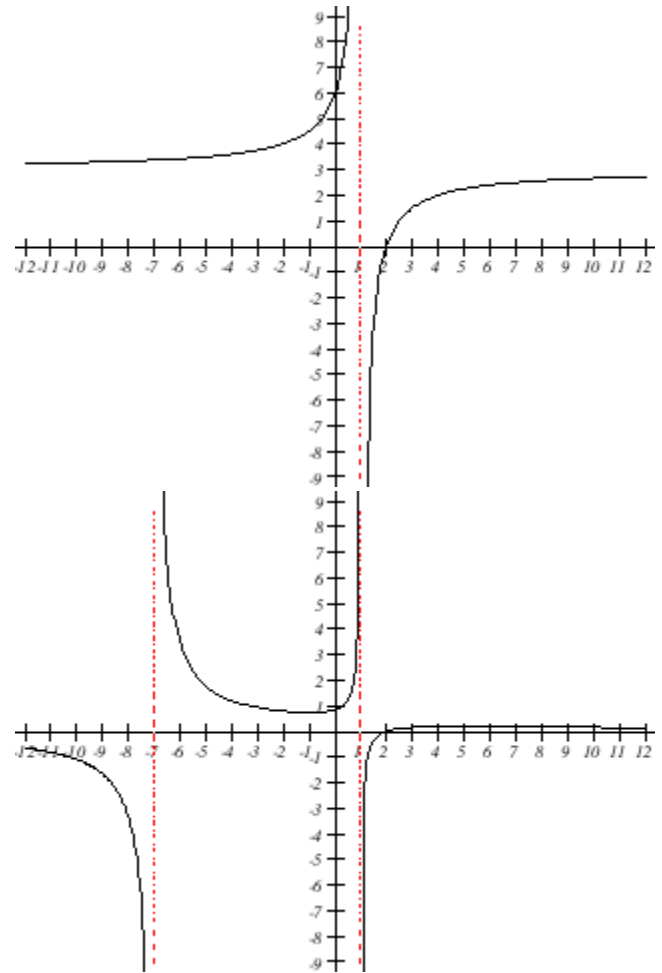
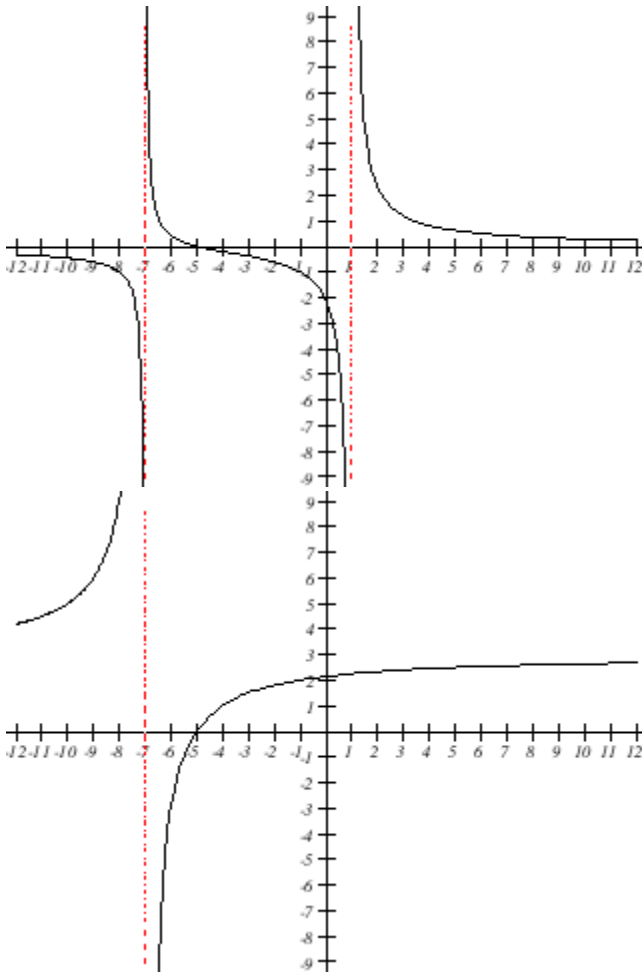
Note that the graph also shows the vertical asymptotes, x-intercepts (zeros of the function), and y-intercept.



18.

Select the choice that is a graph of the function

$$f(x) = 3 \frac{x + 5}{(x + 7)(x - 1)}$$



19.

Draw a graph of $f(x) = \frac{-x+4}{x-1}$, including the horizontal and vertical asymptotes, any intercepts, and an additional point on the graph.

20.

Sketch the graph of $f(x) = \frac{x^2+5x+6}{x^2-2x-8}$.

21.

Sketch the graph of $f(x) = \frac{x^2-6x+9}{x^2-2x-15}$.

22.

Sketch the graph of $f(x) = \frac{4x^2 - 8x + 4}{x^2 - 6x + 9}$.

23.

Sketch the graph of $f(x) = \frac{1}{(x^2 - 8x + 16)(x^2 + 2x + 1)}$.

24.

An ant colony resided in a once-empty home that was recently purchased and treated for pests. The population, in hundreds, of the ant colony is given by the equation $P(t) = \frac{7000t}{7t^2 + 600}$ for $t \geq 9$ where t is the number of hours since the pest treatment. a) Find the population of the colony at 10 hours. Round to the nearest whole number.

Populataion=_____

b) Find the population of the colony at 24 hours. Round to the nearest whole number.

Population=_____

c) Find the population of the colony at 72 hours. Round to the nearest whole number.

Population=_____

d) What value does $P(t)$ approach as time after treatment continues infinitely?

It approaches _____

e) What conclusion can you draw in regards to the ant colony's population at the newly purchased house?

- In time, the population of the ant colony increases towards an infinitely large population.
- In time, the population of the ant colony decreases to 1000.
- In time, the population of the ant colony decreases to 600.
- In time, the ant colony will no longer populate the house.

Lecture 17

Properties of inequalities, Solving linear inequalities, absolute value inequalities

1.

Which of the values below will **not** satisfy the inequality:

$$2x + 9 \leq 37$$

The value that will not satisfy the inequality is:

- 14
- 11
- 9
- 15
- 13

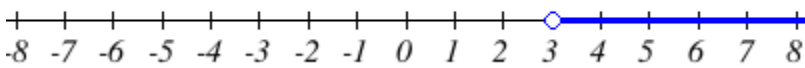
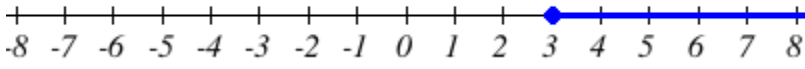
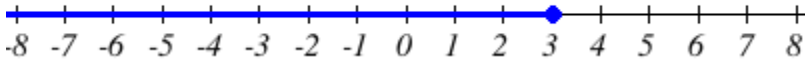
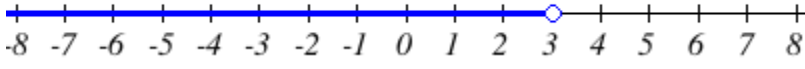
2.

Write the set given below in interval notation and select the graph of the interval.

$$x \leq 3$$

Interval notation: _____

Choose the corresponding graph:

- 
- 
- 
- 

3.

Express the set $-4 < x < -3$ using interval notation.

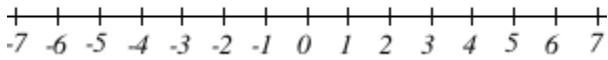
Solution: _____

4.

Solve the following inequality. Write the answer as an inequality, and graph the solution set.

$$3b + 1 > 16$$

Solution: _____

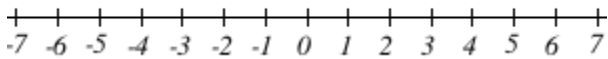


5.

Solve the following inequality. Write the answer as an inequality, and graph the solution set.

$$-3a + 2 \geq 2a - 3$$

Solution: _____



6.

$$\text{Solve: } 4x + 1 > 7x + 2$$

Solution: _____

7.

$$\text{Solve: } -1 < 3 - 4x < 4$$

Solution: _____

8.

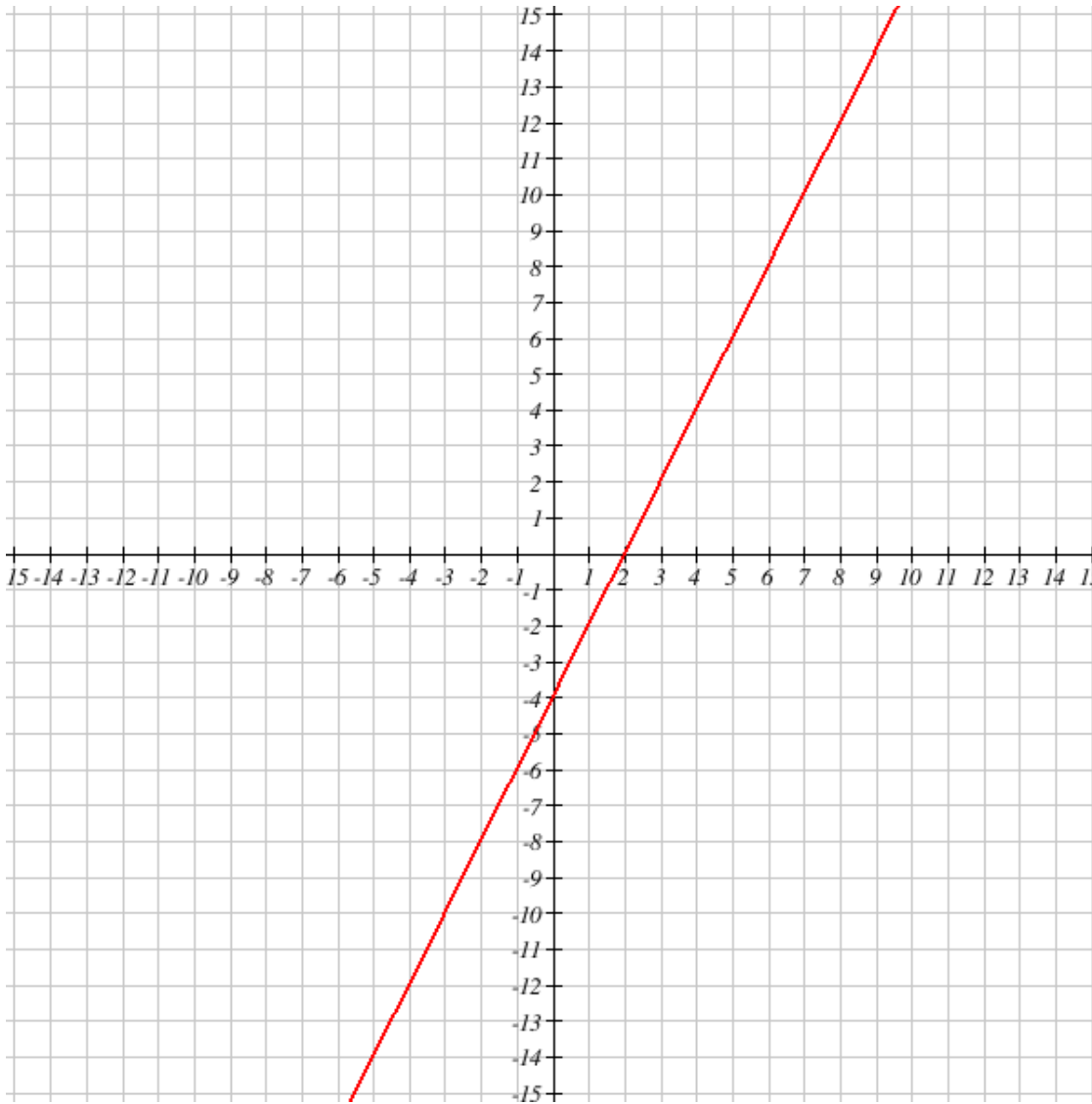
Solve the inequality and write your answer in interval notation.

$$-0.4 + 0.5x \leq 0.4x + 12.6$$

Solution: _____

9.

Given the graph of the function $f(x)$ shown below, solve the inequalities. Express your answer as an inequality.

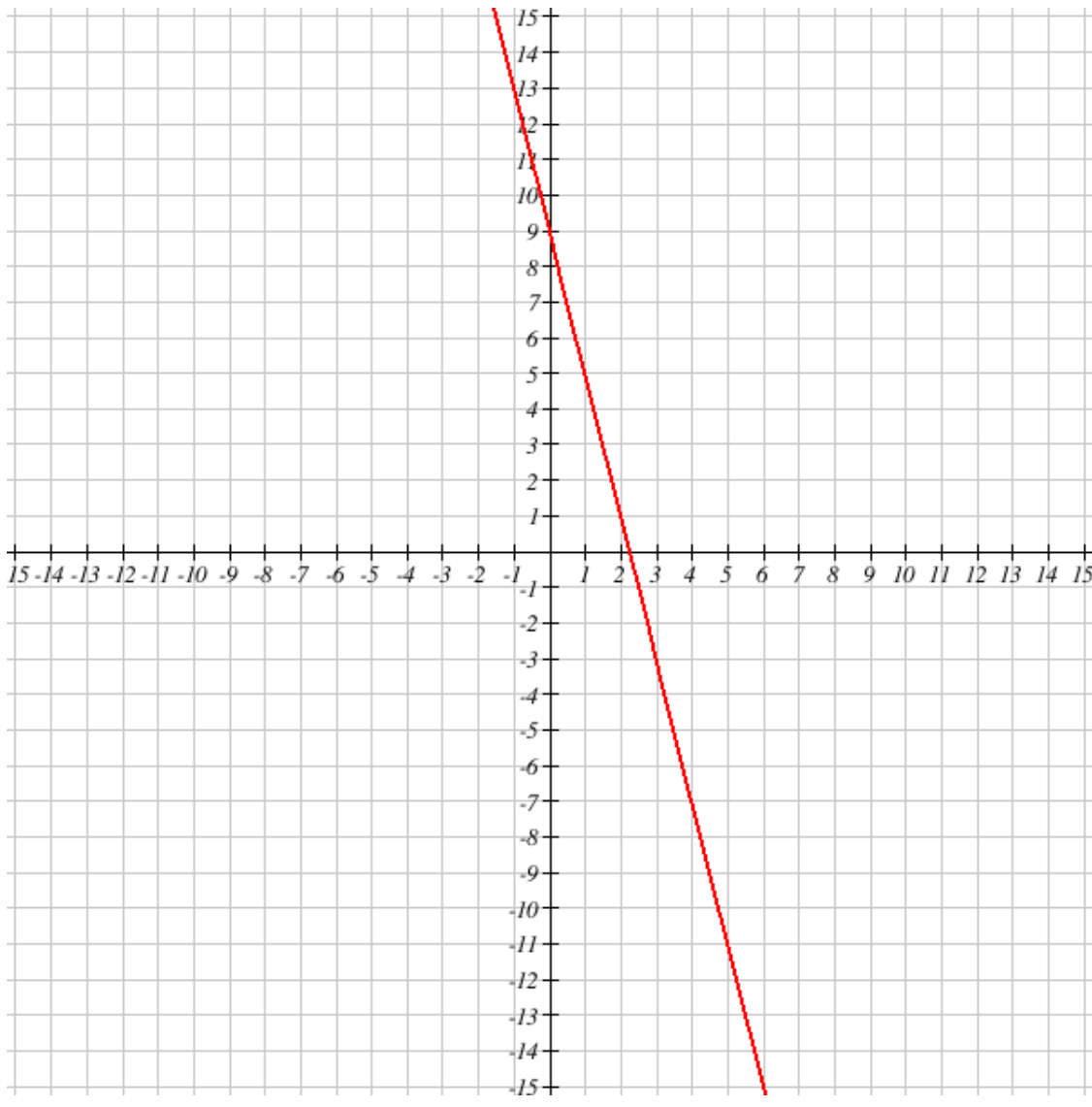


a. $f(x) > -4$ if _____

b. $f(x) < -4$ if _____

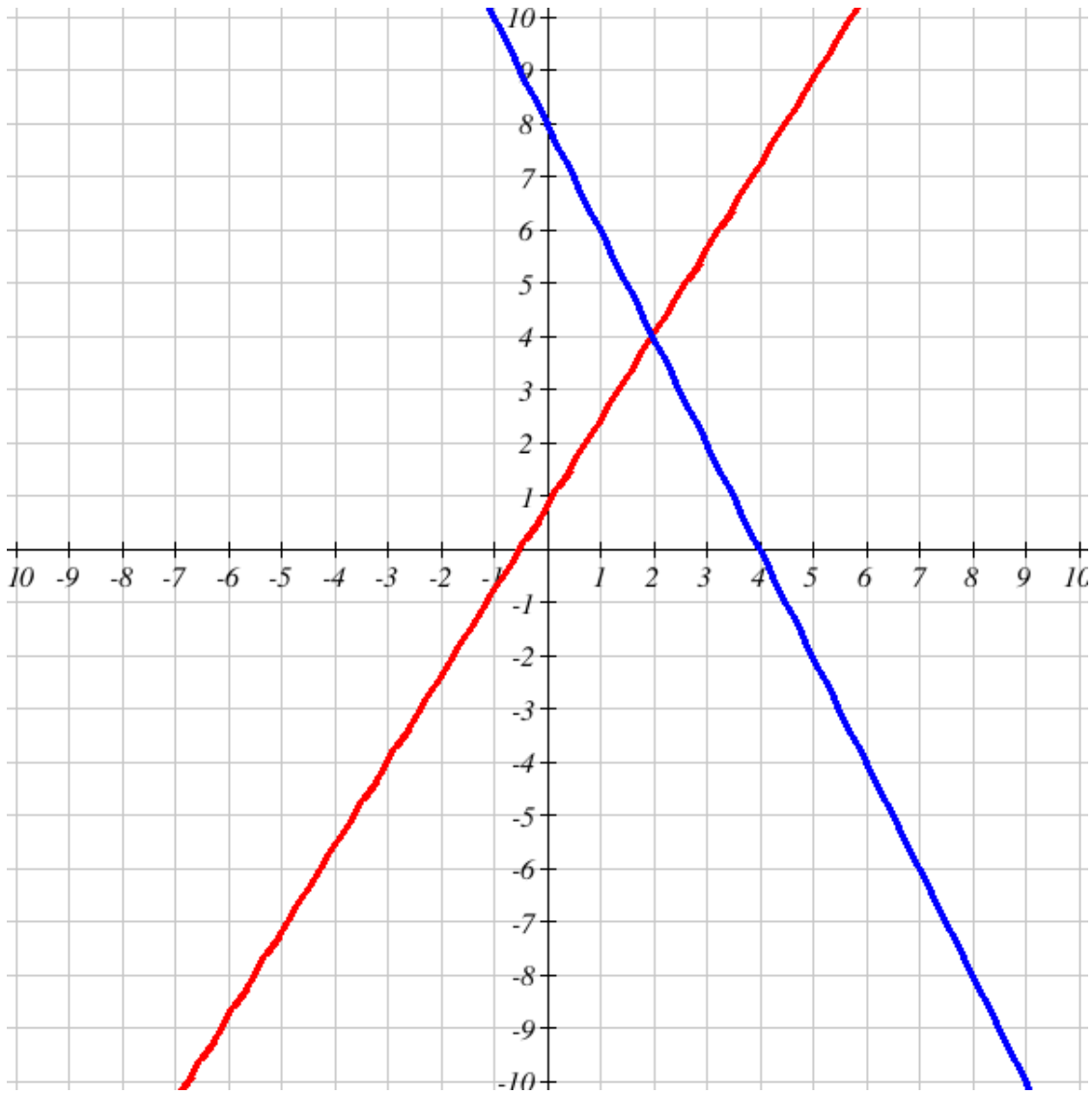
c. $f(x) = -4$ if _____

10.
Given the graph of the function $f(x)$ shown below, solve the inequalities. Express your answer as an inequality



- a. $f(x) > 5$ if _____
- b. $f(x) < 5$ if _____
- c. $f(x) = 5$ if _____

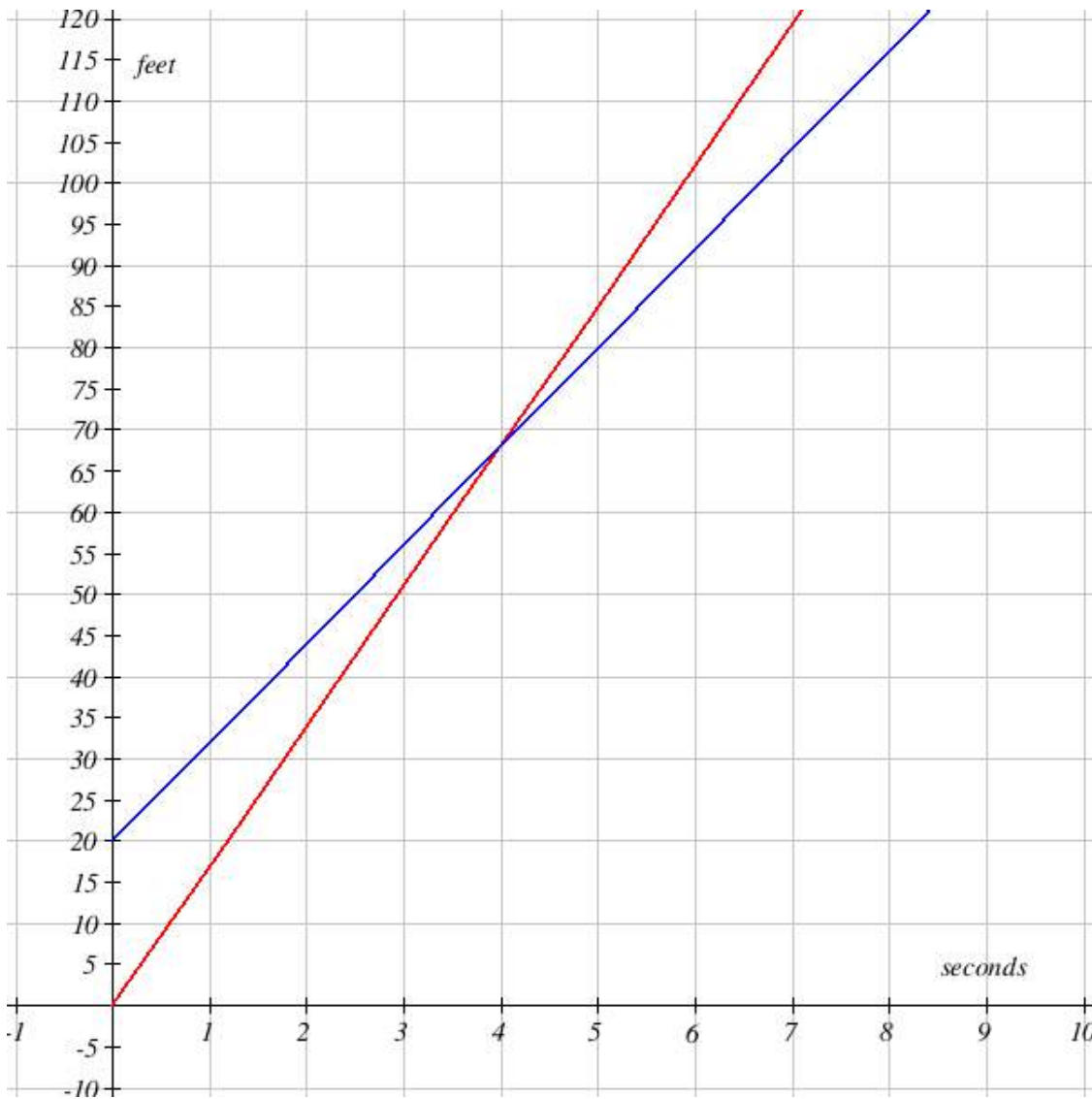
11.
In the graph below, the red line is the graph of Y_1 and the blue line is the graph of Y_2 . Use it to solve the inequality $Y_1 > Y_2$.



Answer: _____

12.

Quinn and Kimberly are going to have a footrace. One of them decides to cheat and gets a head start on the other. The graphs below show Quinn's (blue) and Kimberly's (red) distances (in feet) from the starting line x seconds after the race begins.



How much of a headstart did the cheater get? _____ feet

Who is the faster runner?

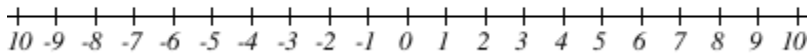
- Quinn
- Kimberly

On what interval of time was Quinn ahead of Kimberly?(Express your answer in interval notation)

Solution: _____

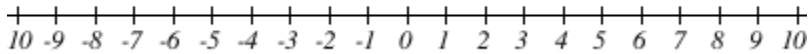
13.

Graph the solution to $|x| \leq 6$.



14.

Graph the solution to $|x| \geq 4$.



15.

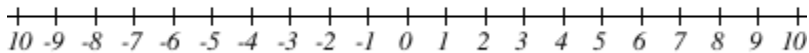
Solve, and give the solution in interval notation. Answer DNE if no solution exists.

$$|6x - 3| \geq 7$$

The solution set is _____

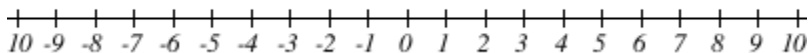
16.

Graph the solution to $|4x - 12| < 16$.



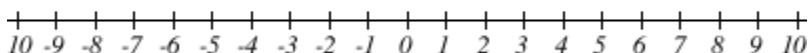
17.

Graph the solution to $|2x - 4| \leq 10$.



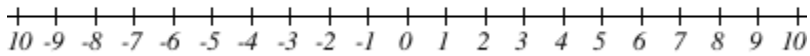
18.

Graph the solution to $|4x - 4| > 24$.



19.

Graph the solution to $|5x + 10| \geq 25$.



20.

Solve, and give the solution in interval notation. Answer DNE is no solution exists.

$$10|6b + 9| + 5 \leq 75$$

The solution is _____

21.

Solve, and give the solution in interval notation. Answer DNE if no solution exists.

$$9|8x - 3| + 4 \geq 85$$

The solution is _____

22.

Solve, and give the solution in interval notation. Answer DNE if no solution exists.

$$7|7b - 8| + 16 \leq 9$$

The solution is _____

23.

Solve, and give the solution in interval notation. Answer DNE if no solution exists.

$$21|9x + 2| + 30 \geq 9$$

The solution is _____

Lecture 18

Solving polynomial and rational inequalities, key numbers, number line analysis

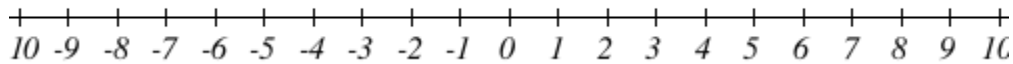
1.

Solve the following inequality and graph the solution:

$$(x + 6)(x - 4) > 0$$

Write the solution as a compound inequality: _____

And draw the solution



2.

Solve the following quadratic inequality: $x^2 - 7x + 12 \geq 0$

Write your answer in interval notation: _____

3.

Solve the following quadratic inequality: $x^2 + 3x - 18 > 0$

Write your answer in interval notation: _____

4.

Solve the following quadratic inequality: $3x^2 + 5x - 2 > 0$

Write your answer in interval notation: _____

5.

Solve the inequality: $r^2 > -r - 12$

Give your answer in interval notation. Answer DNE if there is no solution.

Solution: _____

6.

Solve the inequality and write the solution in interval notation. Leave answers in exact, simplified form.

$$2x^2 + 4 \leq 8x$$

Solution: _____

7.

Solve the polynomial inequality: $4(x - 6)(x + 5)(x + 4) < 0$

Give your answer in interval notation. Answer DNE if there is no solution.

Solution: _____

8.

Solve the inequality: $r^2 \leq r + 72$

Give your answer in interval notation. Answer DNE if there is no solution.

Solution: _____

9.

Solve the polynomial inequality: $x^3(x - 3)^2(x + 7) \geq 0$

Solution: _____

10.

Solve the polynomial inequality: $(x - 4)(x + 5)(x - 2)^2 < 0$

State your answer using interval notation.

Solution: _____

11.

Solve the following inequality: $x^4 - 20x^2 + 64 > 0$

Write your answer in interval notation.

Solution: _____

12.

Solve the inequality $\frac{x+9}{x+8} < 3$

Give your answer in interval notation.

Solution: _____

13.

Solve the given inequality. Present your answer in interval notation.

$$\frac{-6x + 7}{7x - 1} \leq 0$$

Solution: _____

14.

Solve the given inequality. Present your solution in interval notation.

$$\frac{-7x - 1}{x - 6} \leq -\frac{20}{9}$$

Solution: _____

15.

Solve the inequality $\frac{1}{x+1} < 2$

Give your answer in interval notation.

Solution: _____

16.

Solve the following rational inequality $\frac{x-3}{x^2-49} > 0$.

State your answer using interval notation.

Solution: _____

17.

Solve the following rational inequality $\frac{x-4}{x^2-36} < 0$.

Give your answer using interval notation.

Solution: _____

18.

Solve the following inequality: $\frac{x^2-7x+12}{x^2+10x+25} \geq 0$

Give you answer using interval notation.

Solution: _____

19.

Solve the rational inequality. Write your answer in interval notation.

$$\frac{3x}{7-x} < x$$

Solution: _____

20.

Solve the rational inequality. Write your answer in interval notation.

$$\frac{4x}{3-x} \geq 4x$$

Solution: _____

21.

Solve the rational inequality. Write your answer in interval notation.

$$\frac{8}{x-3} > \frac{6}{x-1}$$

Solution: _____

22.

Solve the rational inequality. Write your answer in interval notation.

$$\frac{(x+12)(x-4)}{x-1} \geq 0$$

Solution: _____

Lecture 19

Linear systems and solving techniques, nonlinear systems and solving

1.

Which of these is the definition of solution for a system of equations?

- A value for the variable that makes the equation true.
- Where they intersect.
- An ordered pair that makes both equations true.
- The answer.

2.

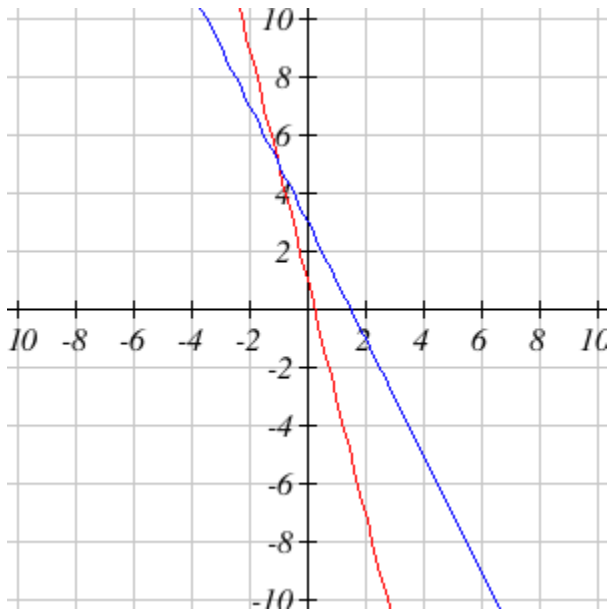
Consider the following system of equations:

$$\begin{cases} -2x - 4y = -40 \\ -6x - 5y = -64 \end{cases}$$

True or False: The point (4,8) is a solution of the system.

3.

Consider the following graph of a system of equations:

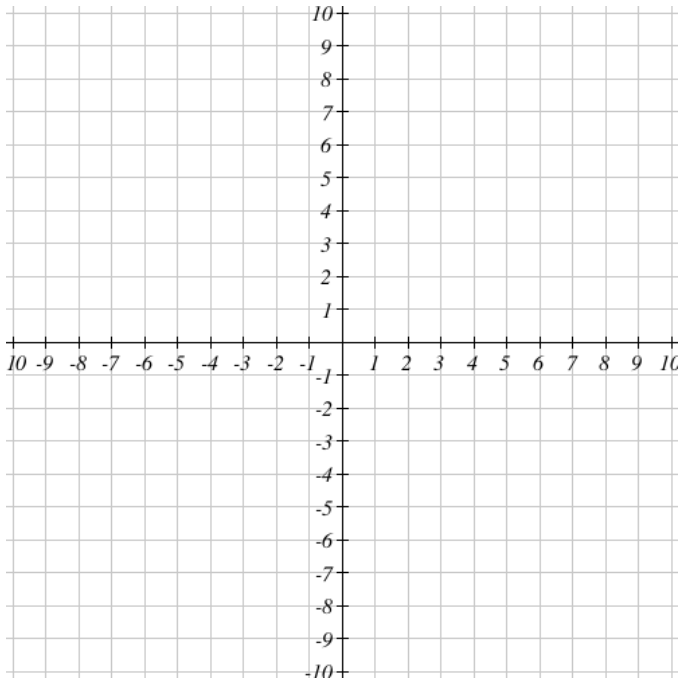


What is the solution of the system? _____

4.

Solve the system of equations by graphing: $\begin{cases} y = -2x + 8 \\ y = 2x + 4 \end{cases}$

First graph each line, then write the solution to the system.

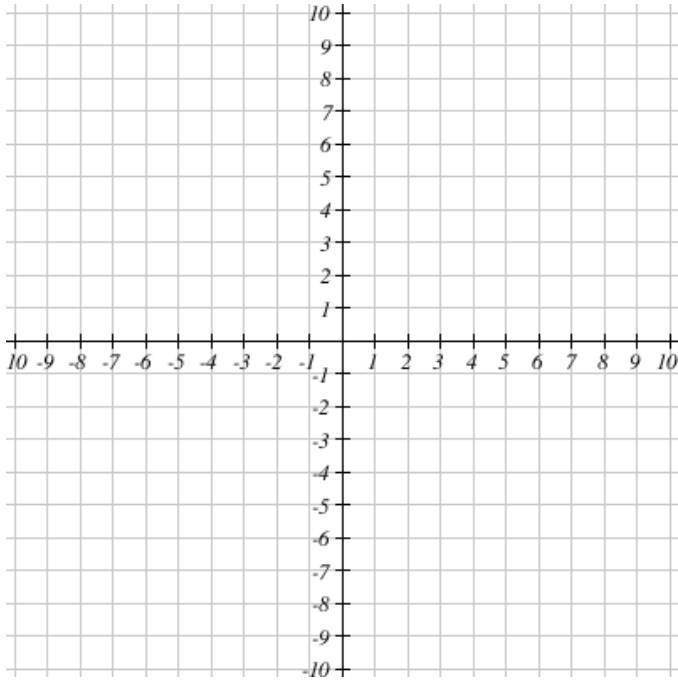


The solution to the system is _____

5.

Solve the system of equations by graphing the equations:

$$\begin{cases} y = 3 \\ x = -7 \end{cases}$$

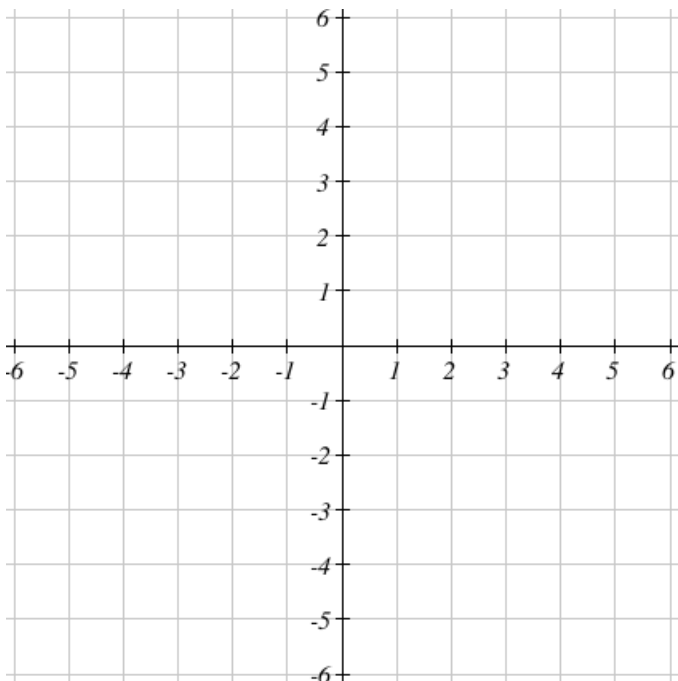


The solution to the system is _____

6.

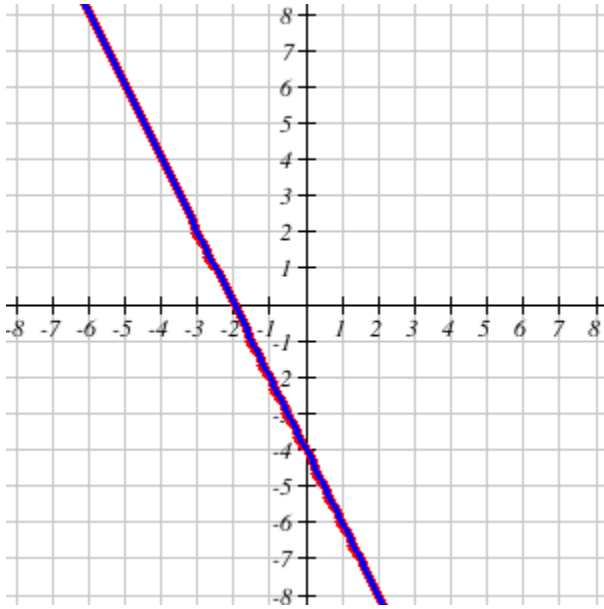
Solve the system of equations by graphing:

$$\begin{cases} 2x + y = -4 \\ 3x + 6y = 12 \end{cases}$$



The solution is _____

7.



1) How many solutions does the system graphed above have?

- No solutions.
- An infinite number of solutions.
- One solution.

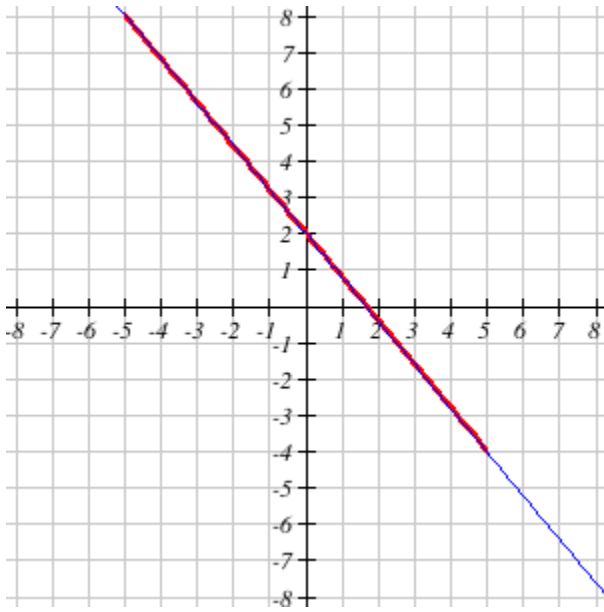
2) Is this system consistent or inconsistent?

- This is a consistent system.
- This is an inconsistent system.

3) Are the equations in this system independent, dependent, or neither?

- The equations are dependent.
- The equations are independent.
- The equations are neither independent nor dependent.

8.



For the system red and blue lines above,
there are...

- no solutions.
- an infinite number of solutions.

9.

Convert the equations in the system below into slope-intercept form and then classify the system.

$$4 \cdot x + y = -4$$

$$8 \cdot x + 2 \cdot y = -8$$

In slope-intercept form, the first equation is $y = \underline{\hspace{2cm}}$

In slope-intercept form, the second equation is $y = \underline{\hspace{2cm}}$

The system is...

- Inconsistent
- Consistent-dependent
- Consistent and Independent

10.

Solve the system by substitution. Give coordinates as integers or reduced fractions.

$$\begin{cases} -4x - 7y = -37 \\ y = -6x + 27 \end{cases}$$

- One or more solutions: _____
- No solution
- Infinite number of solutions

11.

Solve the following system of equations with the substitution method:

$$\begin{cases} x + y = -8 \\ y = 3x \end{cases}$$

Answer: $(x, y) = (\text{_____}, \text{_____})$

Give your answers as integers or as reduced fractions.

12.

Find the point at which the line $f(x) = -5x - 2$ intersects the line $k(x) = -1x - 2$

$(x, y) = (\text{_____}, \text{_____})$

Give coordinates as integers or reduced fractions.

13.

Solve the system using substitution.

$$\begin{cases} -5x + 5y = 35 \\ -7x + y = 31 \end{cases}$$

- One solution: _____
- No solution
- Infinite number of solutions

14.

Find the point at which the line $f(x) = -2.7x - 12.24$ intersects the line $g(x) = 4.3x + 10.16$.

Write the values of x and y in decimal form.

$(x, y) = (\text{_____}, \text{_____})$

15.

Solve the given linear system of equations:

$$\begin{cases} 6x - 9y = 12 \\ -8x + 12y = -16 \end{cases}$$

- One solution: _____
- No solution
- Infinite number of solutions

16.

Solve the system with the addition method:

$$\begin{cases} 8x - 10y = 28 \\ x - 3y = 0 \end{cases}$$

- One solution: _____
- No solution
- Infinite number of solutions

17.

Solve the system by elimination.

$$\begin{cases} 4x - 3y = 20 \\ -5x + 2y = -25 \end{cases}$$

- One solution: _____
- No solution
- Infinite number of solutions

18.

Solve by Elimination/Addition

$$\begin{cases} -2x + 8y = 28 \\ -3x + 12y = 40 \end{cases}$$

- One solution: _____
- No solution
- Infinite number of solutions

19.

Aaron is 2 years older than Diana. In 6 years the sum of their ages will be 40 . How old is Aaron now?

_____ years old

20.

The admission fee at an amusement park is \$2.25 for children and \$6.80 for adults. On a certain day, 364 people entered the park, and the admission fees collected totaled \$1729 . How many children and how many adults were admitted?

There were _____ children admitted.

There were _____ adults admitted

21.

Diana puts x dollars into an investment with an interest rate of 6 percent per year and y dollars into an investment with an interest rate of 9 percent per year. She invests a total of \$8200, and her interest earnings after one year are \$600. From this information, we can create two equations: one for the total investment and one for the interest earned. State both equations, and then solve the system to determine how much Diana invested in each.

The equation that describes the total investment is _____

The equation that describes the interest earned is _____

Amount invested at 6 percent interest is \$ _____

Amount invested at 9 percent interest is \$ _____

22.

A jar contain 800 coins, each of which is gold or silver. A gold coin is valued at \$8.00 while a silver coin is valued at \$4.00 . The total value in the jar is \$5000 . Which of the following can be used to model the information if x represents the number of gold coins and y represents the number of silver coins?

- $\begin{cases} x + y = 5000 \\ 8x + 4y = 800 \end{cases}$
- $\begin{cases} 8x + 4y = 800 \\ 8x + y = 5000 \end{cases}$
- $\begin{cases} x + y = 800 \\ 4x + 8y = 5000 \end{cases}$
- $\begin{cases} x + y = 800 \\ 8x + 4y = 5000 \end{cases}$
- $\begin{cases} x - y = 800 \\ 4x + 8y = 4800 \end{cases}$

23.

A man has 14 coins in his pocket, all of which are dimes (worth \$0.10 each) and quarters (worth \$0.25 each). If the total value of his change is \$2.15 , how many dimes and how many quarters does he have?

He has _____ dimes.

He has _____ quarters.

24.

Carlos needs to produce 2000 milliliters of 50% alcohol solution. At his disposal he has 20% alcohol solution and 60% alcohol solution. How much of each does he need in order to produce his desired solution?

He needs _____ milliliters of 20% solution.

He needs _____ milliliters of 60% solution

25.

Use the elimination method to find all solutions of the system:

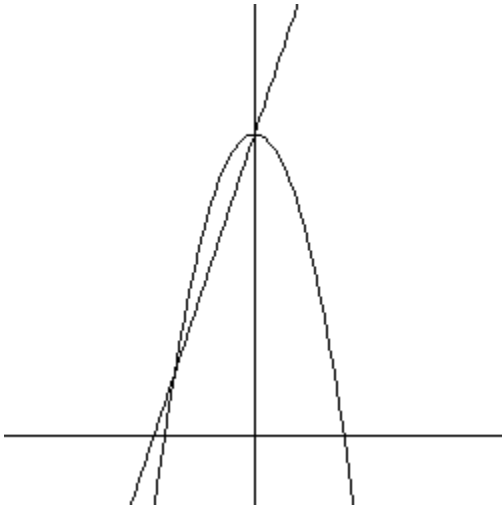
$$\begin{cases} x^2 + y^2 = 5 \\ x^2 - y^2 = 1 \end{cases}$$

The four solutions of the system are: _____

26.

Find the solutions to the system of nonlinear equations given by:

$$\begin{cases} y = 2x + 5 \\ y = -x^2 + 5 \end{cases}$$



Enter your answer as a list of ordered pairs. Solution: _____

27.

Use the elimination method to find all solutions of the system:

$$\begin{cases} x^2 - 2y = 22 \\ x^2 + 5y = 1 \end{cases}$$

The two solutions to the system are: _____

28.

Use the elimination method to find all solutions of the system:

$$\begin{cases} 3x^2 - y^2 = 11 \\ x^2 + 4y^2 = 8 \end{cases}$$

The four solutions of the system are: $(-a, -b)$, $(-a, b)$, $(a, -b)$, and (a, b) .

Using positive numbers find:

$a =$ _____ and $b =$ _____.

29.

Find all solutions of the system:

$$\begin{cases} y = 1 - x^2 \\ y = x^2 - 1 \end{cases}$$

The two solutions of the system are: _____

30.

Use the substitution method to find all solutions of the system:

$$\begin{cases} y = x - 2 \\ xy = 3 \end{cases}$$

The solutions of the system are:

$$x_1 = \text{_____}, y_1 = \text{_____} \text{ and}$$
$$x_2 = \text{_____}, y_2 = \text{_____}$$

31.

A rectangle has an area of 150 cm^2 and a perimeter of 50 cm , and its length is longer than its width.

What are its dimensions?

Its length is _____ cm

Its width is _____ cm.

32.

Construct a consistent and independent system of equations that has $(4,7)$ as its solution. Use x and y as your variables, and put your equations in the form $Ax + By = C$ with $A \neq 0$ and $B \neq 0$. Note that there are many possible correct answers.

Lecture 20

Definitions/Terminology, Graphing, one-to-one property and solving, compound interest, natural base, continuous compounding, applications

1.

Identify whether the following equation represents an exponential function.

$$h(x) = -99x + 71$$

- exponential function
- NOT an exponential function

2.

Determine whether the following equation represents an exponential growth or exponential decay.

$$y = 280 \cdot (1.81)^x$$

- exponential growth
- exponential decay

3.

Determine whether the following equation represents exponential growth or exponential decay.

$$y = \left(\frac{13}{29}\right) \cdot \left(\frac{2}{27}\right)^x$$

- exponential decay
- exponential growth

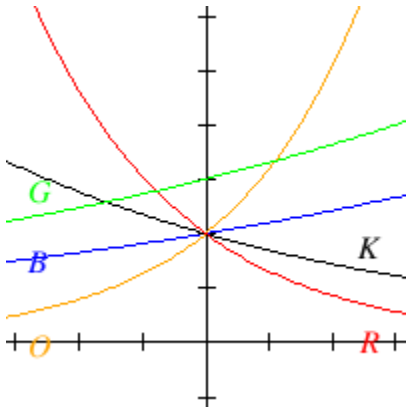
4.

Introduction to Exponential Functions

Which of the following are true regarding $f(x) = a(b)^x$? Assume $a > 0$.

- The Range of the exponential functions is All Real Numbers .
- The Range of the exponential functions is $f(x) > 0$.
- The Domain of the exponential functions is All Real Numbers .
- The Horizontal Asymptote is the point $(0, a)$.
- The Horizontal Asymptote is the line $y = 0$.
- The Horizontal Asymptote is the line $x = 0$.
- The Domain of the exponential functions is $x > 0$.

5.



If all the graphs above have equations with form $y = ab^x$,

Which graph has the largest value for b ?

- blue (B)
- red (R)
- green (G)
- orange (O)
- black (K)

Which graph has the smallest value for b ?

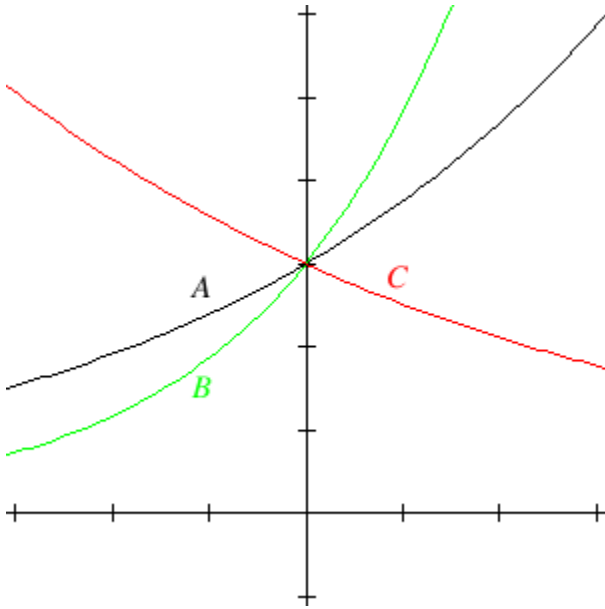
- blue (B)
- black (K)
- orange (O)
- green (G)
- red (R)

Which graph has the largest value for a ?

- blue (B)
- green (G)
- orange (O)
- black (K)
- red (R)

6.

Match the graph with its function



- $3(0.84)^x$
- $3(1.6)^x$
- $3(1.25)^x$

7.

An exponential function $f(x) = ab^x$ passes through the points $(0, 8)$ and $(3, 512)$. What are the values of a and b ?

$a =$ _____
 $b =$ _____

8.

An exponential function $f(x) = ab^x$ passes through the points $(0, 7000)$ and $(3, 5103)$. What are the values of a and b ?

$a =$ _____
 $b =$ _____

9.

Find a formula for the exponential function passing through the points $(-3, \frac{3}{64})$ and $(3, 192)$.

$y =$ _____

10.

The fox population in a certain region has an annual growth rate of 6 percent per year. It is estimated that the population in the year 2000 was 28100.

(a) Find a function that models the population t years after 2000 ($t = 0$ for 2000).

Your answer is $P(t) =$ _____

(b) Use the function from part (a) to estimate the fox population in the year 2008.

Your answer is (the answer should be an integer) _____

11.

A population numbers 12,000 organisms initially and grows by 3.4% each year.

Suppose P represents population and t represents the number of years of growth. An exponential model for the population can be written in the form $P = ab^t$, where

$P =$ _____

12.

A population numbers 19,000 organisms initially and decreases by 8% each year.

Suppose P represents population, and t the number of years of growth. An exponential model for the population can be written in the form $P = ab^t$, where

$P =$ _____

13.

A vehicle purchased for \$ 29800 depreciates at a constant rate of 4 %.

Determine the approximate value of the vehicle 12 years after purchase.

Round to the nearest whole number.

14.

A radioactive substance decays exponentially. A scientist begins with 110 milligrams of a radioactive substance. After 30 hours, 55 mg of the substance remains.

How many milligrams will remain after 39 hours?

_____mg

Give your answer accurate to at least one decimal place.

15.

A car was valued at \$36,000 in the year 1995. The value depreciated to \$10,000 by the year 2000.

A) What was the annual rate of change between 1995 and 2000?

$r =$ _____ *Round the rate of decrease to 4 decimal places.*

B) What is the correct answer to part A written in percentage form?

$r =$ _____ %.

C) Assume that the car value continues to drop by the same percentage. What will the value be in the year 2003 ?

value = \$ _____ *Round to the nearest 50 dollars.*

16.

The population of the world in 2010 was 7 billion and the annual growth rate was estimated at 1.2 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 2057. Round your answer to one decimal place.

The population is estimated to be _____ billion in 2057.

17.

A house was valued at \$105,000 in the year 2012. The value appreciated to \$145,000 by the year 2018.

A) What was the annual growth rate between 2012 and 2018 as a decimal? Round to four decimal places.

$r =$ _____

B) What is the annual growth rate as a percent? Round to two decimal places.

$r =$ _____ %.

C) Assuming that the house value continues to grow at the same rate, what will the value equal in the year 2021? Round to the nearest thousand dollars.

value = \$ _____

18.

A car was valued at \$29,000 in the year 2010. The value depreciated to \$13,000 by the year 2016.

A) What was the annual growth rate between 2010 and 2016 in decimal form? Round to four decimal places.

$r =$ _____

B) What was the annual growth rate in percentage form? Round to two decimal places.

$r =$ _____ %.

C) Assuming that the car's value continues to drop at the same rate, what will the value be in the year 2019? Round to the nearest fifty dollars.

value = \$ _____

19.

In April 1986, a flawed reactor design played a part in the Chernobyl nuclear meltdown. Approximately 14252 becquereles (Bqs), units of radioactivity, were initially released into the environment. Only areas with less than 800 Bqs are considered safe for human habitation. The function $f(x) = 14252(0.5)^{\frac{x}{32}}$ describes the amount, $f(x)$, in becquereles, of a radioactive element remaining in the area x years after 1986.

Find $f(40)$ to determine the amount of becquereles in 2026. _____

Determine if the area is safe for human habitation in the year 2026.

- No, because by 2026, the radioactive element remaining in the area is greater than 800 Bqs.
- No, because by 2026, the radioactive element remaining in the area is less than 800 Bqs.
- Yes, because by 2026, the radioactive element remaining in the area is greater than 800 Bqs.
- Yes, because by 2026, the radioactive element remaining in the area is less than 800 Bqs.

20.

The half-life of caffeine in the human body is about 5.6 hours. A cup of coffee has about 110 mg of caffeine.

- a. Write an equation for the amount of caffeine in a person's body after drinking a cup of coffee? Let C be the milligrams of caffeine in the body after t hours.

$C(t) =$ _____

- b. How much caffeine will remain after 10 hours?

_____ mg

- c. Estimate the time until there are only 20 mg remaining

_____ hours

21.

Starting with the graph of $f(x) = 9^x$, write the equation of the graph that results when:

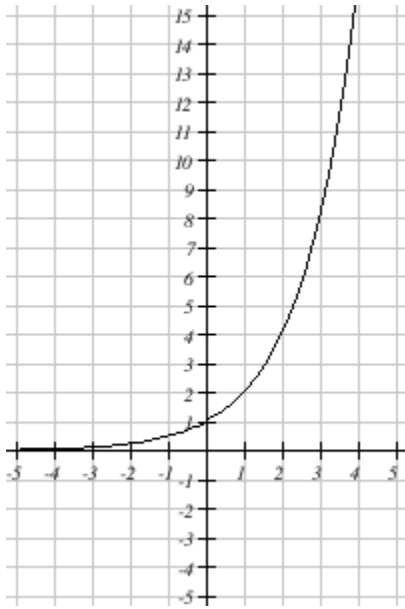
(a) $f(x)$ is shifted 4 units upward. $y =$ _____

(b) $f(x)$ is shifted 1 units to the right. $y =$ _____

(c) $f(x)$ is reflected about the x-axis. $y =$ _____

22.

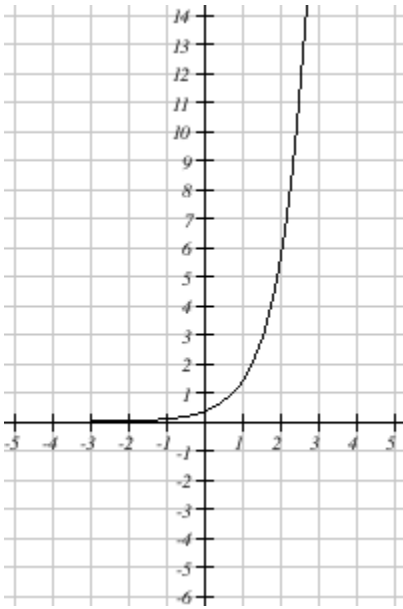
What function is graphed below?



- $f(x) = \left(\frac{1}{5}\right)^x$
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = 2^x$
- $f(x) = 3^x$
- $f(x) = 5^x$
- $f(x) = \left(\frac{1}{3}\right)^x$
- $f(x) = \left(\frac{1}{4}\right)^x$
- $f(x) = 4^x$

23.

The function below has the form $f(x) = a \cdot b^x$.

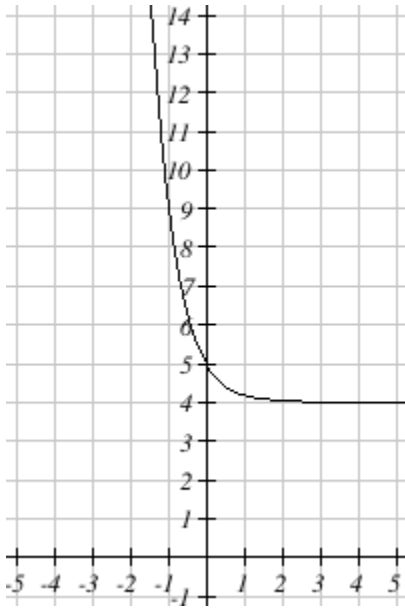


Which of the following functions is shown on the graph?

- $f(x) = 3 \cdot 4^x$
- $f(x) = 3 \cdot \left(\frac{1}{4}\right)^x$
- $f(x) = \frac{1}{3} \cdot 4^x$
- $f(x) = -\frac{1}{3} \cdot 4^x$
- $f(x) = -3 \cdot 4^x$
- $f(x) = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$
- $f(x) = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$
- $f(x) = -3 \cdot \left(\frac{1}{4}\right)^x$

24.

The function below has the form $f(x) = b^x + k$.

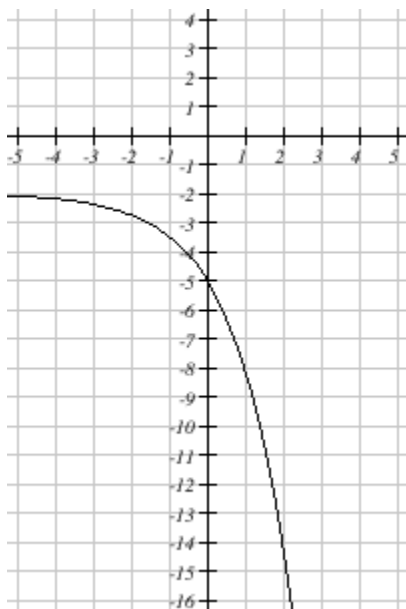


Which of the following functions is shown on the graph?

- $f(x) = \left(\frac{1}{5}\right)^x + 3$
- $f(x) = 5^x + 5$
- $f(x) = \left(\frac{1}{5}\right)^x + 5$
- $f(x) = 5^x + 3$
- $f(x) = \left(\frac{1}{5}\right)^x + 4$
- $f(x) = \left(\frac{1}{5}\right)^x - 4$
- $f(x) = 5^x + 4$
- $f(x) = 5^x - 4$

25.

The function below has the form $f(x) = a \cdot b^x + k$.

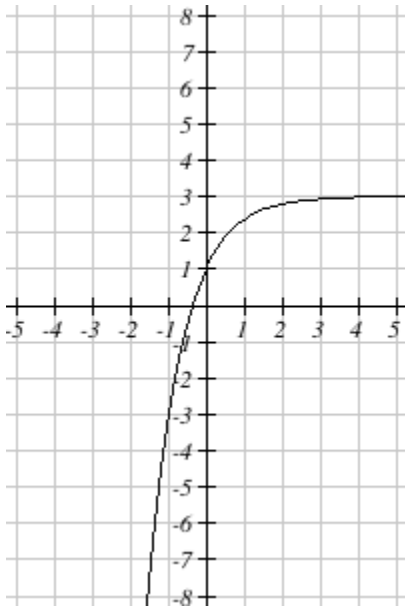


Which of the following functions is shown on the graph?

- $f(x) = 3 \cdot 2^x + 2$
- $f(x) = -\frac{1}{3} \cdot 2^x - 1$
- $f(x) = 3 \cdot 2^x - 2$
- $f(x) = -3 \cdot \left(\frac{1}{2}\right)^x - 3$
- $f(x) = 3 \cdot \left(\frac{1}{2}\right)^x + 2$
- $f(x) = -\frac{1}{3} \cdot \left(\frac{1}{2}\right)^x + 2$
- $f(x) = -3 \cdot 2^x - 2$
- $f(x) = 3 \cdot 2^x - 3$

26.

Find an equation for the graph sketched below



$f(x) = \underline{\hspace{2cm}}$

27.

Describe the long run behavior of $f(n) = 2(4)^n + 3$

As $n \rightarrow -\infty$, $f(n) \rightarrow$

- ∞
- $-\infty$
- 0
- 3

As $n \rightarrow \infty$, $f(n) \rightarrow$

- ∞
- $-\infty$
- 0
- 3

28.

Describe the long run behavior of $f(t) = 2\left(\frac{1}{3}\right)^t - 1$

As $t \rightarrow -\infty$, $f(t) \rightarrow$

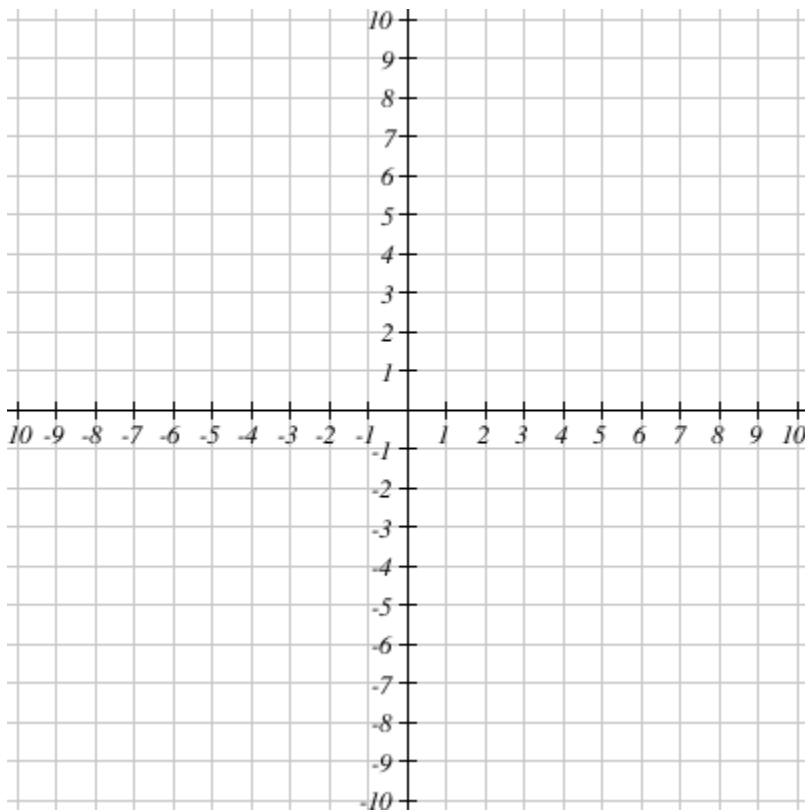
- ∞
- $-\infty$
- 0
- -1

As $t \rightarrow \infty$, $f(t) \rightarrow$

- ∞
- $-\infty$
- 0
- -1

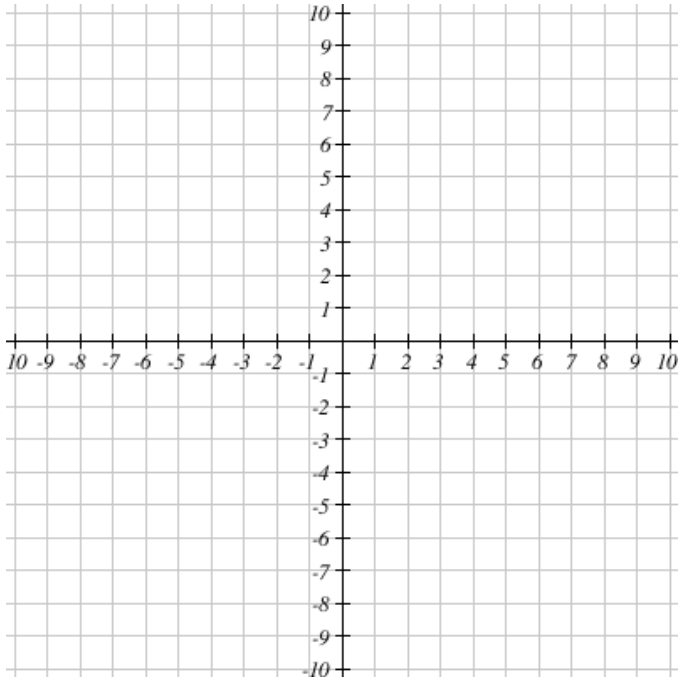
29.

Draw the graph of $f(x) = 3^{x-3}$



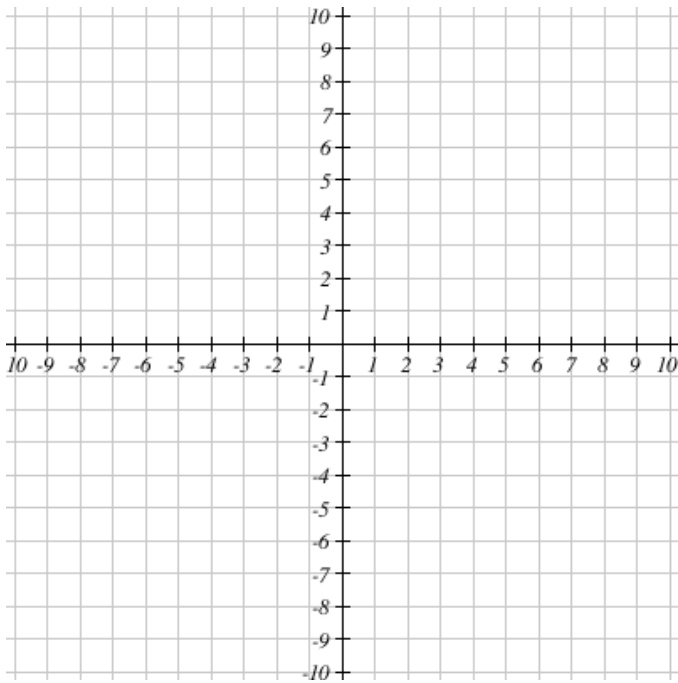
30.

Draw the graph of $f(x) = 4^{x+4}$



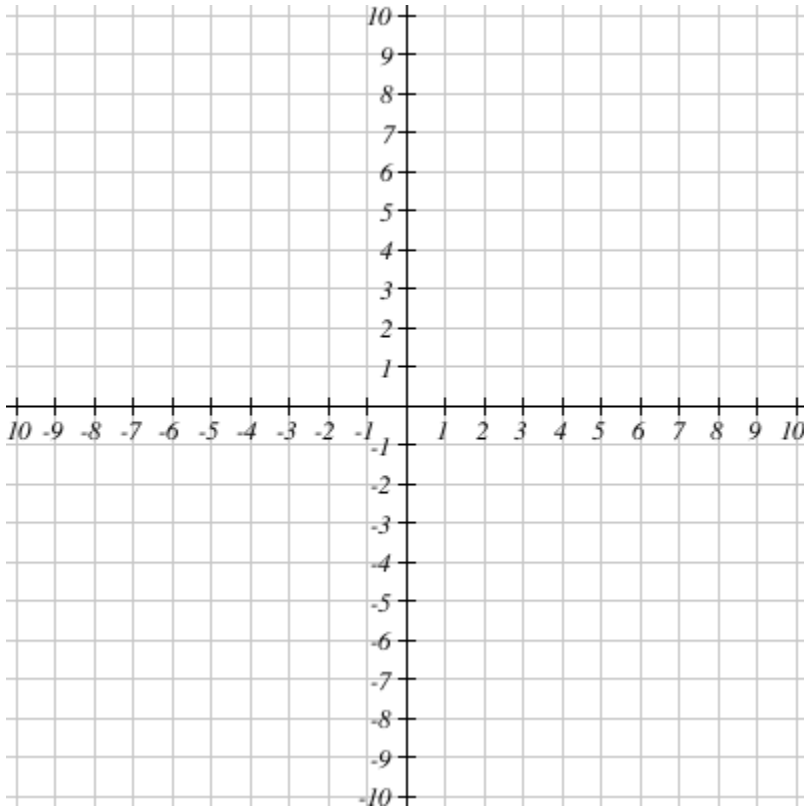
31.

Draw the graph of $f(x) = \left(\frac{1}{5}\right)^{x+2}$



32.

Draw the graph of $f(x) = -\left(\frac{1}{6}\right)^x$



33.

The temperature, $f(t)$ of a cup of coffee, in degrees Celsius, after t minutes can be determined by the equation $f(t) = 58(0.89)^t + 20$.

Estimate the temperature after 5 minutes:
_____ degrees

Determine the asymptote for the function:
The asymptote is _____

Interpret the asymptote in the context of the problem. Use complete sentences and correct units.

34.

Use the like-bases property and exponents to solve the equation

$$\left(\frac{1}{10}\right)^n = 10000$$

$$n = \underline{\hspace{2cm}}$$

35.

Use the like-bases property and exponents to solve the equation

$$10^{n+10} = 10^{6n+6}$$

$$n = \underline{\hspace{2cm}}$$

36.

Use the like-bases property and exponents to solve the equation

$$6(10)^x - 23 = 59977$$

$$x = \underline{\hspace{2cm}}$$

37.

Use the like bases property to solve the equation. Give your answer as an integer or reduced fraction.

$$\left(\frac{1}{64}\right) = 2^{5x-8}$$

$$x = \underline{\hspace{2cm}}$$

38.

Solve the equation $125^{z+5} = 25^{z+4}$.

$$z = \underline{\hspace{2cm}}$$

39.

$$9 \cdot 5^{x+4} = 625 \cdot 3^{x+2}$$

$$x = \underline{\hspace{2cm}}$$

40.

$$2 \cdot 2^{x-1} + 2^{x+8} - \frac{257}{16} = 0$$

$$x = \underline{\hspace{2cm}}$$

Lecture 21

Definitions/Terminology, Graphing, one-to-one property and solving, compound interest, natural base, continuous compounding, applications

1.

Approximate e^3 to 4 decimal places. _____

Approximate $e^{-1.48}$ to 4 decimal places. _____

2.

A) $e^5 \cdot e^8 = e^p$ where $p =$ _____

B) $(e^5)^8 = e^r$ where $r =$ _____

3.

The expression

$$\frac{e^7(2e)^6}{e^3}$$

equals ce^f where

the coefficient c is _____, the exponent f is _____.

4.

Solve the given equation. Enter the exact value of the solution, in simplified form.

$$e^{4b+3} = 1$$

$$b = \underline{\hspace{2cm}}$$

5.

Determine whether the following equation represents an exponential growth or exponential decay.

$$y = 164.5 \cdot (e)^{-1.52 \cdot x}$$

- exponential decay
- exponential growth

6.

Determine whether the following equation represents an exponential growth or exponential decay.

$$y = 43 \cdot (e)^{1.53 \cdot x}$$

- exponential growth
- exponential decay

7.

\$4000 are invested in a bank account at an interest rate of 9 percent per year.

Find the amount in the bank after 7 years if interest is compounded annually. _____

Find the amount in the bank after 7 years if interest is compounded quarterly. _____

Find the amount in the bank after 7 years if interest is compounded monthly. _____

Finally, find the amount in the bank after 7 years if interest is compounded continuously. _____

8.

A bank features a savings account that has an annual percentage rate of $r = 2.2\%$ with interest compounded quarterly. Stephanie deposits \$2,500 into the account.

The account balance can be modeled by the exponential formula $A(t) = a \left(1 + \frac{r}{k}\right)^{kt}$, where A is account value after t years, a is the principal (starting amount), r is the annual percentage rate, k is the number of times each year that the interest is compounded.

(A) What values should be used for a , r , and k ?

$$a = \underline{\hspace{2cm}}, \quad r = \underline{\hspace{2cm}}, \quad k = \underline{\hspace{2cm}}$$

(B) How much money will Stephanie have in the account in 7 years?

$$\text{Amount} = \$ \underline{\hspace{2cm}}$$

Round answer to the nearest penny.

(C) What is the annual percentage yield (APY) for the savings account? (The APY is the actual or effective annual percentage rate which includes all compounding in the year).

$$\text{APY} = \underline{\hspace{2cm}}\%$$

Round answer to 3 decimal places.

9.

The fox population in a certain region has a continuous growth rate of 8 percent per year. It is estimated that the population in the year 2000 was 15400.

(a) Find a function that models the population t years after 2000 ($t = 0$ for 2000).

Your answer is $P(t) =$ _____

(b) Use the function from part (a) to estimate the fox population in the year 2008.

Your answer is _____ (the answer must be an integer)

10.

Convert the equation $f(t) = 336e^{0.2t}$ to the form $f(t) = ab^t$

$a =$ _____

$b =$ _____

Give answers accurate to three decimal places

11.

Convert the equation $f(t) = 178e^{0.835t}$ to the form $f(t) = a(b)^t$. Round b to three decimal places.

$f(t) =$ _____

12.

The number of bacteria in a culture is given by the function

$$n(t) = 960e^{0.25t}$$

where t is measured in hours.

(a) What is the relative rate of growth of this bacterium population?

Your answer is _____ percent

(b) What is the initial population of the culture (at $t=0$)?

Your answer is _____

(c) How many bacteria will the culture contain at time $t=5$?

Your answer is _____

13.

For each nominal exponential growth/decay described below, find the effective annual growth rate and express it as a percentage rounded to one decimal place.

A quantity's size after t years is given by $A(t) = (1.07)^t$. Its effective growth rate is _____% per year.

A quantity shrinks at a continuous rate of 40% per year. Its effective growth rate is _____% per year.

A quantity grows at a rate of 20% compounded monthly. Its effective growth rate is _____% per year.

A quantity has a half-life of 12 years. Its effective annual growth rate is _____% per year.

A quantity has a tripling time of 7 years. Its effective annual growth rate is _____% per year.

14.

A bacteria culture initially contains 2500 bacteria and doubles every half hour.

Find the size of the bacterial population after 40 minutes. _____

Find the size of the bacterial population after 8 hours. _____

15.

The half-life of Radium-226 is 1590 years. If a sample contains 400 mg, how many mg will remain after 2000 years? _____

16.

You want to have \$700,000 when you retire in 30 years. If you can earn 8% interest compounded monthly, how much would you need to deposit now into the account to reach your retirement goal?

\$ _____

17.

A radioactive substance decays exponentially. A scientist begins with 160 milligrams of a radioactive substance. After 25 hours, 80 mg of the substance remains. How many milligrams will remain after 45 hours?

_____ mg

Give your answer accurate to at least one decimal place

18.

Let $P(t) = 25(1 - e^{-kt}) + 53$ represent the expected score for a student who studies t hours for a test. Suppose $k = 0.17$ and test scores must be integers.

What is the highest score the student can expect? _____

If the student does not study, what score can he expect? _____

Lecture 22

Definitions/Terminology, Properties (Basic, inverse, one-to-one), Graphs, Natural log, Common log

1.

Write the equation in exponential form.

Assume that all constants are positive and not equal to 1.

$$\log_y(m) = b$$

2.

Write the equation in exponential form.

Assume that all constants are positive and not equal to 1.

$$\log(s) = c$$

3.

Write the equation in logarithmic form.

Assume that all constants are positive and not equal to 1.

$$9^r = n$$

4.

Write the equation in logarithmic form.

Assume that all constants are positive and not equal to 1.

$$10^c = v$$

5.

Fill in each box below with an integer or a reduced fraction.

(a) $\log_2 8 = 3$ can be written in the form $2^A = B$ where
 $A = \underline{\hspace{2cm}}$ and $B = \underline{\hspace{2cm}}$

(b) $\log_5 25 = 2$ can be written in the form $5^C = D$ where
 $C = \underline{\hspace{2cm}}$ and $D = \underline{\hspace{2cm}}$

6.

Fill in each box below with an integer or a reduced fraction.

(a) $\log_4 2 = \frac{1}{2}$ can be written in the form $A^B = C$ where
 $A = \underline{\hspace{2cm}}$, $B = \underline{\hspace{2cm}}$, and $C = \underline{\hspace{2cm}}$

(b) $\log_2 \left(\frac{1}{4}\right) = -2$ can be written in the form $D^E = F$ where
 $D = \underline{\hspace{2cm}}$, $E = \underline{\hspace{2cm}}$, and $F = \underline{\hspace{2cm}}$

7.

Express the equation in logarithmic form:

(a) $e^x = 4$ is equivalent to $\ln A = B$. Then

$A = \underline{\hspace{2cm}}$

and

$B = \underline{\hspace{2cm}}$

(b) $e^3 = x$ is equivalent to $\ln C = D$. Then

$C = \underline{\hspace{2cm}}$

and

$D = \underline{\hspace{2cm}}$

8.

Express the following equations in logarithmic form:

(a) $3^4 = 81$ is equivalent to the logarithmic equation: $\underline{\hspace{2cm}}$

(b) $10^{-3} = 0.001$ is equivalent to the logarithmic equation: $\underline{\hspace{2cm}}$

9.

Express equation equation in logarithmic form.

(a) $e^x = 8$ is equivalent to the logarithmic equation: $\underline{\hspace{2cm}}$

(b) $e^5 = x$ is equivalent to the logarithmic equation: $\underline{\hspace{2cm}}$

10.

Write the equation in exponential form. Assume that all constants are positive and not equal to 1.

$$\log_{343}(7) = \frac{1}{3}$$

11.

Simplify without a Calculator

$$\log_8(512) = \underline{\hspace{2cm}}$$

12.

Simplify without a Calculator

$$\log\left(\frac{1}{10}\right) = \underline{\hspace{2cm}}$$

13.

Find the logarithm.

$$\log_5\left(\frac{1}{5}\right) = \underline{\hspace{2cm}}$$

14.

Find the logarithm.

$$\log_5\left(\frac{1}{3125}\right) = \underline{\hspace{2cm}}$$

15.

Evaluate the following expressions. Your answers must be exact and in simplest form.

(a) $\ln e^{-5} = \underline{\hspace{2cm}}$

(b) $e^{\ln 3} = \underline{\hspace{2cm}}$

(c) $e^{\ln\sqrt{2}} = \underline{\hspace{2cm}}$

(d) $\ln\left(\frac{1}{e^5}\right) = \underline{\hspace{2cm}}$

16.

Evaluate the following expressions without using a calculator.

(a) $\ln\left(\frac{1}{e^7}\right) = \underline{\hspace{2cm}}$

(b) $\ln(\sqrt[5]{e^2}) = \underline{\hspace{2cm}}$

(c) $\ln(e^5) = \underline{\hspace{2cm}}$

(d) $e^{\ln(5)} = \underline{\hspace{2cm}}$

(e) $e^{\ln(\sqrt{3})} = \underline{\hspace{2cm}}$

17.

Solve: $\log_6(t) = 7$

$t = \underline{\hspace{2cm}}$

18.

If $\log_2(5x + 2) = 6$, then $x = \underline{\hspace{2cm}}$.

19.

If $\ln(5x + 5) = 4$, then $x = \underline{\hspace{2cm}}$.

20.

Solve for x :

$\log_2(x^9) = 8$

$x = \underline{\hspace{2cm}}$

21.

Solve for x :

$(\log_4(\log_3 x)) = -3$

$x = \underline{\hspace{2cm}}$

22.

Simplify

$$\log_z(z^9) = \underline{\hspace{2cm}}$$

23.

Solve for x:

$$\log_x(2) = 1$$

$$x = \underline{\hspace{2cm}}$$

24.

Find the logarithm.

$$\log_3\left(3^{\frac{1}{5}}\right) = \underline{\hspace{2cm}}$$

25.

Find the logarithm.

$$\log(10,000) = \underline{\hspace{2cm}}$$

26.

Find the logarithm.

$$\log\left(\frac{1}{100,000}\right) = \underline{\hspace{2cm}}$$

27.

Evaluate the following expressions.

$$(a) \log_7 7^{12} = \underline{\hspace{2cm}}$$

$$(b) \log_2 32 = \underline{\hspace{2cm}}$$

$$(c) \log_4 1024 = \underline{\hspace{2cm}}$$

$$(d) \log_4 4^4 = \underline{\hspace{2cm}}$$

28.

Simplify

$$\log_x \left(\frac{1}{(x^6)^7} \right) = \underline{\hspace{2cm}}$$

29.

Solve for x in each equation below. It may be helpful to convert the equation into exponential form.

(A) $\log_a a^{-2} = x$

$$x = \underline{\hspace{2cm}}$$

(B) $\log_a a^4 = x$

$$x = \underline{\hspace{2cm}}$$

(C) $\log_a a = x$

$$x = \underline{\hspace{2cm}}$$

(D) $\log_a a^n = x$

$$x = \underline{\hspace{2cm}}$$

30.

Evaluate using your calculator, giving at least 3 decimal places:

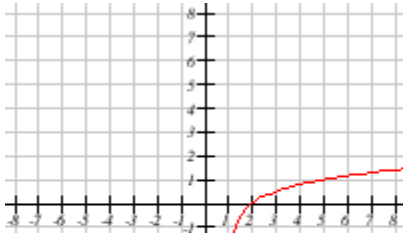
$$\log(680) = \underline{\hspace{2cm}}$$

31.

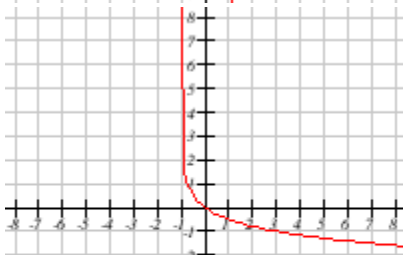
Find the equation of the vertical asymptote $f(x) = \log(x - 4)$

32.

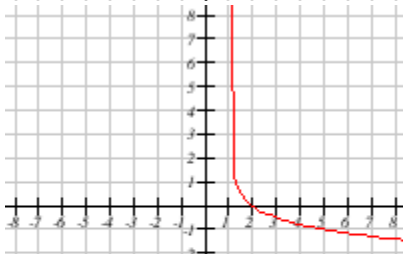
Choose the graph of $y = \log_{\frac{1}{4}}(x + 1)$



•



•

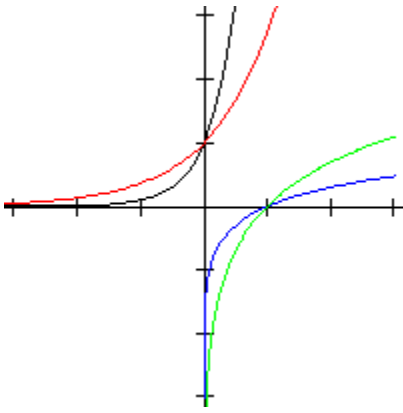


•



•

33.



Match each equation with a graph above:

- $\ln(x)$
 - e^x
 - $\log(x)$
 - 10^x
1. black
 2. red
 3. blue
 4. green

34.

Find the domain of $y = \log(5 - 5x)$.

The domain is: _____

35.

Find the domain of $\log(x^2 + x - 2)$ in interval notation.

36.

Find the domain of $\log(x^2 + 2x - 3)$ in interval notation.

37.

Find the domain and range of $y = \log_2(5 + 2x)$.

The domain is: _____

The range is : _____

38.

The Richter Scale reading of an earthquake is based on a logarithmic equation:

$$R = \log\left(\frac{A}{A_0}\right)$$

where

A - the measure of the amplitude of the earthquake wave

A_0 the amplitude of the smallest detectable wave (or standard wave).

An earthquake is measured with a wave amplitude $A = 0.0982$ while the smallest detectable was A_0 is measured at 0.0001 cm. What is the magnitude of this earthquake using the Richter scale, to the nearest tenth?

The earthquake registered _____ on the Richter scale.

39.

The pH scale for acidity is defined by $\text{pH} = -\log_{10}[\text{H}^+]$ where $[\text{H}^+]$ is the concentration of hydrogen ions measured in moles per liter (M).

A solution has a pH of 12.6.

Calculate the concentration of hydrogen ions in moles per liter (M).

The concentration of hydrogen ions is _____ moles per liter.

40.

The pH reading of a sample of each substances is given. Calculate the hydrogen ion concentration of the substance.

(a) Vinegar: pH = 3.0.

Your answer is _____.

(b) Milk: pH = 6.5.

Your answer is _____.

Lecture 23

Properties (Multiplication, division, exponent), expanding/combining expressions, common mistakes, change of base formula, measuring with logs

1.

Find the exact value for the following expression without a calculator

$$\frac{\log_2(625) - \log_2(625)}{\log_2(125) - \log_2(125)} = \underline{\hspace{2cm}}$$

2.

Which of the following statements is TRUE? Select ALL that apply.

- $\log(x + y) = \log(x) + \log(y)$
- $\log_7(xy) = \log_7(x) + \log_7(y)$
- $\log_2\left(\frac{x}{y}\right) = \log_2(x) - \log_2(y)$
- $\ln\left(\frac{x}{y}\right) = \frac{\ln(x)}{\ln(y)}$
- $\log_4(xy) = \log_4(x) \cdot \log_4(y)$
- $\log_b\left(\frac{1}{5}\right) = -\log_b(5)$

3.

Which of the following are equivalent to $\log_b(3)$? Choose all that apply.

- $-\log_b\left(\frac{1}{3}\right)$
- $\log_b\left(\frac{1}{10}\right) + \log_b(30)$
- $\frac{1}{2}\log_b(9)$
- $\frac{1}{3}\log_b(27)$
- $\log_b(21) - \log_b(7)$

4.

Simplify

$$\log_y\left(\frac{y^7}{y^7}\right) = \underline{\hspace{2cm}}$$

5.

Simplify the given expression.

$$e^{\ln(N)} = \underline{\hspace{2cm}}$$

6.

Write the following as the sum and/or difference of logarithms. Assume all variables are positive.

$$\log\left(\frac{5c}{11}\right) = \underline{\hspace{2cm}}$$

7.

Write expression $\log\left(\frac{x^{20}y^{11}}{z^2}\right)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$$\log\left(\frac{x^{20}y^{11}}{z^2}\right) = \underline{\hspace{2cm}}$$

8.

Expand the given logarithm and simplify. Assume when necessary that all quantities represent positive real numbers. Be sure to factor out any common factors in your final answer.

$$\ln(x^{139}y^{-41}) = \underline{\hspace{2cm}}$$

9.

Expand the given logarithm and simplify. Assume when necessary that all quantities represent positive real numbers. Be sure to factor out any common factors in your final answer.

$$\log_9\left(\frac{729}{x^2-9}\right) = \underline{\hspace{2cm}}$$

10.

Write the expression $\ln\left(\frac{w^{20}x^8}{\sqrt[5]{z+2}}\right)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$$\ln\left(\frac{w^{20}x^8}{\sqrt[5]{z+2}}\right) = \underline{\hspace{2cm}}$$

11.

Write the expression $\log_3(9x^{15}y^8)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$$\log_3(9x^{15}y^8) = \underline{\hspace{2cm}}$$

12.

Write the expression $\log \sqrt[3]{\frac{y^3 w^{11}}{x^{10}}}$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$$\log \sqrt[3]{\frac{y^3 w^{11}}{x^{10}}} = \underline{\hspace{2cm}}$$

13.

Let $\log(A) = 12$, $\log(B) = 7$, and $\log(C) = 6$. Evaluate the following logarithms using logarithmic properties.

- $\log\left(\frac{A^2}{\sqrt{B}}\right) = \underline{\hspace{2cm}}$
- $\log\left(\frac{B}{C^5}\right) = \underline{\hspace{2cm}}$
- $\log\left(\frac{A}{B^3 C}\right) = \underline{\hspace{2cm}}$

14.

Evaluate using your calculator and round to 4 decimal places.

$$\log_{19} 44 = \underline{\hspace{2cm}}$$

15.

Simplify the expressions given. Use exact values. (Hint: Using the change-base formula might make these problems quite easy.)

$$\log_8(32) = \underline{\hspace{2cm}}$$

$$\log_8(\sqrt{32}) = \underline{\hspace{2cm}}$$

$$\log_8\left(\frac{1}{\sqrt{32}}\right) = \underline{\hspace{2cm}}$$

16.

Given that f is defined by $f(t) = -5(2^t) + 6$, which of the following is a formula for f^{-1} ?

- $f^{-1}(t) = \frac{1}{-5(2^t)+6}$
- $f^{-1}(t) = -5\left(\frac{\ln t}{\ln 2}\right) + 6$
- $f^{-1}(t) = \frac{\ln\left(\frac{t-6}{-5}\right)}{\ln 2}$
- $f^{-1}(t) = \ln(-5(2^t) + 6)$
- $f^{-1}(t) = \frac{t-6}{\ln 2}$

17.

Write the following as a single logarithm. Assume all variables are positive.

$$\log_3(5) + 5\log_3(b) = \underline{\hspace{2cm}}$$

18.

Write the following sum as a single logarithm. Assume all variables are positive.

$$\log_3(b) + \log_3(b + 5) = \underline{\hspace{2cm}}$$

19.

Simplify the following into a single logarithm:

$$3\log(7) - 1\log(x)$$

- $\log(7^3 x^1)$
- $\log\left(\frac{7^3}{x^1}\right)$
- $\log\left(\frac{3 \cdot 7}{1x}\right)$
- $\log(3 \cdot 7 \cdot 1x)$
- $\log(3 \cdot 7 \cdot x^1)$

20.

Write the expression $20\log_4(w) + 5\log_4(x)$ as a single logarithm.

$$20\log_4(w) + 5\log_4(x) = \underline{\hspace{2cm}}$$

21.

Write the expression $4\log_3(w) + 9\log_3(x) - \frac{1}{2}\log_3(y + 17)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$$4\log_3(w) + 9\log_3(x) - \frac{1}{2}\log_3(y + 17) = \underline{\hspace{2cm}}$$

Lecture 24

Solving exponential equations, solving logarithmic equations, finding inverses of exponential/log functions

1.

Use a calculator to find the natural logarithm.

$$\ln(4.53)$$

2.

Simplify mentally using the properties of logarithms

$$\ln(e^8)$$

3.

Simplify mentally using the properties of logarithms.

$$e^{\ln(2)}$$

4.

Use the like-bases property and exponents to solve the equation

$$\left(\frac{1}{5}\right)^{n+3} = 5^{3n-4}$$

$$n = \underline{\hspace{2cm}}$$

5.

If $\ln x + \ln(x - 2) = \ln(3x)$, then $x = \underline{\hspace{2cm}}$.

6.

5000 dollars is invested in a bank account at an interest rate of 9 percent per year, compounded continuously. Meanwhile, 36000 dollars is invested in a bank account at an interest rate of 3 percent compounded annually.

To the nearest year, When will the two accounts have the same balance?

The two accounts will have the same balance after $\underline{\hspace{2cm}}$ years.

7.

If 7000 dollars is invested in a bank account at an interest rate of 7 per cent per year, compounded continuously. How many years will it take for your balance to reach 10000 dollars?

NOTE: Give your answer to the nearest tenth of a year.

8.

Find the time required for an investment of 5000 dollars to grow to 8400 dollars at an interest rate of 7.5 percent per year, compounded quarterly.

Your answer is $t =$ _____ years.

9.

Solve for x :

$$11(1.09^x) = 18(1.12^x)$$

$$x = \underline{\hspace{2cm}}$$

10.

Solve for x :

$$9(1.17^x) = 20(1.09^x)$$

$$x = \underline{\hspace{2cm}}$$

11.

Find the solution of the exponential equation $4e^x - 3 = 13$

The exact solution, in terms of the natural logarithm is: $x =$ _____

The approximate solution, accurate to 4 decimal places is: $x =$ _____

12.

A computer purchased for \$1,250 loses 19% of its value every year. The computer's value can be modeled by the function $v(t) = a \cdot b^t$, where v is the dollar value and t the number of years since purchase.

(A) Give the function that models the decrease in value of the computer: $v(t) =$ _____

(B) In how many years will the computer be worth half its original value? *Round answer to 1 decimal place.* _____ years

13.

Find the solution of the exponential equation $18e^{x+5} = 5$

The exact solution (using natural logarithms) is: $x =$ _____

The approximate solution, rounded to 4 decimal places is: $x =$ _____

14.

Solve correct to 2 decimal places.

$$40257 = \frac{2600((1.09)^n - 1)}{\frac{9}{100}}$$

$n =$ _____

15.

Solve for x : $4^x = 23$

The exact solution is $x =$ _____

The solution rounded to 4 decimal places is $x =$ _____

16.

Find the solution of the exponential equation $e^{4x-1} = 13$

The exact solution is: $x =$ _____

The approximate solution, correct to four decimal places is $x =$ _____

17.

Solve the equation $4^{\frac{x}{3}} = 3$

The exact solution is $x =$ _____

The solution, rounded to 4 decimal places is $x =$ _____

18.

Solve for x . Round answers to four decimal places.

$$\ln(x) = 6$$

19.

Solve for x . Round answers to four decimal places.

$$3\ln(x + 4) = 9$$

20.

Solve for x . Round answers to four decimal places.

$$5e^x + 3 = 2e^x + 338$$

21.

A culture of bacteria grows according to the continuous growth model

$$B = f(t) = 200e^{0.058t}$$

where B is the number of bacteria and t is in hours.

Find

$$f(0) = \underline{\hspace{2cm}}$$

To the nearest whole number, find the number of bacteria after 6 hours.

To the nearest tenth of an hour, determine how long it will take for the population to grow to 600 bacteria.

22.

Solve the equation, enter your answer as a decimal approximation.

$$5e^{14t} = 12 + 11e^{14t}$$

- One solution:
- No solution

23.

Solve:

$$20(4^{3x}) = 14$$

$$x = \underline{\hspace{2cm}} \text{ (Answer DNE if no solution exists)}$$

24.

Suppose $\ln n = 10$. Find $\log_{14} n$.

$$\log_{14} n = \underline{\hspace{2cm}}$$

25.

Solve: $e^{6x} - 1e^{3x} = 12$.

$$x = \underline{\hspace{2cm}} \text{ (If no solution exists, answer DNE)}$$

26.

Solve: $9^x - 3^{x+3} = 324$.

$$x = \underline{\hspace{2cm}} \text{ (If no solution exists, answer DNE)}$$

27.

Solve $5^{7x+4} = 3^{x+2}$ for x . Give both the exact answer and the decimal approximation to the nearest hundredth. If logarithms are needed to solve the equation, use "ln."

Exact answer: $\underline{\hspace{2cm}}$

Decimal approximation: $\underline{\hspace{2cm}}$

28.

Solve $6^{8x+5} = 5^{x-3}$ for x . Give both the exact answer and the decimal approximation to the nearest hundredth. If logarithms are needed to solve the equation, use "ln."

Exact answer: $\underline{\hspace{2cm}}$

Decimal approximation: $\underline{\hspace{2cm}}$

29.

Solve exactly, then give the approximate decimal solution rounded to four decimal places.

$$7^{x-9} = 6$$

$$x = \underline{\hspace{2cm}}$$

$$x \approx \underline{\hspace{2cm}}$$

30.

If $\ln x + \ln(x - 7) = \ln(2x)$, then $x =$ _____.

31.

If $\log_2(6x + 5) = 4$, then $x =$ _____.

32.

Solve for n in the equation below. It may be helpful to convert the equation into exponential form. Write answer as an integer or reduced fraction.

$$-\log_6(n) + 25 = 23$$

$n =$ _____

33.

Solve the equation for x : $\log_{12} x + \log_{12}(x - 1) = 1$.

Answer DNE if there is no solution

34.

Solve for x :

$$\left(\log_4(\log_4 x)\right) = -4$$

$x =$ _____

Lecture 25

Exponential growth/decay examples, Doubling time and half-life

1.

A population of bacteria is growing according to the equation $P(t) = 1650e^{0.1t}$. Estimate when the population will exceed 2161.

$t =$ _____

Give your answer accurate to at least one decimal place.

2.

The amount of money in an investment is modeled by the function $A(t) = 800(1.0471)^t$. The variable A represents the investment balance in dollars, and t the number of years.

What is the doubling time for the investment? *Round answer to 1 decimal place.*

Answer = _____ years

3.

Find the time required for an investment of 5000 dollars to grow to 7900 dollars at an interest rate of 7.5 percent per year, compounded quarterly.

Your answer is $t =$ _____ years.

Round to 2 decimal places.

4.

A bacteria culture initially contains 2500 bacteria and doubles every half hour.

Find the size of the bacterial population after 100 minutes. _____

Find the size of the bacterial population after 8 hours. _____

5.

A computer purchased for \$1,600 loses 15% of its value every year.

The computer's value can be modeled by the function $v(t) = a \cdot b^t$, where v is the dollar value and t the number of years since purchase.

(A) In the exponential model $a =$ _____ and $b =$ _____.

(B) In how many years will the computer be worth half its original value? *Round answer to 1 decimal place.*

The answer is _____ years

6.

You go to the doctor and he gives you 12 milligrams of radioactive dye. After 24 minutes, 7.25 milligrams of dye remain in your system. To leave the doctor's office, you must pass through a radiation detector without sounding the alarm.

If the detector will sound the alarm if more than 2 milligrams of the dye are in your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived?

Give your answer to the nearest minute.

You will spend _____ minutes at the doctor's office.

7.

The half-life of Radium-226 is 1590 years. If a sample contains 200 mg, how many mg will remain after 1000 years?

_____ mg

Give your answer accurate to at least 2 decimal places.

8.

A wooden artifact from an ancient tomb contains 50 percent of the carbon-14 that is present in living trees.

How long ago, to the nearest year, was the artifact made? (The half-life of carbon-14 is 5730 years.)

_____ years

9.

An object with initial temperature $130^{\circ}F$ is submerged in large tank of water whose temperature is $60^{\circ}F$. Find a formula for $F(t)$, the temperature of the object after t minutes, if the cooling constant is $k = -0.2$

$F(t) =$ _____

10.

The temperature, $f(t)$ of a cup of coffee, in degrees Celsius, after t minutes can be determined by the equation $f(t) = 60(0.88)^t + 21$.

Estimate the temperature after 35 minutes:

_____ degrees

Determine the asymptote for the function:

The asymptote is _____

Interpret the asymptote in the context of the problem. Use complete sentences and correct units.

11.

You currently have \$8,200 (Present Value) in an account that has an interest rate of 6% per year compounded semi-annually (2 times per year). You want to withdraw all your money when it reaches \$12,300 (Future Value). In how many years will you be able to withdraw all your money?

The number of years is _____.

Round your answer to 1 decimal place.

12.

The function $f(t) = \frac{575,000}{1+4000e^{-t}}$ describes the number of people, $f(t)$, who have become ill with ebola t weeks after the initial outbreak in a particular community.

How many people became ill with ebola when the epidemic began? _____

Round to the nearest whole number of people.

How many people were infected 6 weeks after the initial breakout? Round to the nearest whole number of people. _____

What is the limiting size of the infected population? _____

Round to the nearest whole number of people.

13.

The function $P(x) = \frac{120}{1+372e^{-0.133x}}$ models the percentage, $P(x)$, of Americans who are x years old and have some degree of heart disease. What is the percentage, to the nearest tenth, of 29-year olds with some degree of heart disease?

_____ %

14.

A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a carrying capacity of 1000 animals and that the growth of the herd will follow the logistic curve $P = \frac{1000}{1+9e^{-0.165t}}$ where t is measured in months.

What is the population after 5 months? _____

Lecture 26

Definitions (standard position, central angle, positive/negative), coterminal angles, radian measure and conversion, complementary, supplementary, arc length, sector area, linear and angular speed

1.

In which quadrant does an angle of $\frac{25}{6}\pi$ terminate? Assume the vertex of the angle is at the origin and one leg of the angle is on the positive x -axis. Find the quadrant of the other leg of the angle.

2.

Let $\theta = 1$ rad be the measure of an angle. State the exact value and the approximate value of the angle measure in degrees. For the approximate value, round your answer to two decimal places.

Exact Value: _____ degrees

Approximate Value: _____ degrees

Conversion Formula

Let $\theta = x$ rad be the measure of an angle. State the conversion formula to degree measure.

x rad = _____ degrees.

Let $\theta = 4$ rad be the measure of an angle. State the exact value and the approximate value of the angle measure in degrees. For the approximate value, round your answer to two decimal places.

Exact Value: _____ degrees

Approximate Value: _____ degrees

3.

Convert the angle $\frac{11\pi}{6}$ from radians to degrees.

_____ degrees

4.

Convert the angle $\frac{-7\pi}{6}$ from radians to degrees:

$\frac{-7\pi}{6} = \text{_____}^\circ$

5.

Convert the angle 120° to radians. Give the exact value.

6.

Convert 220 degrees to radians

_____radians

Convert $\frac{4\pi}{9}$ to degrees

_____degrees

Give a exact answers.

7.

Find the angle between 0° and 360° and is coterminal with a standard position angle measuring 1376° .

8.

Find an angle between 0° and 360° that is coterminal with the given angle.

439° is coterminal to _____ $^\circ$

-125° is coterminal to _____ $^\circ$

-945° is coterminal to _____ $^\circ$

11199° is coterminal to _____ $^\circ$

9.

Write an expression describing *all* the angles that are coterminal with 266° . (Please use the variable k in your answer. Give your answer in degrees.)

_____degrees

10.

Given an angle with measure $\frac{-2\pi}{3}$ find the following.

Find a coterminal angle between -4π and -2π . _____

Find a coterminal angle between 0 and 2π . _____

11.

Find an angle between 0° and 360° that is coterminal with a standard position angle measuring -240° .

Find an angle between -720° and -360° that is coterminal with a standard position angle measuring -240° .

12.

The angle between 0 and 2π in radians that is coterminal with the angle $-\frac{11\pi}{3}$ radians is _____.

13.

Find the complement of each of the following angles.

48° is complement to _____ $^\circ$

1° is complement to _____ $^\circ$

60° is complement to _____ $^\circ$

53° is complement to _____ $^\circ$

6° is complement to _____ $^\circ$

14.

$m\angle X = 35^\circ$. $\angle X$ and $\angle Y$ are supplementary angles.

$m\angle Y =$ _____ $^\circ$

15.

Find the length of an arc that subtends a central angle of 5° in a circle of radius 11 in.

arc-length = _____in

Answer must be exact.

16.

In a circle of radius 2 miles, the length of the arc that subtends a central angle of 3 radians is _____ miles.

17.

On a circle of radius 8 feet, give the degree measure of the angle that would subtend an arc of length 5 feet. Round your answer to the nearest hundredth, or two decimal places. degrees

18.

A bicycle with 18-in.-diameter wheels has its gears set so that the chain has a 7-in. radius on the front sprocket and 3-in. radius on the rear sprocket. The cyclist pedals at 190 rpm.

Find the linear speed of the bicycle in in/min (correct to at least **two decimal places**) _____ in/min

How fast is the bike moving in mph (to two decimal places)? _____ mph

19.

A truck's 44-in.-diameter wheels are turning at 530 rpm.

Find the linear speed of the truck in mph:

_____miles/hour

Write answer as an exact expression using pi for π .

20.

Your car's speedometer is geared to accurately give your speed using a certain tire size: 14.5" diameter wheels (the metal part) and 4.5" tires (the rubber part).

(a) If your car's instruments are properly calibrated, how many times should your tire rotate per second if you are travelling at 45 mi/hr?

rotations = _____

Report answer accurate to 3 decimal places.

(b) You buy new 5.4" tires and drive at a constant speed of 55 mph (according to your car's instrument). However, a cop stops you and claims that you were speeding. How fast did the radar gun clock you moving?

actual speed = _____ mph

Report answer accurate to the nearest whole number.

(c) Then you replace your tires with 3.8" tires. When your speedometer reads 30 mph, how fast are you really moving?

actual speed = _____ mph

Report answer accurate to 1 decimal places.

21.

A saw uses a circular blade 10 inches in diameter that spins at 3480 rpm. How quickly are the teeth of the saw blade moving? Express your answer in several forms:

In **exact** feet per second: _____ ft/sec

In **approximate** feet per second, rounded to 1 decimal place: _____ ft/sec

In **exact** miles per hour: _____ mph

In **approximate** miles per hour, rounded to 1 decimal place: _____ mph

22.

Find the area of a sector with a central angle of 0.5 rad in a circle of radius 10.9 m.

area = _____sq-m

Report answer accurate to 4 decimal places.

23.

The area of a sector of a circle with a central angle of $\frac{4}{5}\pi$ rad is 39 mm².

Find the radius of the circle.

$r =$ _____mm

Give an exact value.

24.

A sector of a circle has a central angle of 150° . Find the area of the sector if the radius of the circle is 17 cm.

_____cm²

Lecture 27

Unit circle, Special angles and their unit circle coordinates, definition of trig functions,
Domain/range/period of sin/cos

1.

From the information given, find the quadrant in which the terminal point determined by t lies. Input I, II, III, or IV.

- (a) $\sin(t) < 0$ and $\cos(t) < 0$, quadrant _____;
- (b) $\sin(t) > 0$ and $\cos(t) < 0$, quadrant _____;
- (c) $\sin(t) > 0$ and $\cos(t) > 0$, quadrant _____;
- (d) $\sin(t) < 0$ and $\cos(t) > 0$, quadrant _____;

2.

A point on the unit circle lies in quadrant III and has y -coordinate $\frac{9}{11}$. What is its x -coordinate? The x -coordinate is _____

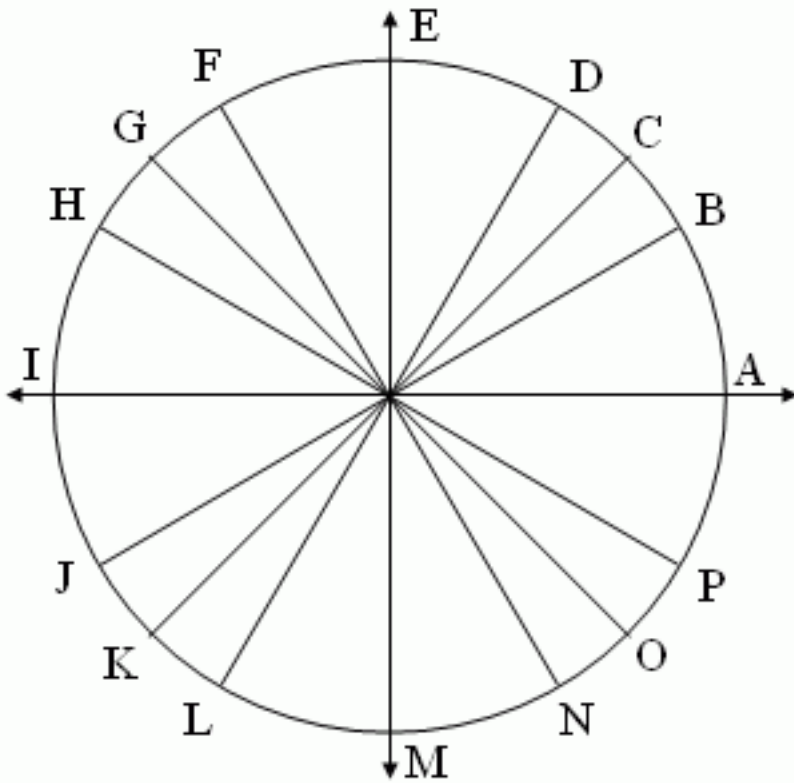
3.

A point on the unit circle lies in quadrant II and has y -coordinate $\frac{7}{11}$. What is its x -coordinate? The x -coordinate is _____

4.

A point on the unit circle lies in quadrant I and has y -coordinate $\frac{7}{13}$. What is its x -coordinate? The x -coordinate is _____

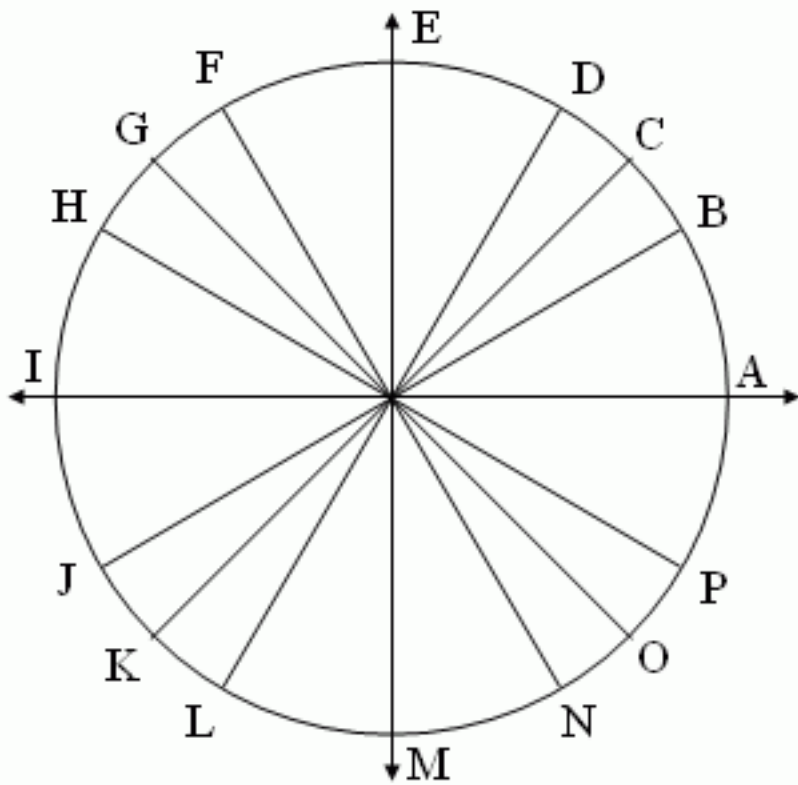
5.



Identify the special angles above. Give your answers in radians, using **pi** for π . No decimal values allowed.

- A: _____
- B: _____
- C: _____
- D: _____
- E: _____
- F: _____
- G: _____
- H: _____
- I: _____
- J: _____
- K: _____
- L: _____
- M: _____
- N: _____
- O: _____
- P: _____

6.

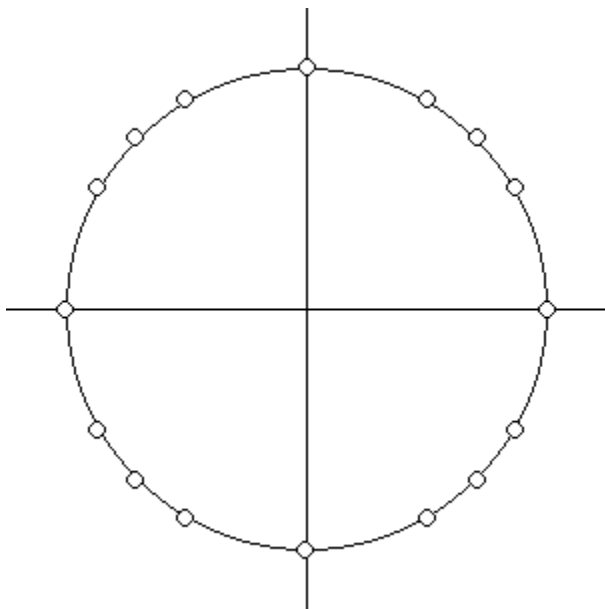


Identify the special angles above. Give your answers in degrees.

- A: _____
- B: _____
- C: _____
- D: _____
- E: _____
- F: _____
- G: _____
- H: _____
- I: _____
- J: _____
- K: _____
- L: _____
- M: _____
- N: _____
- O: _____
- P: _____

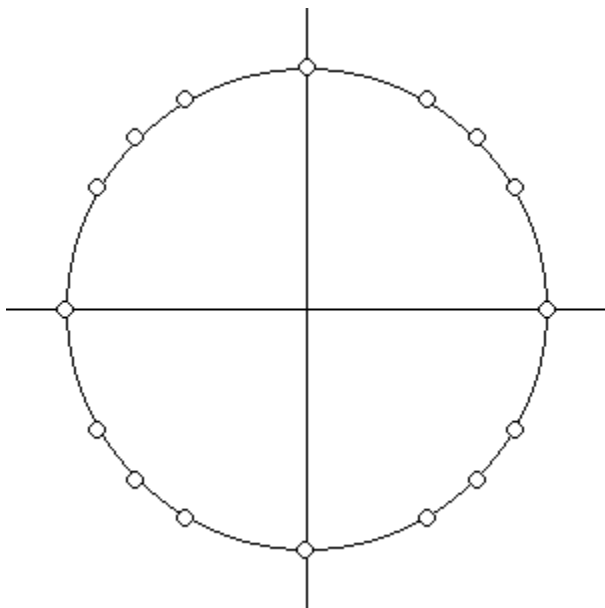
7.

Plot the point(s) where $\tan \theta = \sqrt{3}$



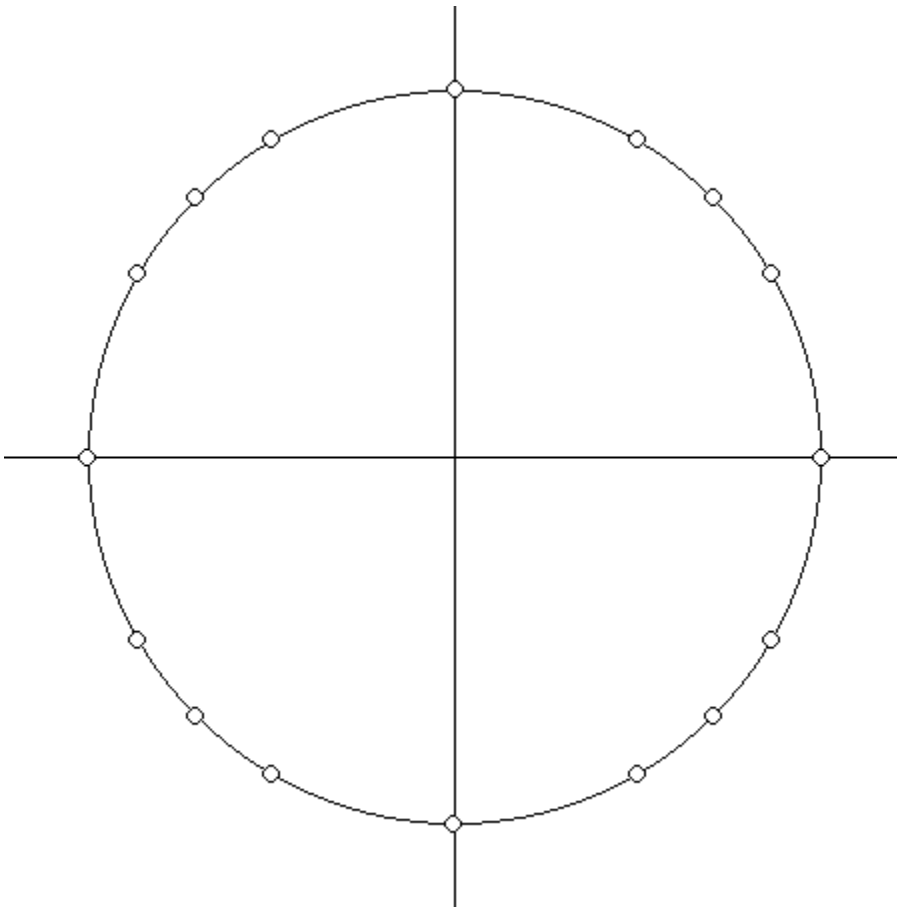
8.

Plot the point(s) where $\cot \theta = 1$



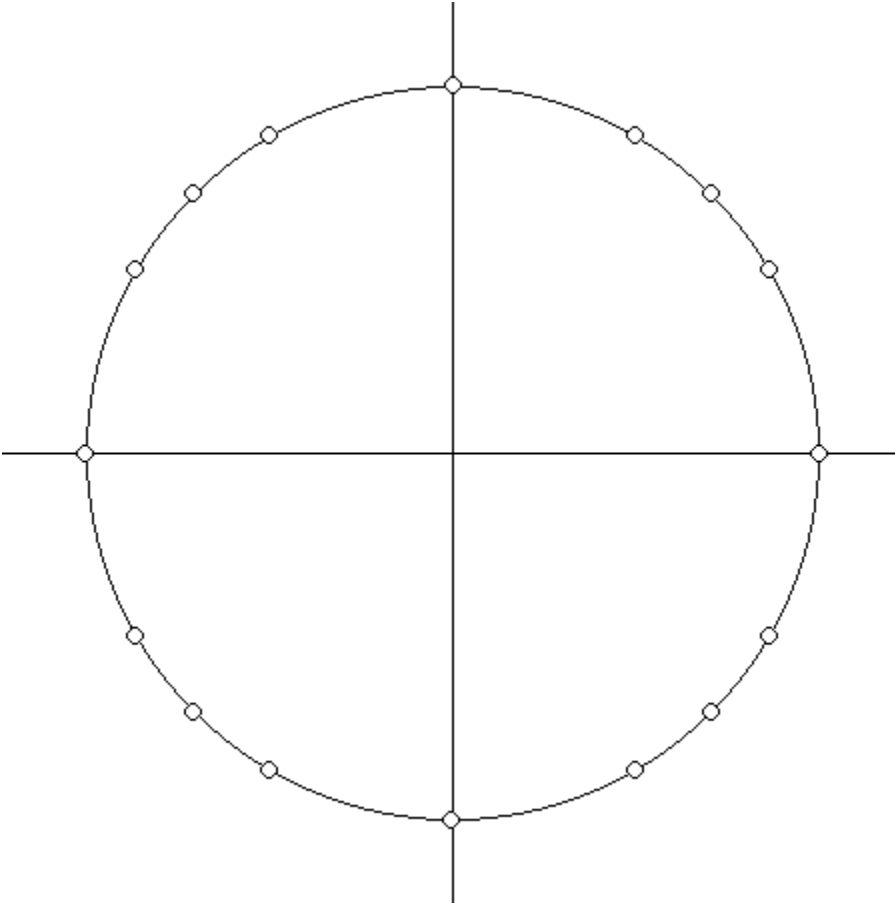
9.

Plot the point where $\theta = -\frac{2\pi}{3}$



10.

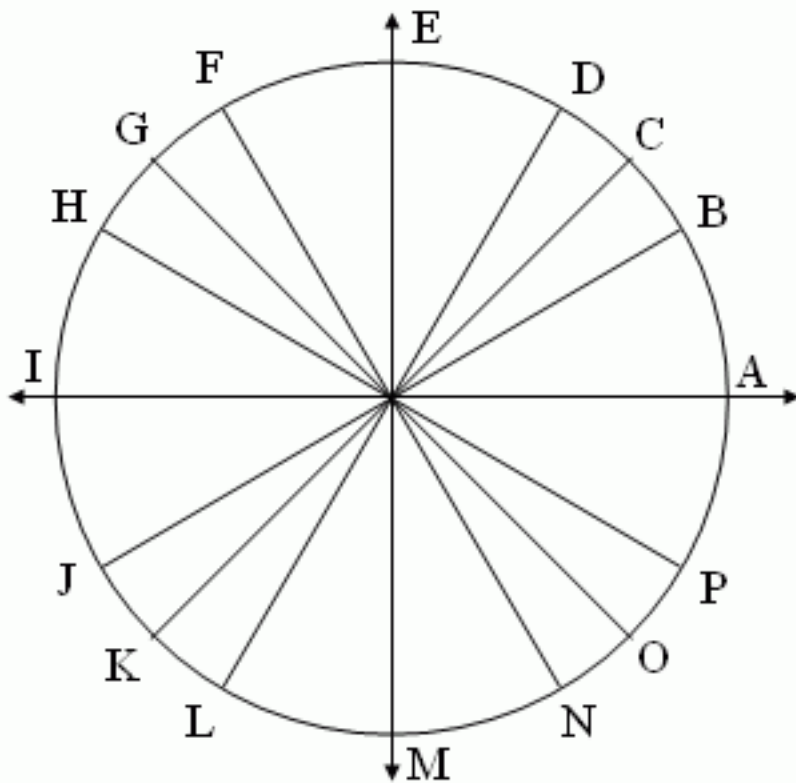
Plot the point where $\theta = -330^\circ$



11.

What angle is at position $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$? _____^o

12.



Use the above figure to find exact values of the sine and cosine of the special angles listed below.

Enter \sqrt{w} as `sqrt(w)`.

$\sin(K) = \underline{\hspace{2cm}}$ $\cos(K) = \underline{\hspace{2cm}}$

$\sin(C) = \underline{\hspace{2cm}}$ $\cos(C) = \underline{\hspace{2cm}}$

$\sin(B) = \underline{\hspace{2cm}}$ $\cos(B) = \underline{\hspace{2cm}}$

13.

State the exact value of $\cos\left(\frac{\pi}{6}\right) = \underline{\hspace{2cm}}$

14.

State the exact value of $\tan\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}}$

15.

State the exact value of $\sin\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}}$

16.

State the exact value of $\tan\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}}$

17.

State the exact value of $\tan\left(\frac{\pi}{4}\right) =$ _____

18.

$\csc 150^\circ$ equals _____

19.

$\tan 60^\circ$ equals _____

20.

Find an angle θ with $0^\circ < \theta < 360^\circ$ that has the same:

Sine as 60° : $\theta =$ _____ degrees

Cosine as 60° : $\theta =$ _____ degrees

21.

Find an angle θ with $0^\circ < \theta < 360^\circ$ that has the same:

Sine function value as 220°

$\theta =$ _____ degrees

Cosine function value as 220°

$\theta =$ _____ degrees

22.

To show that the point $\left(\frac{3}{5}, \frac{4}{5}\right)$ is on the unit circle, we need to prove that $\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 =$ _____.

Lecture 28

Trig functions in right triangles, Identities(Cofunction, Reciprocal, Quotient, Pythagorean), Reference angles, Angles of elevation/depression

1.

The reference angle of 227 degrees is _____ degrees.

The reference angle of 327 degrees is _____ degrees.

The reference angle of -143 degrees is _____ degrees.

2.

The reference angle of $\frac{12}{5}\pi$ radians is _____ radians.

The reference angle of $-\frac{3}{10}$ radians is _____ radians.

The reference angle of $\frac{7}{5}\pi$ radians is _____ radians.

3.

If $\cos(38^\circ) = \sin(\theta)$ and $0^\circ < \theta < 90^\circ$, then

$\theta =$ _____ degrees

4.

If $\sin\left(\frac{\pi}{6}\right) = \cos(\theta)$ and $0^\circ < \theta < \frac{\pi}{2}$, then

$\theta =$ _____

5.

Find the **exact value** of each of the following.

$$\sin(30^\circ) = \underline{\hspace{2cm}}$$

$$\cos(30^\circ) = \underline{\hspace{2cm}}$$

$$\tan(30^\circ) = \underline{\hspace{2cm}}$$

$$\csc(30^\circ) = \underline{\hspace{2cm}}$$

$$\sec(30^\circ) = \underline{\hspace{2cm}}$$

$$\cot(30^\circ) = \underline{\hspace{2cm}}$$

6.

Find the **exact value** of each of the following.

$$\tan(60^\circ) = \underline{\hspace{2cm}}$$

$$\csc(30^\circ) = \underline{\hspace{2cm}}$$

$$\cos(45^\circ) = \underline{\hspace{2cm}}$$

7.

For $0 < \theta < \frac{\pi}{2}$, find the values of the trigonometric functions based on the given one

(give your answers with THREE DECIMAL PLACES or as expressions, e.g. you can enter 3/5).

If $\sin(\theta) = \frac{8}{9}$ then

$$\cos(\theta) = \underline{\hspace{2cm}}$$

$$\sec(\theta) = \underline{\hspace{2cm}}$$

$$\csc(\theta) = \underline{\hspace{2cm}}$$

$$\tan(\theta) = \underline{\hspace{2cm}}$$

$$\cot(\theta) = \underline{\hspace{2cm}}$$

8.

Suppose that θ is an angle in quadrant I and $\sin(\theta) = \frac{2}{13}$. Find the values of the other five trigonometric functions for θ . Give exact answers, but do not rationalize denominators.

$$\cos(\theta) = \underline{\hspace{2cm}}$$

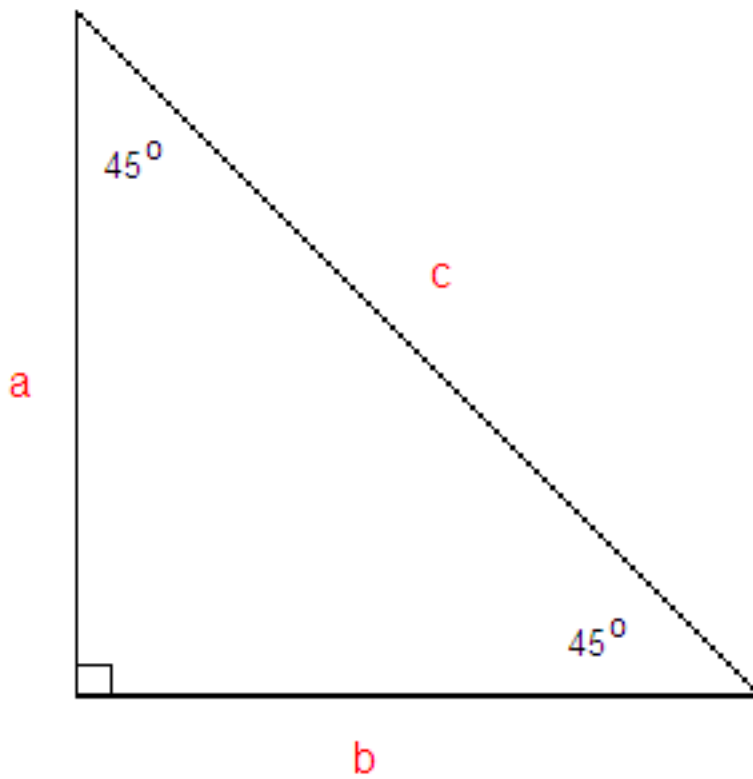
$$\tan(\theta) = \underline{\hspace{2cm}}$$

$$\csc(\theta) = \underline{\hspace{2cm}}$$

$$\sec(\theta) = \underline{\hspace{2cm}}$$

$$\cot(\theta) = \underline{\hspace{2cm}}$$

9.



Note: Triangle may not be drawn to scale.

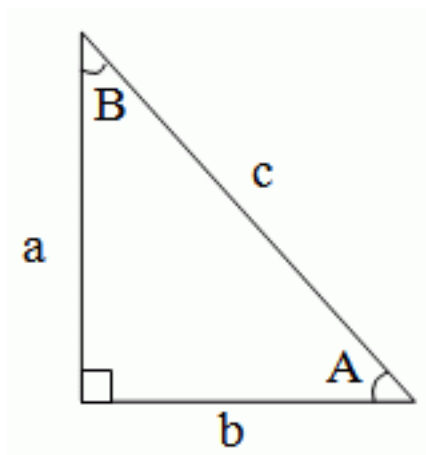
Suppose $b = 4$

Find exact values for the other sides.

$a =$ _____

$c =$ _____

10.



Note: Triangle may not be drawn to scale.

Suppose $a = 8$ and $b = 10$.

Find an exact value or give at least two decimal places:

$$\sin(A) = \underline{\hspace{2cm}}$$

$$\cos(A) = \underline{\hspace{2cm}}$$

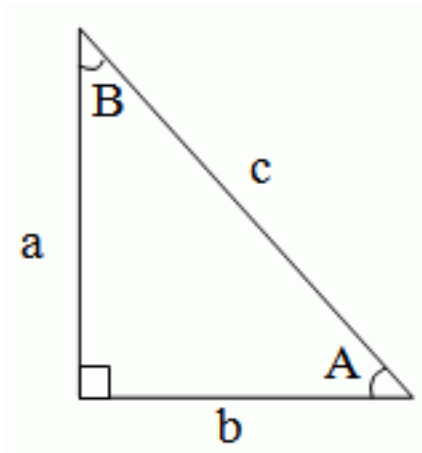
$$\tan(A) = \underline{\hspace{2cm}}$$

$$\sec(A) = \underline{\hspace{2cm}}$$

$$\csc(A) = \underline{\hspace{2cm}}$$

$$\cot(A) = \underline{\hspace{2cm}}$$

11.



Note: Triangle may not be drawn to scale.

Suppose $a = 6$ and $A = 30$ degrees.

Find:

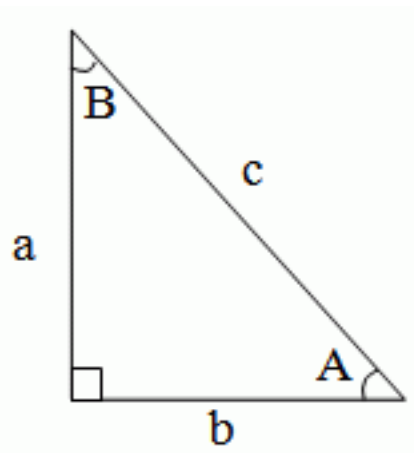
$$b = \underline{\hspace{2cm}}$$

$$c = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}} \text{ degrees}$$

Give all answers to at least one decimal place. Give angles in **degrees**

12.



Note: Triangle may not be drawn to scale.

Suppose $c = 10$ and $A = 15$ degrees.

Find:

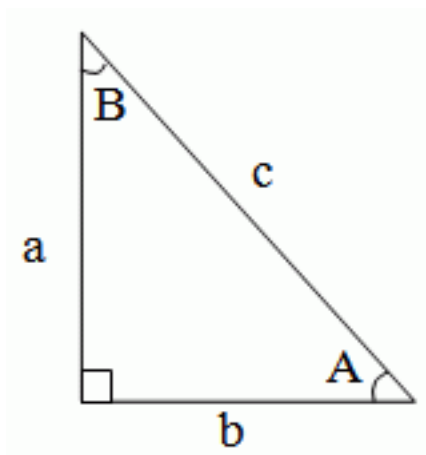
$a =$ _____

$b =$ _____

$B =$ _____ degrees

Give all answers to at least one decimal place. Give angles in **degrees**

13.



Note: Triangle may not be drawn to scale.

Suppose $a = 120$ and $b = 119$ and $c = 169$.

Find an exact value (report answer as a fraction):

$$\sin(A) = \underline{\hspace{2cm}}$$

$$\cos(A) = \underline{\hspace{2cm}}$$

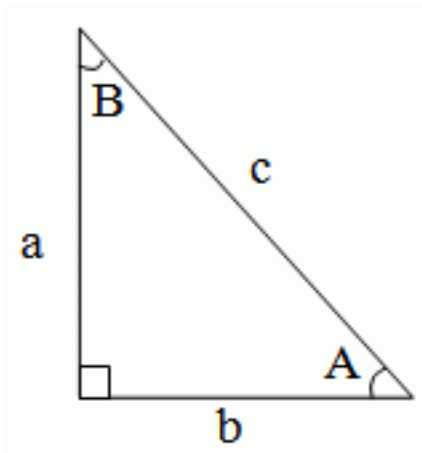
$$\tan(A) = \underline{\hspace{2cm}}$$

$$\sec(A) = \underline{\hspace{2cm}}$$

$$\csc(A) = \underline{\hspace{2cm}}$$

$$\cot(A) = \underline{\hspace{2cm}}$$

14.



Note: Triangle may not be drawn to scale.

Suppose $a = 7$ and $b = 10$.

Find an exact value for each of the following trig functions.

$$\sin(A) = \underline{\hspace{2cm}}$$

$$\cos(A) = \underline{\hspace{2cm}}$$

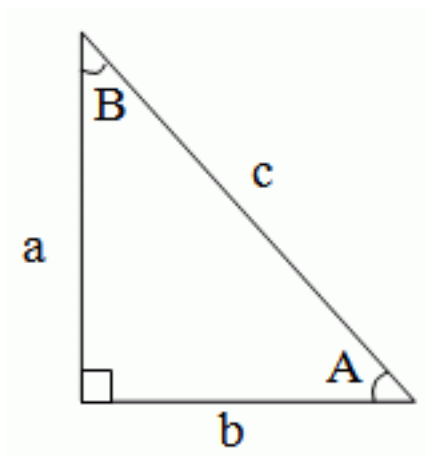
$$\tan(A) = \underline{\hspace{2cm}}$$

$$\sec(A) = \underline{\hspace{2cm}}$$

$$\csc(A) = \underline{\hspace{2cm}}$$

$$\cot(A) = \underline{\hspace{2cm}}$$

15.



Note: Triangle may not be drawn to scale.

Suppose $a = 105$ and $b = 88$.

Find an exact value (report answer as a fraction):

$$\sin(B) = \underline{\hspace{2cm}}$$

$$\cos(B) = \underline{\hspace{2cm}}$$

$$\tan(B) = \underline{\hspace{2cm}}$$

$$\sec(B) = \underline{\hspace{2cm}}$$

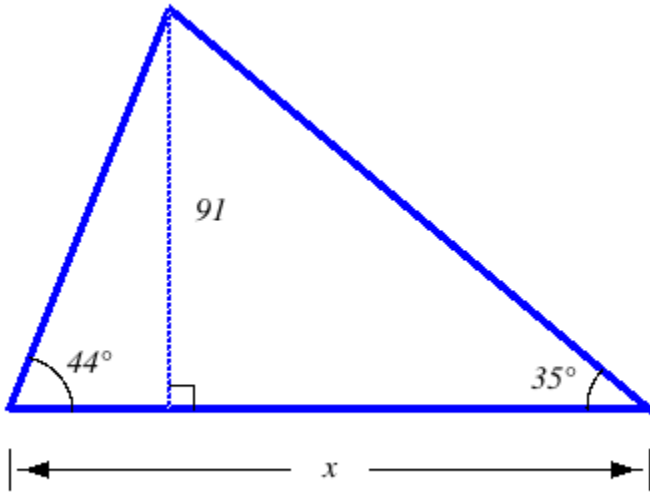
$$\csc(B) = \underline{\hspace{2cm}}$$

$$\cot(B) = \underline{\hspace{2cm}}$$

16.

Find x correct to 2 decimal places.

NOTE: The triangle is NOT drawn to scale.

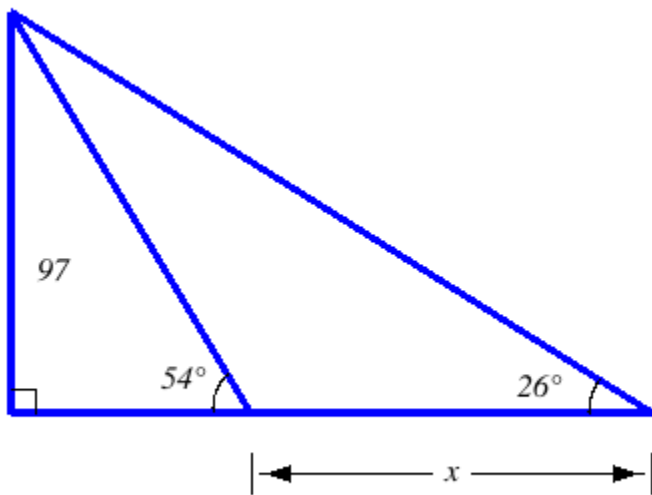


$x =$ _____

17.

Find x correct to 2 decimal places.

NOTE: The triangle is NOT drawn to scale.



$x =$ _____

18.

A 32 -ft ladder leans against a building so that the angle between the ground and the ladder is 81° .

How high does the ladder reach on the building? _____ft

19.

From the top of a 153-ft lighthouse, the angle of depression to a ship in the ocean is 28° . How far is the ship from the base of the lighthouse?

distance = _____ feet

Report answer accurate to 2 decimal places.

20.

A survey team is trying to estimate the height of a mountain above a level plain. From one point on the plain, they observe that the angle of elevation to the top of the mountain is 26° . From a point 2000 feet closer to the mountain along the plain, they find that the angle of elevation is 31° .

How high (in feet) is the mountain?

21.

The angle of elevation to the top of a Building in New York is found to be 3 degrees from the ground at a distance of 2 miles from the base of the building. Using this information, find the height of the building. Round to the tenths. Hint: 1 mile = 5280 feet

Your answer is _____ feet.

22.

A radio tower is located 275 feet from a building. From a window in the building, a person determines that the angle of elevation to the top of the tower is 32° and that the angle of depression to the bottom of the tower is 28° . How tall is the tower?

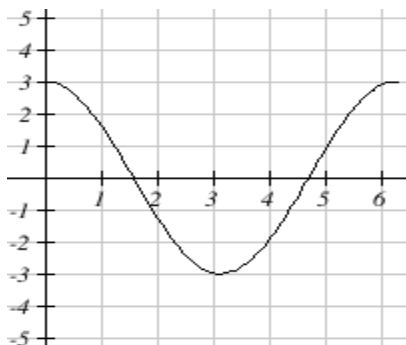
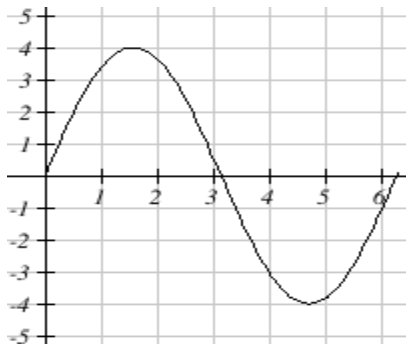
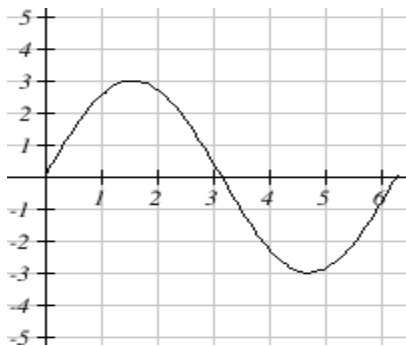
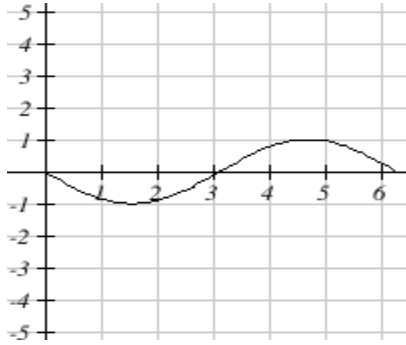
_____ feet

Lecture 29

Period, Amplitude, Phase shift, Vertical translation

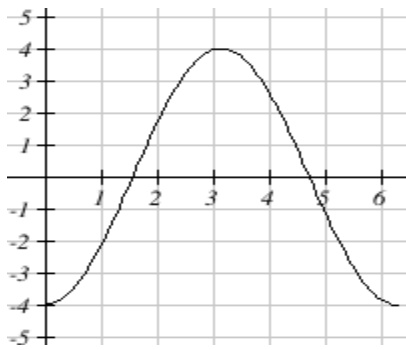
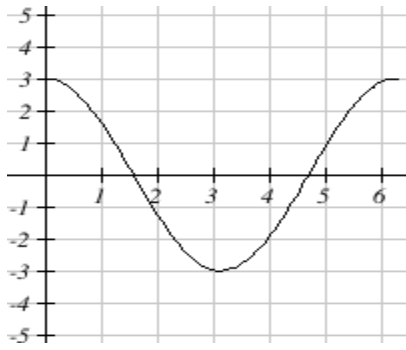
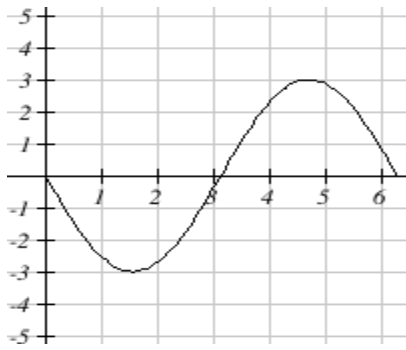
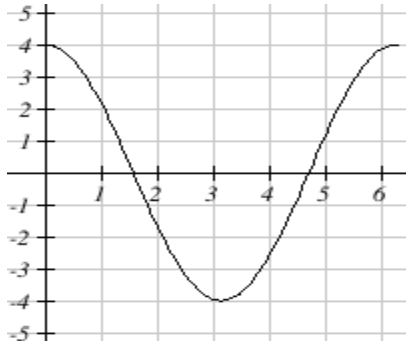
1.

Which of the following graphs is the correct plot of $y = 3\sin(x)$?



2.

Which of the following graphs is the correct plot of $y = 4\cos(x)$?



3.

Find the equation of a sine wave that is obtained by shifting the graph of $y = \sin(x)$ to the right 5 units and downward 8 units and is vertically stretched by a factor of 8 when compared to $y = \sin(x)$.

$f(x) =$ _____

4.

For $y = 6\sin 6x$,

its amplitude is _____

its period is _____

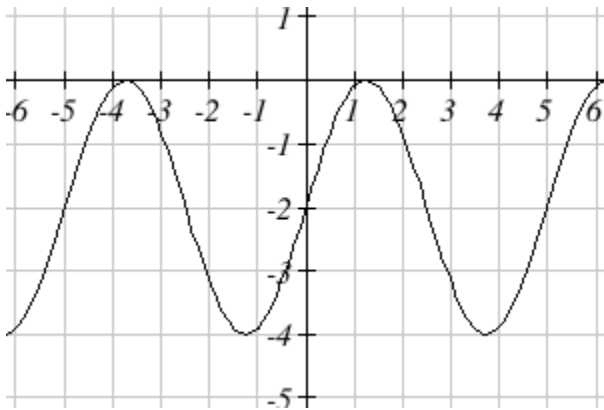
5.

For $y = -2\cos \frac{1}{8}x$,

its amplitude is _____

its period is _____

6.



Based on the graph above, determine the amplitude, midline, and period of the function

Amplitude: _____

Period: _____

Midline: $y =$ _____

7.

Given the equation $y = 5\sin(8(x - 3)) + 4$

The amplitude is: _____

The period is: _____

The horizontal shift is: _____ units to the (Right/ Left)

The midline is: $y =$ _____

8.

Given the equation $y = 5\sin(6x - 42) + 3$

The amplitude is: _____

The period is: _____

The horizontal shift is: _____ units to the (Right/ Left)

The midline is: $y =$

9.

Given the equation $y = 5\sin\left(\frac{\pi}{6}x + \frac{\pi}{3}\right) + 3$

The amplitude is: _____

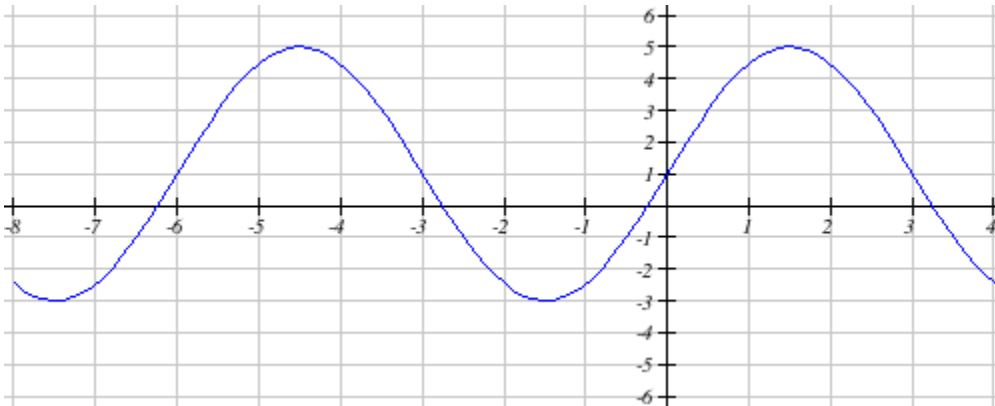
The period is: _____

The horizontal shift is: _____ units to the (Right/ Left)

The midline is: $y =$ _____

10.

Find a function of the form $y = A\sin(kx) + C$ or $y = A\cos(kx) + C$ whose graph matches the function shown below:

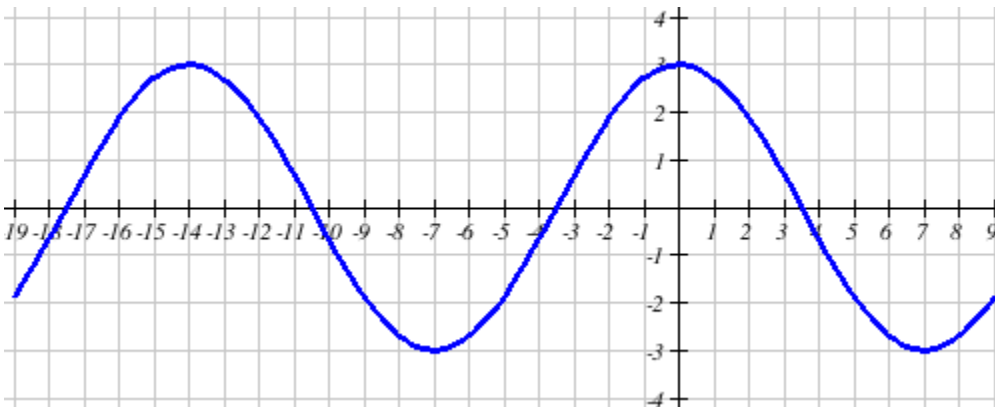


Leave your answer in exact form.

$y =$ _____

11.

Find a function of the form $y = A\sin(kx) + C$ or $y = A\cos(kx) + C$ whose graph matches this one:

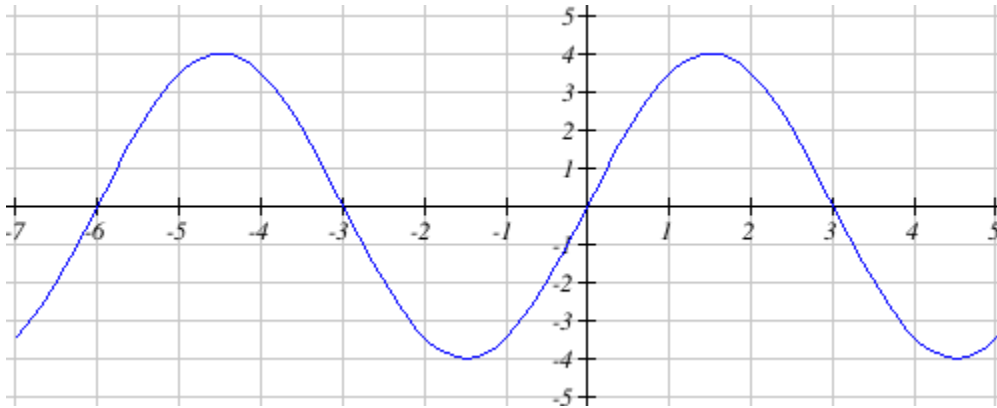


Leave your answer in exact form.

$y =$ _____

12.

Find a function of the form $y = A\sin(kx)$ or $y = A\cos(kx)$ whose graph matches the function shown below:



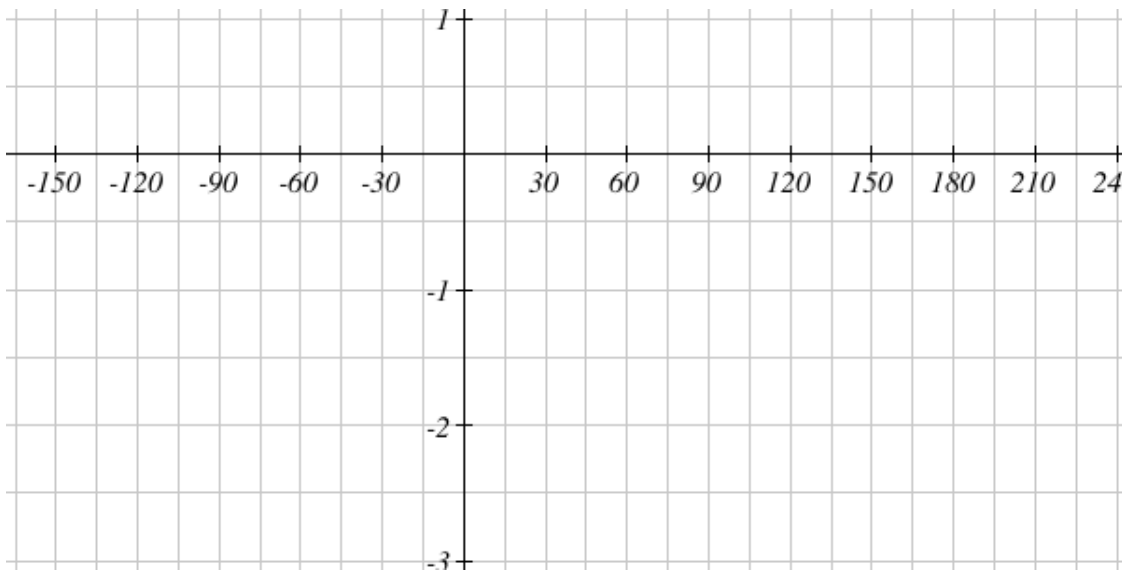
Leave your answer in exact form.

$y =$ _____

13.

Draw the following graph on the interval $-165^\circ < x < 240^\circ$:

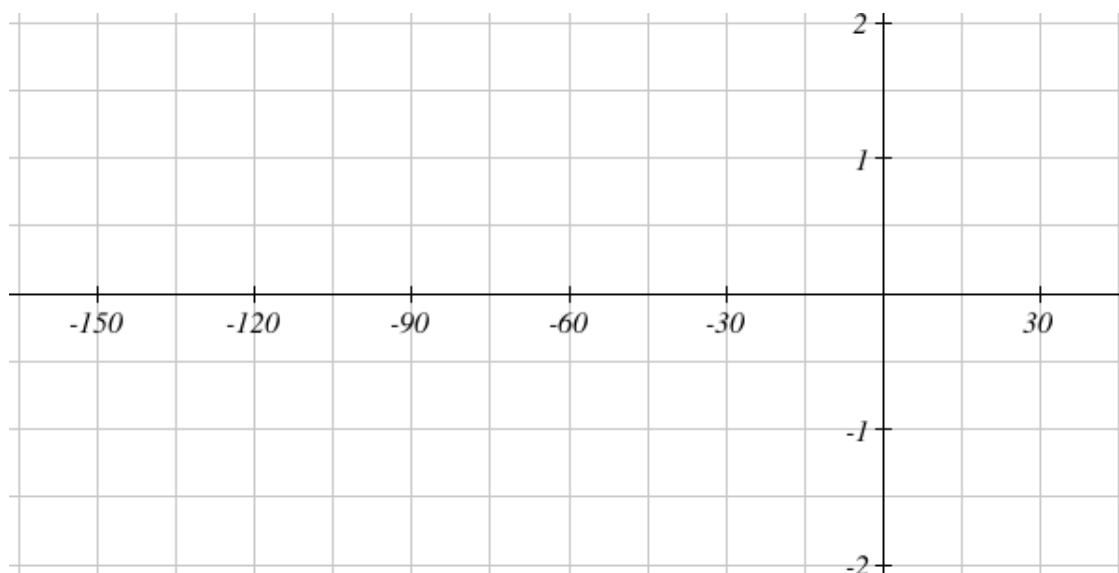
$y = -\sin(x) - 1$



14.

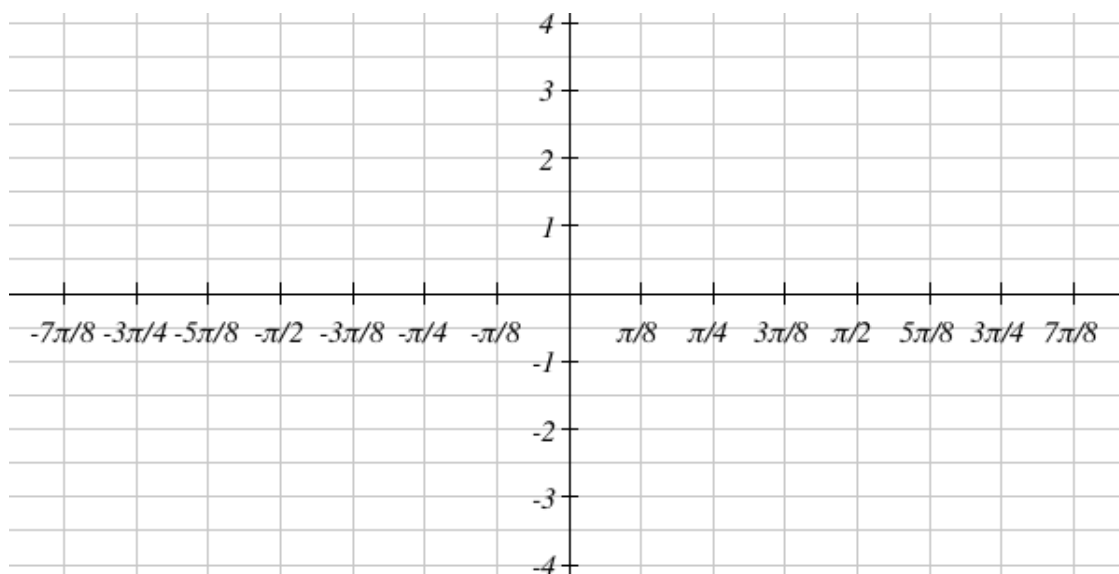
Draw the following graph on the interval $-165^\circ < x < 45^\circ$:

$$y = \sin(2x)$$



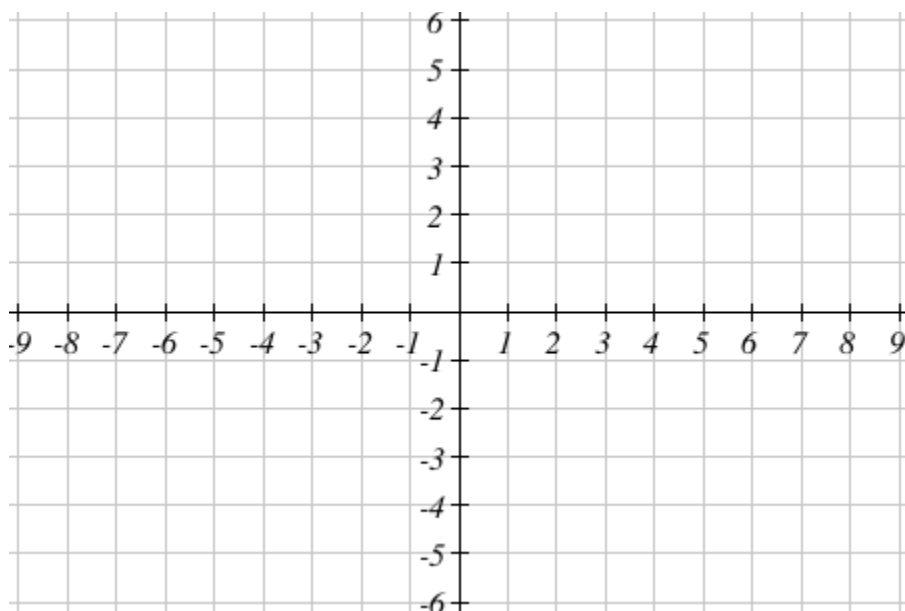
15.

Sketch a graph of the function $f(x) = -2\sin(2x)$.



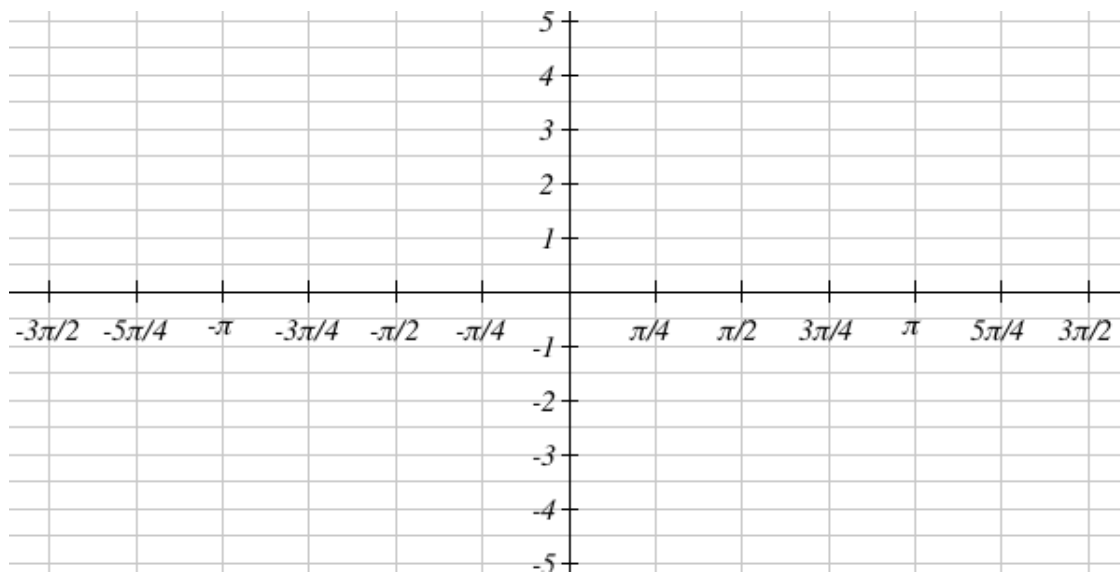
16.

Sketch a graph of the function $f(x) = 3\sin\left(\frac{\pi}{2}x\right) - 2$



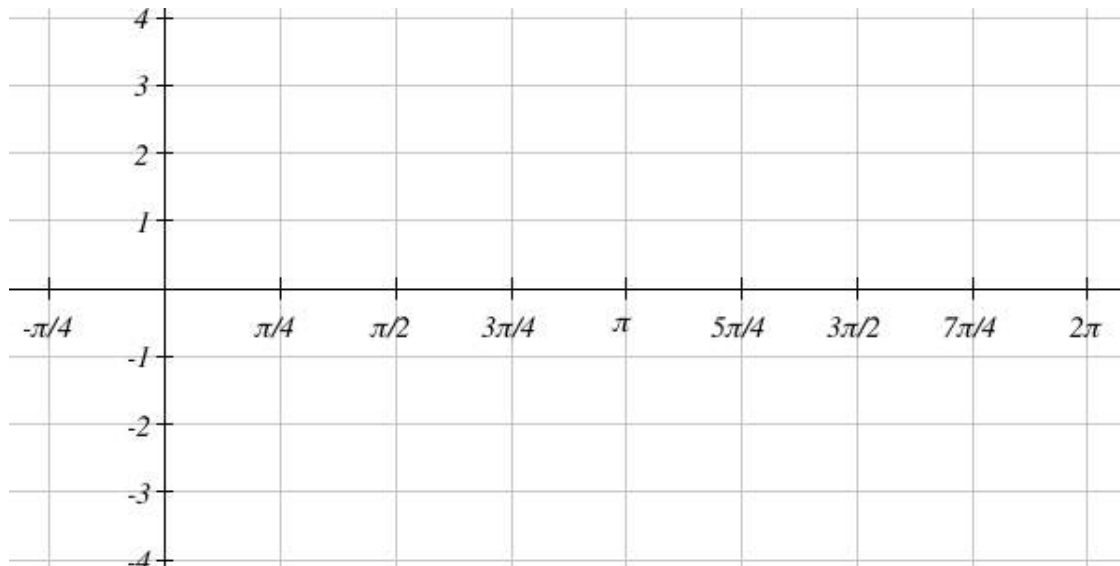
17.

Sketch a graph of the function $f(x) = -2\sin\left(x - \frac{3\pi}{4}\right) + 1$.



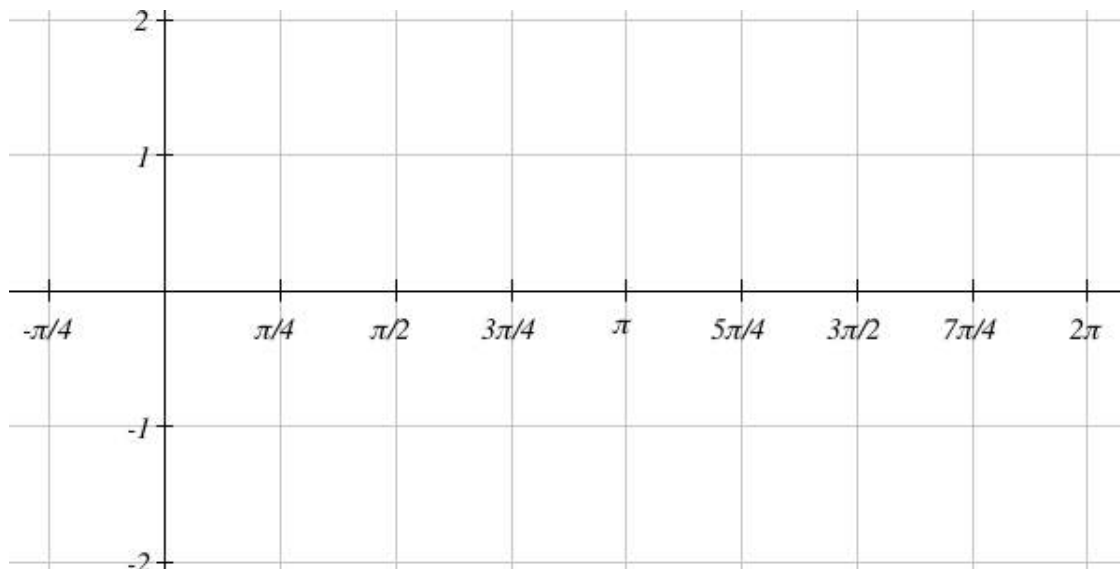
18.

Sketch a graph of the function $f(x) = \sin(x) + 1$.



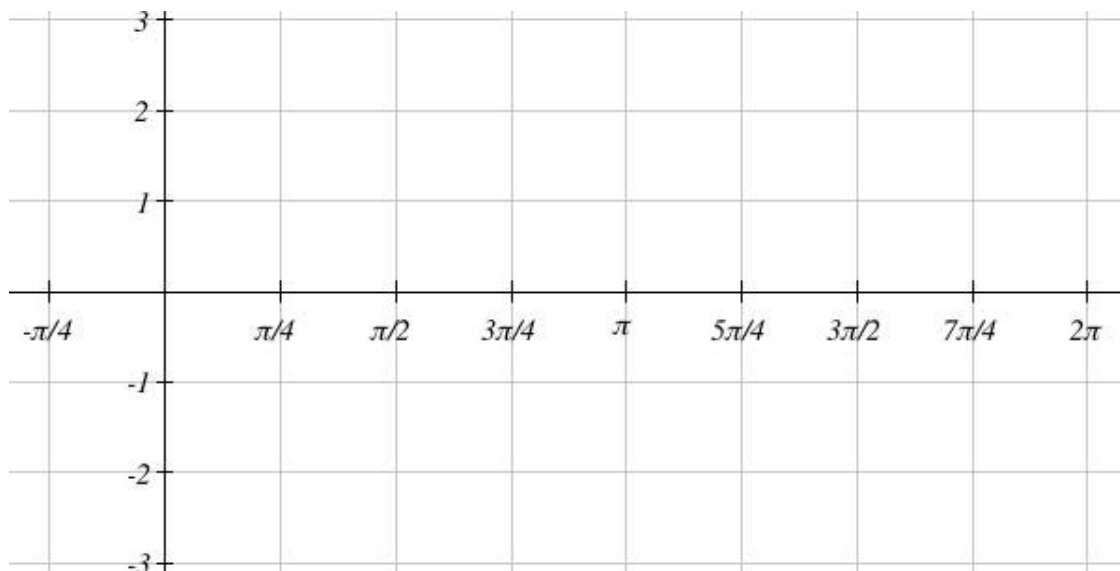
19.

Sketch a graph of the function $f(x) = \sin(x)$.



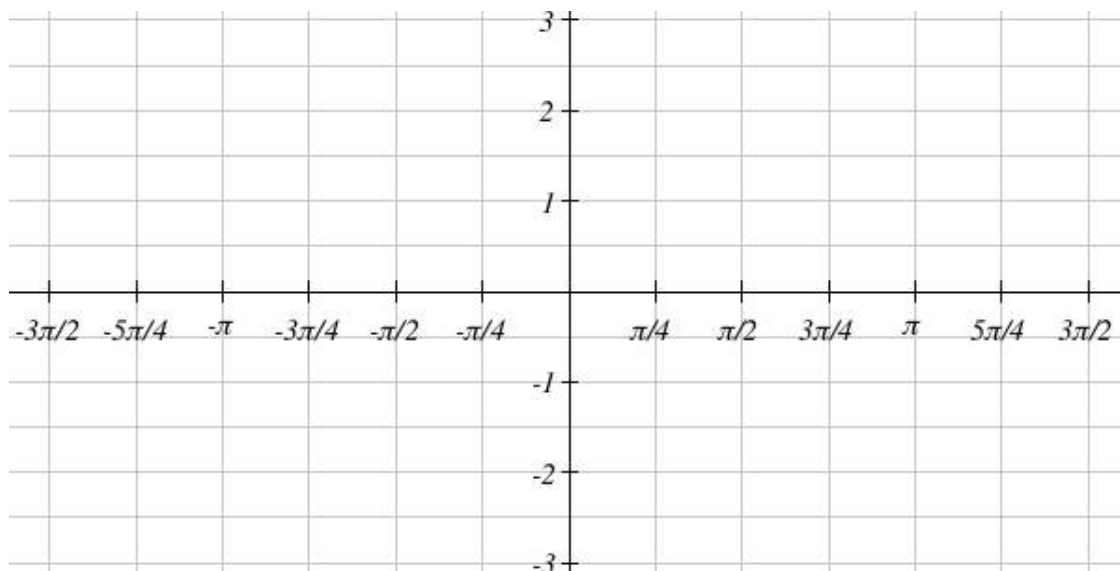
20.

Sketch a graph of the function $f(x) = -2\sin(x)$.



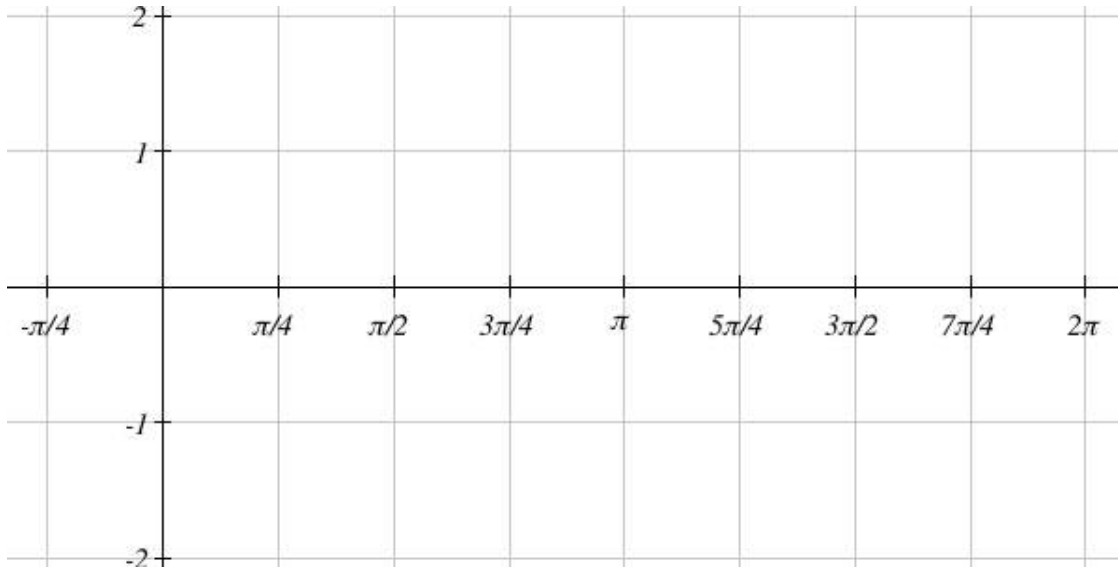
21.

Sketch a graph of the function $f(x) = \sin\left(x + \frac{3\pi}{4}\right)$.



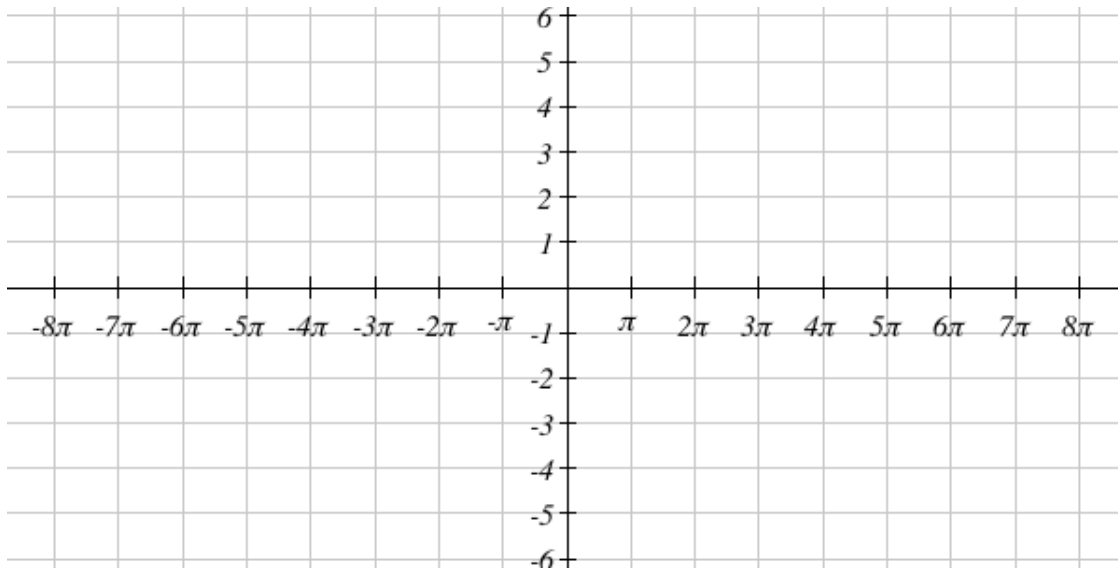
22.

Sketch a graph of the function $f(x) = \cos(x)$.



23.

Sketch a graph of the function $f(x) = -4\cos\left(\frac{2}{5}x\right)$.



24.

Outside temperature over a day can be modeled as a sinusoidal function. Suppose you know the temperature is 70 degrees at midnight and the high and low temperature during the day are 81 and 59 degrees, respectively. Assuming t is the number of hours since midnight, find an equation for the temperature, D , in terms of t .

$D(t) =$ _____

25.

A ferris wheel is 15 meters in diameter and boarded from a platform that is 4 meters above the ground. The six o'clock position on the ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 4 minutes. The function $h = f(t)$ gives your height in meters above the ground t minutes after the wheel begins to turn.

What is the Amplitude? _____ meters

What is the Midline? $y =$ _____ meters

What is the Period? _____ minutes

How High are you off of the ground after 2 minutes? _____ meters

26.

A ferris wheel is 30 meters in diameter and boarded from a platform that is 4 meters above the ground. The six o'clock position on the ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 4 minutes. The function $h = f(t)$ gives your height in meters above the ground t minutes after the wheel begins to turn. Write an equation for $h = f(t)$.

$f(t) =$ _____

Lecture 30

Sec/csc, tan/cot, periods, asymptotes, transformations

1.

On the interval $[0, 2\pi)$ determine which angles are not in the domain of the tangent function, $f(\theta) = \tan(\theta)$

What angles are NOT in the domain of the tangent function on the given interval? _____

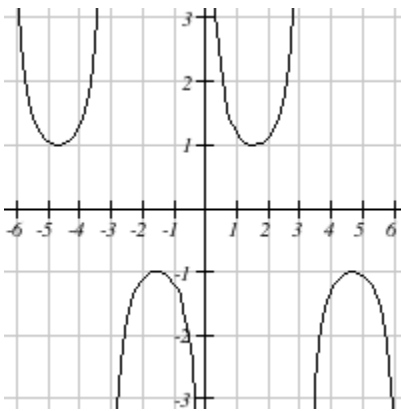
2.

On the interval $[0, 2\pi)$ determine which angles are not in the domain of the given functions.

What angles are NOT in the domain of the secant function on the given interval? _____

What angles are NOT in the domain of the cosecant function on the given interval? _____

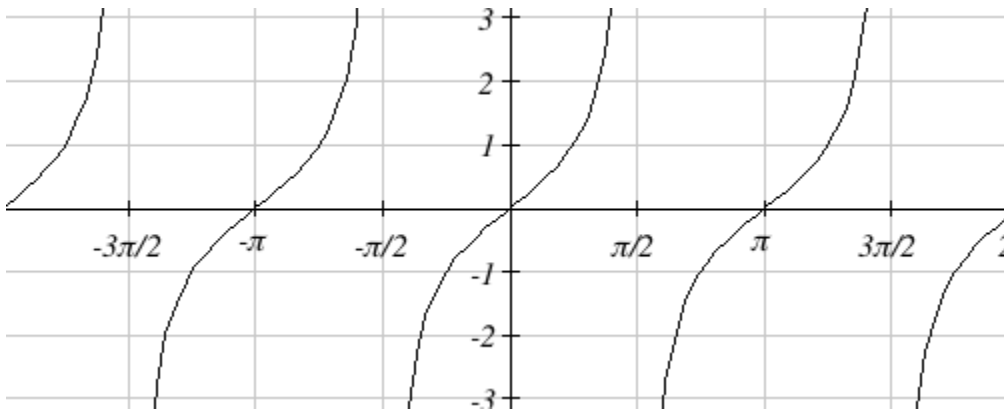
3.



The graph above is a graph of what function?

- $y = \sin(x)$
- $y = \csc(x)$
- $y = \cos(x)$
- $y = \sec(x)$
- $y = \tan(x)$
- $y = \cot(x)$

4.

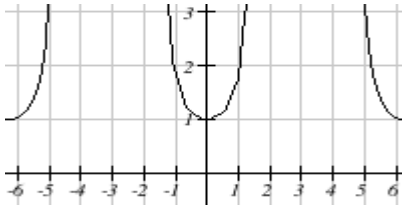


The graph above is a graph of what function?

- $y = \sin(x)$
- $y = \sec(x)$
- $y = \cot(x)$
- $y = \cos(x)$
- $y = \csc(x)$
- $y = \tan(x)$

5.

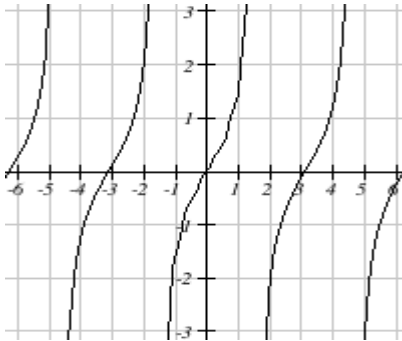
Match each graph with its equation. Not all equations will be used.



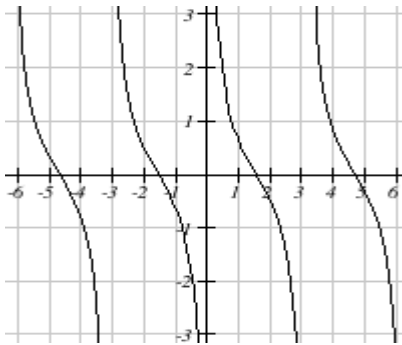
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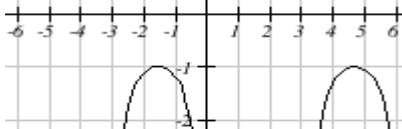
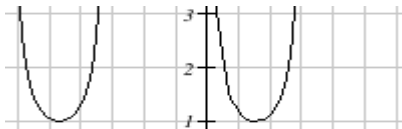
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•



- a. $y = \cot(x)$
- b. $y = \csc(x)$
- c. $y = \tan(x)$
- d. $y = \cos(x)$
- e. $y = \sec(x)$
- f. $y = \sin(x)$

6.

What is the period of the graph of the function $y = \tan\left(\frac{7\pi}{5}x\right)$?

period = _____

7.

What is the period of the graph of the function $y = \csc\left(\frac{5x}{8}\right)$?

period = _____

8.

What is the period of the graph of the function $y = \sec\left(\frac{9\pi}{2}x - 7\right)$?

period = _____

9.

Given the equation $y = 6\tan(3x - 6)$

The exact period (in terms of π) is: _____

The phase shift is: _____ units to the (Left/Right)

10.

Given the equation $y = 2\sec(3x + 24)$

The exact period (in terms of π) is: _____

The phase shift is: _____ units to the (Left/Right)

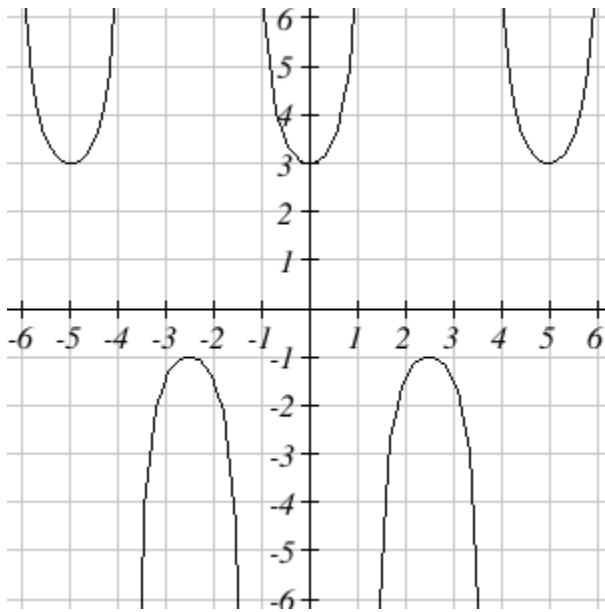
11.

Given the equation $y = 7\csc\left(\frac{2\pi}{3}x + \frac{10\pi}{3}\right)$

The exact period (give as an integer or fraction) is: _____

The horizontal shift is: _____ units to the (Left/Right)

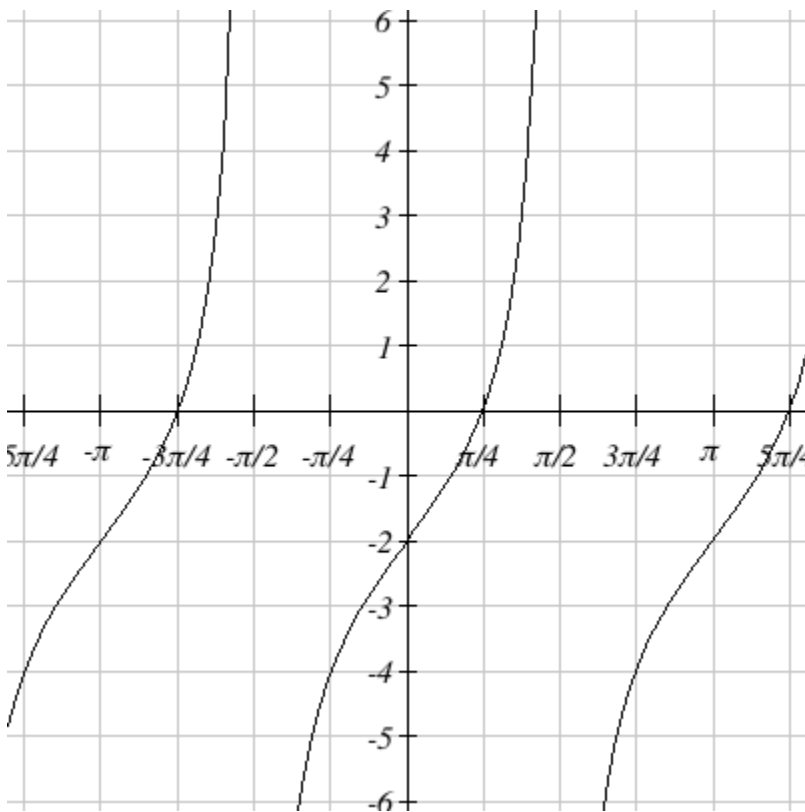
12.



Write an equation for the function graphed above. (There are multiple correct answers)

$y =$ _____

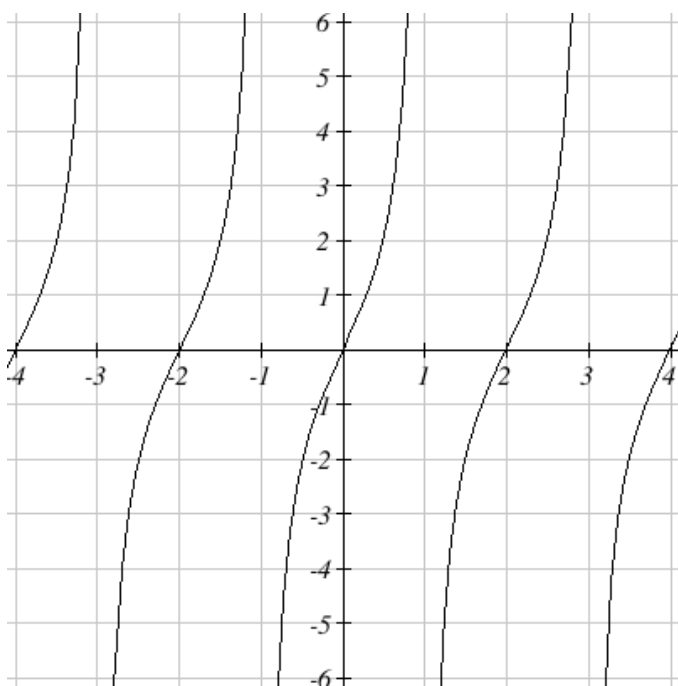
13.



Identify the function whose graph appears above. (There are multiple correct answers.)

$f(x) =$ _____

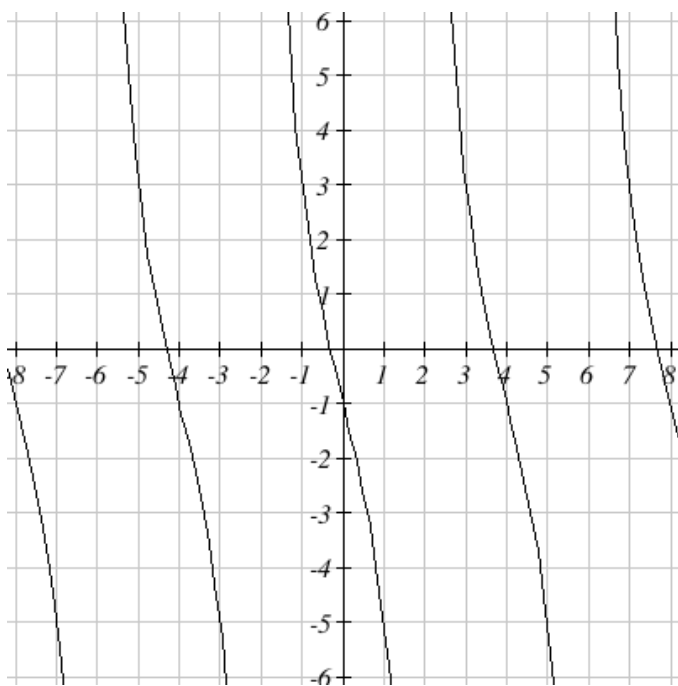
14.



Identify the function whose graph appears above. (There are multiple correct answers.)

$f(x) =$ _____

15.



Give the equation for the function whose graph appears above. (There are multiple correct answers.)

$f(x) =$ _____

Lecture 31

Inverse sin/cos/tan, restrictions, properties, composition, examples

1.

Evaluate the following expressions. Your answer must be an angle in radians.

(a) $\sin^{-1}(-1) = \underline{\hspace{2cm}}$

(b) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$

(c) $\sin^{-1}(1) = \underline{\hspace{2cm}}$

2.

Evaluate the following expressions. Your answer must be an exact angle in radians.

(a) $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$

(b) $\cos^{-1}\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$

(c) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$

3.

Evaluate the following expressions. Your answer must be an exact angle in radians.

(a) $\tan^{-1}(1) = \underline{\hspace{2cm}}$

(b) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \underline{\hspace{2cm}}$

(c) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \underline{\hspace{2cm}}$

4.

Use your calculator to evaluate $\cos^{-1}(0.58)$ to at least 3 decimal places. Give the answer in radians.

5.

Suppose $\sin\theta = -\frac{1}{10}$, and θ is an angle in standard position.

Then the terminal side of θ could be in (choose all that apply):

- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

$\arcsin\left(-\frac{1}{10}\right)$ is an angle whose terminal side is in (choose all that apply):

- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

6.

Suppose $\cos\theta = -\frac{9}{10}$, and θ is an angle in standard position.

Then the terminal side of θ could be in (choose all that apply):

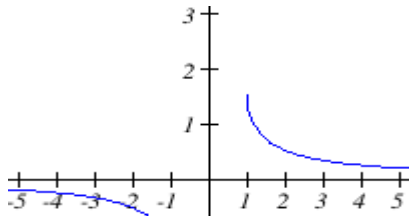
- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

$\arccos\left(-\frac{9}{10}\right)$ is an angle whose terminal side is in (choose all that apply):

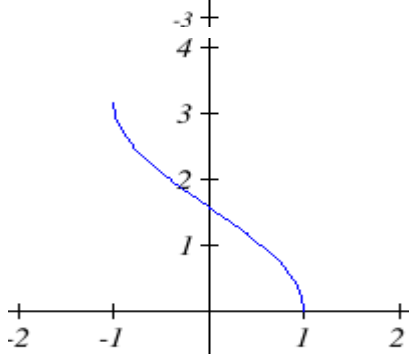
- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

7.

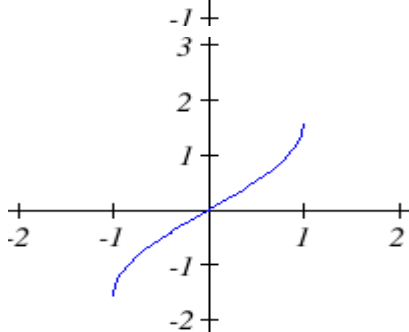
Match each graph with its equation. Not all equations will be used.



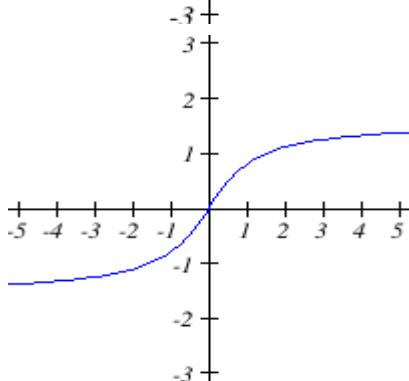
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•

- a. $y = \operatorname{arccot}(x)$
- b. $y = \operatorname{arccsc}(x)$
- c. $y = \operatorname{arcsec}(x)$
- d. $y = \arcsin(x)$
- e. $y = \arccos(x)$
- f. $y = \arctan(x)$

8.

Evaluate the following expression.

$$\sin(\arcsin(0.6)) = \underline{\hspace{2cm}}$$

9.

Evaluate the following expression.

$$\cos(\arccos(2.3)) = \underline{\hspace{2cm}}$$

10.

Evaluate the following expression.

$$\arcsin\left(\sin\left(\frac{-3\pi}{7}\right)\right) = \underline{\hspace{2cm}}$$

11.

Evaluate the following expression.

$$\arcsin\left(\sin\left(\frac{19\pi}{12}\right)\right) = \underline{\hspace{2cm}}$$

12.

Evaluate the following expression.

$$\sin^{-1}\left(\sin\left(\frac{-7\pi}{4}\right)\right) = \underline{\hspace{2cm}}$$

13.

Find the exact value or state that it is undefined. In the latter case, enter "DNE".

$$\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) = \underline{\hspace{2cm}}$$

14.

Find $\arcsin(\sin 230^\circ)$.

$\arcsin(\sin 230^\circ) = \underline{\hspace{2cm}}$ degrees.

15.

Evaluate the following expression.

$\tan(\arctan(1.3)) = \underline{\hspace{2cm}}$

16.

Evaluate the following expression.

$\arctan\left(\tan\left(\frac{32\pi}{9}\right)\right) = \underline{\hspace{2cm}}$

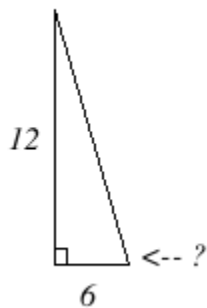
17.

Find the exact value or state that it is undefined. In the latter case, enter "DNE".

$\tan(\tan^{-1}(1)) = \underline{\hspace{2cm}}$

18.

For the right triangle below, find the measure of the angle.
Figure is not to scale.



$\underline{\hspace{2cm}}$ degrees

19.

Evaluate the expression: $\cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right) = \underline{\hspace{2cm}}$

20.

Evaluate the expression: $\sin^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right) = \underline{\hspace{2cm}}$

21.

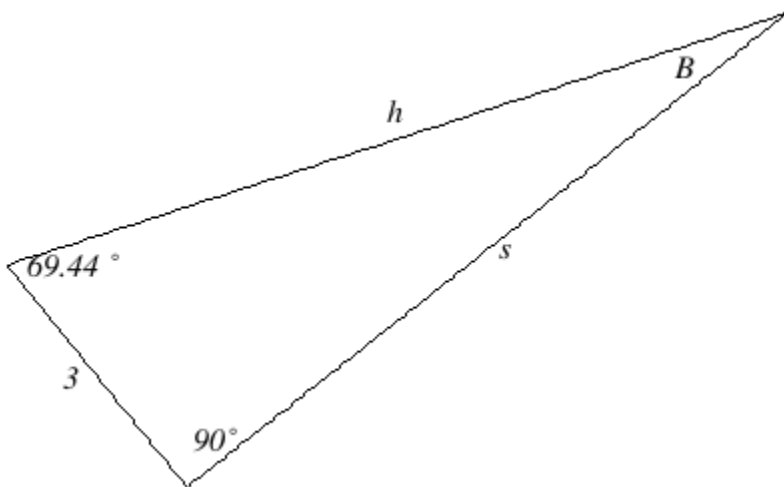
Evaluate: $\sin\left(\cos^{-1}\left(\frac{1}{3}\right)\right) = \underline{\hspace{2cm}}$

22.

Find an algebraic expression for $\cos\left(\tan^{-1}\left(\frac{a}{3}\right)\right)$

23.

Find the unknowns in the graph below:



$B = \underline{\hspace{2cm}}$

$h = \underline{\hspace{2cm}}$

$s = \underline{\hspace{2cm}}$

24.

Write the given expression in algebraic form.

$$\cos(\tan^{-1}(y)) = \underline{\hspace{2cm}}$$

25.

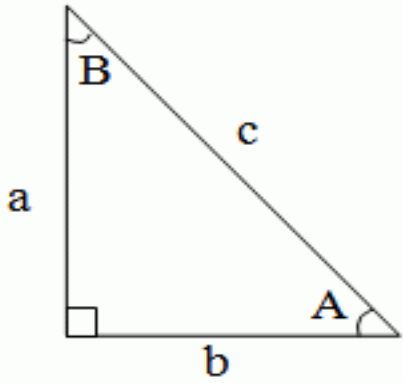
Write the given expression in algebraic form.

$$\cot(\sin^{-1}(x)) = \underline{\hspace{2cm}}$$

Lecture 32

Solving triangles, bearing, other applications

1.



Suppose $a = 8$ and $c = 13$.

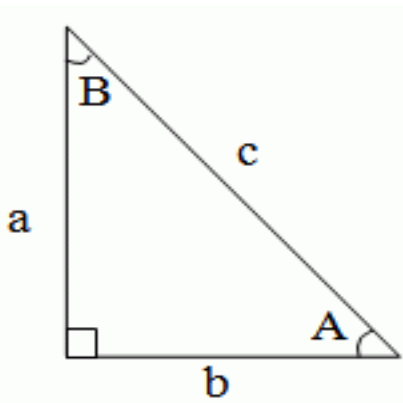
Find:

$b =$ _____

$A =$ _____ degrees

$B =$ _____ degrees

2.



Suppose $a = 7$ and $b = 2$.

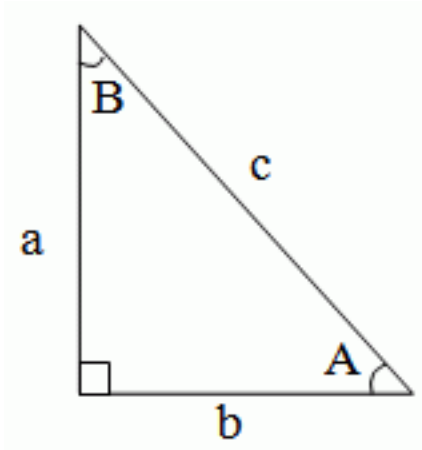
Find:

$c =$ _____

$A =$ _____ degrees

$B =$ _____ degrees

3.



Suppose $a = 5$ and $A = 70$ degrees.

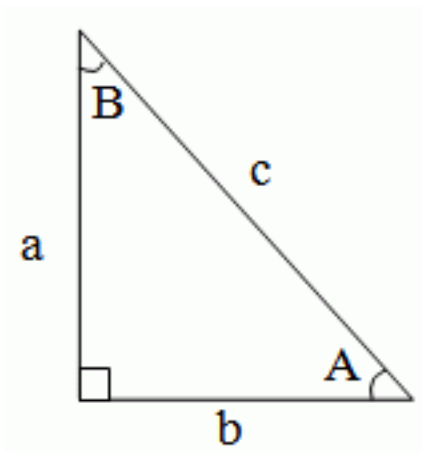
Find:

$$b = \underline{\hspace{2cm}}$$

$$c = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}} \text{ degrees}$$

4.



Suppose $c = 7$ and $A = 10$ degrees. Find:

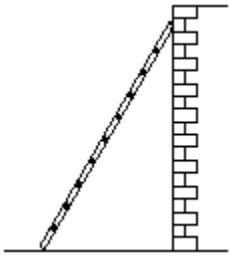
$$a = \underline{\hspace{2cm}}$$

$$b = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}} \text{ degrees}$$

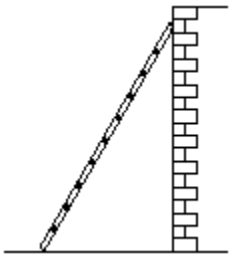
5.

The proper angle for a ladder is about 75° from the ground. Suppose you have a 12 foot ladder. How far from the house should you place the base of the ladder? _____ feet

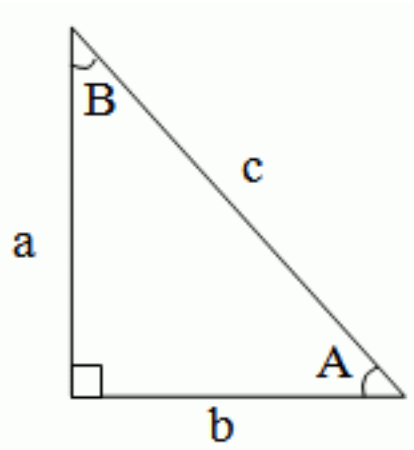


6.

The proper angle for a ladder is about 75° from the ground. Suppose you have a 13 foot ladder. How high can it reach? _____ feet



7.



Suppose $\angle A = 30^\circ$ and $a = 22$.

$\angle B = \quad^\circ$

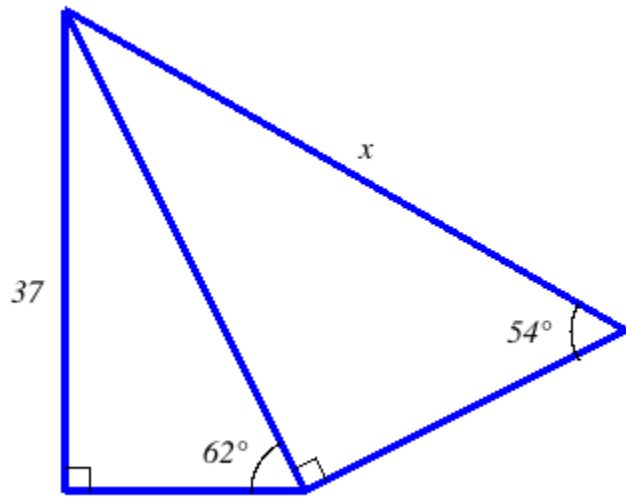
Find an exact value (report answer as a fraction, use sqrt if necessary):

$c =$ _____ feet

8.

Find x correct to 2 decimal places.

NOTE: The triangle is NOT drawn to scale.



$x =$ _____ feet

9.

To measure the height of the cloud cover at an airport, a worker shines a spotlight upward at an angle of 65° from the horizontal. An observer 682 m away measures the angle of elevation to the spot of light to be 41° . Find the height of the cloud cover.

height = _____ m

10.

From the top of a 183-ft lighthouse, the angle of depression to a ship in the ocean is 27° . How far is the ship from the base of the lighthouse?

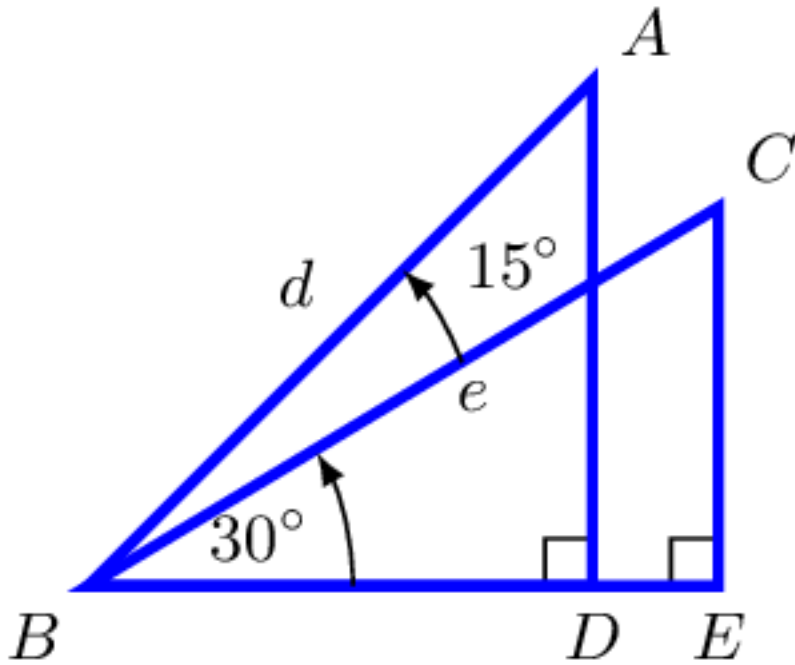
distance = _____ feet

11.

A smokestack is 160 feet high. A guy wire must be fastened to the stack 20 feet from the top. The guy wire makes an angle of 40° with the ground. Find the length of the guy wire rounded to the nearest foot.

_____ feet

12.



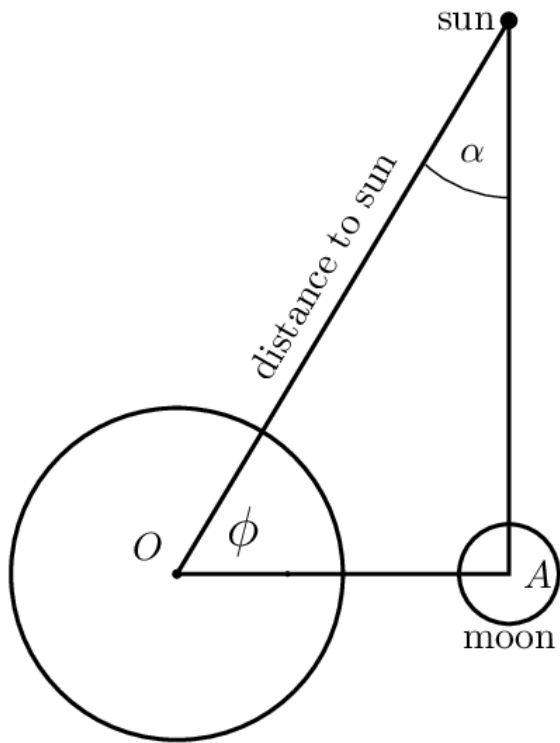
Using the special triangles, determine the exact value of segment DE. Segments $d = BA$ and $e = BC$ have length 2. Express your answer in simplified radical form.

DE = _____

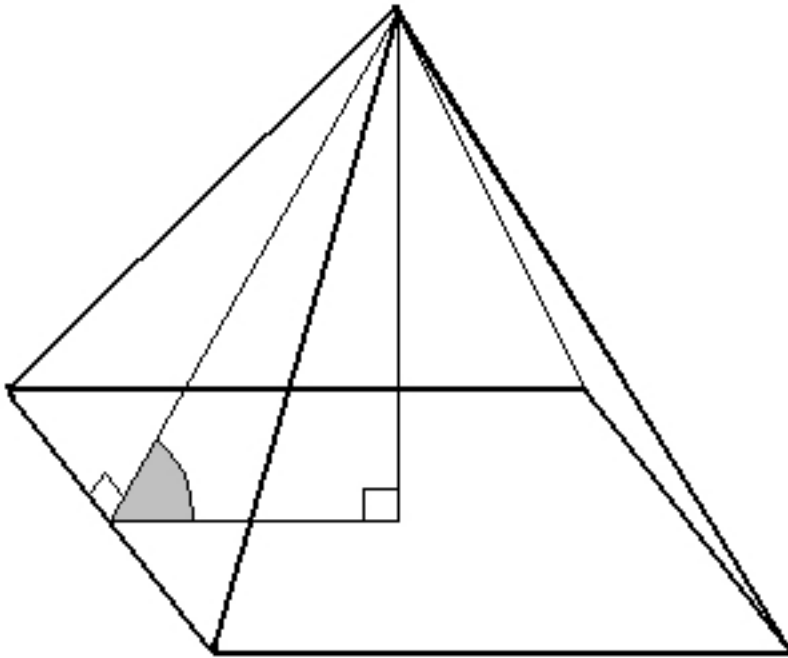
13.

An alien on a distant planet realizes that using trigonometry and the distance to one of its moons it is possible to calculate the distance to the nearby sun. Let O be the center of the planet and let A be the center of the moon. The alien begins with the premise that, during a half moon, the moon forms a right triangle with the Sun and the planet. By observing the angle between the Sun and Moon, $\phi = 89.43$ degrees and knowing the distance to the moon is about 179000 km estimate the distance from the planet to the sun using these values. Round to the nearest 1000 km

distance =



14.



Consider a square-based straight pyramid. Suppose that the base is a square with sides 6 cm long, and all other edges are 7 cm long. Find an approximate value of the angle formed between the base and a triangular face. Present your answer in degrees, accurate up to four or more decimal places.

$\alpha = \underline{\hspace{2cm}}^\circ$ (degrees)

15.

From a fire tower 200 feet above level ground in the Sasquatch National Forest, a ranger spots a fire off in the distance. The angle of depression to the fire is 2.7° . How far away from the base of the tower is the fire? Round to the nearest foot.

 ft

16.

From the observation deck of the lighthouse at Sasquatch Point 48 feet above the surface of Lake Ippizuti, a lifeguard spots a boat out on the lake sailing directly toward the light house. The first sighting had a angle of depression of 8.2° and the second sighting had an angle of depression of 26° . How far had the boat traveled between the sightings?

 ft

Lecture 33

Identities: Reciprocal, Quotient, Pythagorean, Even/odd, Cofunction

Simplifying, Factoring expressions, Combining using identities, Trig substitution, Simplifying log expressions

1.

Simplify $\sin(t)\sec(t)$ to a single trig function or constant.

2.

Simplify $\frac{\csc(t)}{\sec(t)}$ to a single trig function.

3.

Simplify $\frac{\cot(t)}{\csc(t)-\sin(t)}$ to a single trig function.

4.

Simplify $\frac{1+\csc(t)}{1+\sin(t)}$ to a single trig function.

5.

Simplify $\frac{\cos^2(t)}{1-\cos^2(t)}$ to an expression involving a single trig function with no fractions.

6.

Fill in the blanks:

1. If $\tan x = -3$ then $\tan(-x) =$ _____

2. If $\sin x = 0.1$ then $\sin(-x) =$ _____

3. If $\cos x = 0.7$ then $\cos(-x) =$ _____

4. If $\tan x = -3.5$ then $\tan(\pi + x) =$ _____

7.
Simplify to an expression involving a single trigonometric function with no fractions.

$$\cot(-x)\cos(-x) + \sin(-x) = \underline{\hspace{2cm}}$$

8.
Simplify and write the trigonometric expression in terms of sine and cosine:
 $\tan^2 x - \sec^2 x = \underline{\hspace{2cm}}$.

9.
Determine the value of $\sin^2 x + \cos^2 x$ for $x = 50$ degrees.

10.
Simplify and write the trigonometric expression in terms of sine and cosine:

$$\cot(-x)\cos(-x) + \sin(-x) = -\frac{1}{f(x)}$$
$$f(x) = \underline{\hspace{2cm}}.$$

11.
If $\tan^2 t - \sin^2 t = \frac{\sin^a t}{\cos^b t}$, then
the positive power $a = \underline{\hspace{2cm}}$,
the positive power $b = \underline{\hspace{2cm}}$.

12.
Simplify and write the trigonometric expression in terms of sine and cosine:

$$\frac{2 + \tan^2 x}{\sec^2 x} - 1 = g(x)$$
$$g(x) = \underline{\hspace{2cm}}.$$

13.

Simplify $\frac{1+\csc(t)}{1+\sin(t)}$ to a single trig function.

14.

Simplify and write the trigonometric expression without any fractions:

$$\tan u + \cot u = \underline{\hspace{2cm}}$$

15.

Factor: $2\sin^2(x) - 3\sin(x) + 1 = \underline{\hspace{2cm}}$

16.

Factor: $2\sin^2(x) - \sin(x) - 1 = \underline{\hspace{2cm}}$

17.

Suppose that α is an acute angle with $\tan \alpha = \frac{11}{10}$. Compute the exact value of $\sec \alpha$. You do not have to rationalize the denominator.

$$\sec \alpha = \underline{\hspace{2cm}}$$

18.

Use a substitution $x = f(t)$ to re-express $\sqrt{49 - x^2}$ as a trigonometric expression in terms of t . State the function $f(t)$ used for substitution and the new expression.

$$f(t) = \underline{\hspace{2cm}}$$

$$\sqrt{49 - x^2} \text{ can be rewritten as } \underline{\hspace{2cm}}$$

19.

Use a substitution $x = f(t)$ to re-express $\sqrt{x^2 + 4}$ as a trigonometric expression in terms of t . State the function $f(t)$ used for substitution and the new expression.

$$f(t) = \underline{\hspace{2cm}}$$

$$\sqrt{x^2 + 4} \text{ can be rewritten as } \underline{\hspace{2cm}}$$

20.

Use a substitution $x = f(t)$ to re-express $\sqrt{x^2 - 25}$ as a trigonometric expression in terms of t . State the function $f(t)$ used for substitution and the new expression.

$$f(t) = \underline{\hspace{2cm}}$$

$$\sqrt{x^2 - 25} \text{ can be rewritten as } \underline{\hspace{2cm}}$$

Lecture 34

Linear, quadratic, multiple angle, and using inverse trig functions

1.

Find all solutions to $2\sin(\theta) = 1$ on the interval $0 \leq \theta < 2\pi$

$\theta =$ _____

Give your answers as exact values, as a list separated by commas.

2.

Find all solutions to $2\sin(\theta) = \sqrt{2}$ on the interval $0 \leq \theta < 2\pi$

$\theta =$ _____

Give your solutions as exact values, separating multiple solutions by commas.

3.

Find all solutions to $2\sin(\theta) = -\sqrt{2}$ on the interval $0 \leq \theta < 2\pi$.

$\theta =$ _____

Give your answers as exact values in a list separated by commas.

4.

Find all solutions to $2\cos(\theta) = \sqrt{3}$ on the interval $0 \leq \theta < 2\pi$.

$\theta =$ _____

Give your answers as exact values in a list separated by commas.

5.

Solve $\sin(x) = 0.42$ on $0 \leq x < 2\pi$.

There are two solutions, A and B, with $A < B$.

A = _____

B = _____

Give your answers accurate to 3 decimal places.

6.

Solve $\cos(x) = 0.31$ on $0 \leq x < 2\pi$.

There are two solutions, A and B, with $A < B$.

A = _____

B = _____

Give your answers accurate to 3 decimal places.

7.

Find all solutions of the equation $2\cos x - 1 = 0$.

_____ + $2k\pi$ where k is any integer

8.

Solve $5\cos(w) = 0$ for all solutions.

$w =$ _____ where k is any integer

9.

Solve $2\sin(x) = 2$ for all solutions.

$x =$ _____ where k is any integer

10.

Without using a calculator, find all the solutions of

$$\tan(t) = 1$$

$$t = \underline{\hspace{2cm}} \text{ where } -\pi < t \leq \pi .$$

11.

Find the exact solutions to $\sin(x) = \cos(x)$ in the interval $[0, 2\pi)$. If the equation has no solutions, answer DNE.

12.

Solve $2\sin^2(t) + 3\sin(t) + 1 = 0$ for all solutions $0 \leq t < 2\pi$.

$$t = \underline{\hspace{2cm}}$$

Give your answers as exact values in a list separated by commas.

13.

Solve $2\cos^2(w) + 3\cos(w) + 1 = 0$ for all solutions.

$$w = \underline{\hspace{2cm}} + 2k\pi \text{ where } k \text{ is any integer}$$

Give your answers as exact values in a list separated by commas.

14.

Solve $2\cos^2(x) - 7\cos(x) + 5 = 0$ for all solutions.

$$x = \underline{\hspace{2cm}} \text{ where } k \text{ is any integer}$$

15.

Find all solutions of $\sin^2(x) - 4\cos(x) = 4$.

$$x = \underline{\hspace{2cm}} \text{ where } n \text{ is any integer}$$

16.

Find all solutions of $\sin^2(x) - 8\cos(x) = -8$.

$x =$ _____ where n is any integer.

17.

Find all solutions on the interval $[0, 2\pi)$. Give exact answers.

$$\sin^2(x) - \cos^2(x) + \sin(x) = 0$$

$x =$ _____

18.

Solve for the exact solutions in the interval $[0, 2\pi)$. If the equation has no solutions, respond with DNE.

$$\sec(x) = 2\csc(x)$$

$x =$ _____

19.

REMOVED

20.

$$\text{Suppose } \sin 3x = -\frac{\sqrt{3}}{2}.$$

Find all solutions $0 \leq x \leq 2\pi$. Give exact values in radians.

$x =$ _____

21.

Find all solutions in the interval $[0, 360^\circ)$. List your answers in degrees. If there is no real solution, answer DNE.

$$\tan 3x = 0$$

$$x = \underline{\hspace{2cm}}$$

22.

Find all solutions in the interval $[0, 360^\circ)$. List your answers in degrees. If there is no real solution, answer DNE.

$$\sin 2x = -\frac{1}{2}$$

$$x = \underline{\hspace{2cm}}$$

Lecture 35

Conditional vs identity equations, guidelines for verifying

1.

If $(\tan x + \sec x)^2 = \frac{A + \sin x}{B - \sin x}$, then

$$A = \underline{\hspace{2cm}},$$

$$B = \underline{\hspace{2cm}}.$$

2.

The expression $3\tan(x)\sin(x) + 5\sec(x)$ simplifies to $A\sec(x) - B\cos(x)$, determine A and B .

$$A = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}}$$

3.

$\tan(x) + \sec(x)$ simplifies to $\frac{f(x)}{\cos(x)}$ where

$$f(x) = \underline{\hspace{2cm}}$$

4.

Simplify $\frac{1 + \cos(t)}{1 + \sec(t)}$ to a single trig function.

5.

Simplify $\frac{\sin^2(t) + \cos^2(t)}{\sin^2(t)}$ to an expression involving a single trig function with no fractions.

6.

Simplify and write the trigonometric expression in terms of sine and cosine:

$$\frac{1 + \cos y}{1 + \sec y} = \underline{\hspace{2cm}}.$$

7.

Simplify the lefthandside so that $LHS = RHS$:

$$-\frac{\cos(x)}{1 - \sin(x)} + \frac{\cos(x)}{1 + \sin(x)} = -2\tan(x)$$

_____ =

_____ =

_____ =

_____ =

_____ = $-2\tan(x)$

8.

Simplify the lefthandside so that $LHS = RHS$:

$$\left(\frac{1}{\tan(y)} + \frac{1}{\sin(y)}\right)^2 = \frac{1 + \cos(y)}{1 - \cos(y)}$$

_____ =

_____ =

_____ =

_____ =

_____ = $\frac{1 + \cos(y)}{1 - \cos(y)}$

9.

Prove the given identity.

$$\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 2 \tan x \sec x$$

10.

Prove the given identity.

$$\frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$$

11.

Prove the given identity.

$$\frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

12.

Prove the given identity.

$$\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$$

Lecture 36

Sum/difference formulas for sin/cos/tan

1.

Use the sum or difference formula for cosine to find the exact value for $\cos(75^\circ)$

$$\cos(75^\circ) = \underline{\hspace{2cm}}$$

2.

Use the sum or difference formula for tangent to find the exact value for $\tan(15^\circ)$

$$\tan(15^\circ) = \underline{\hspace{2cm}}$$

3.

Use the sum or difference formula for sine to find the exact value for $\sin\left(\frac{5\pi}{12}\right)$

$$\sin\left(\frac{5\pi}{12}\right) = \underline{\hspace{2cm}}$$

4.

Find the exact value of $\cos(75^\circ)$

5.

Use the sum and difference identities to find the exact value. You may have need of the quotient, reciprocal or even/odd identities as well.

$$\cos\left(\frac{11\pi}{12}\right) = \underline{\hspace{2cm}}$$

6.

Use an addition or subtraction formula to find the exact value of $\tan 75^\circ = \frac{\sqrt{A+1}}{\sqrt{B-1}}$.

$$A = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}}$$

7.

Use an addition or subtraction formula to write the expression as a trigonometric function of one number:

$$\cos \frac{3\pi}{7} \cos \frac{2\pi}{21} + \sin \frac{3\pi}{7} \sin \frac{2\pi}{21} = \cos \frac{\pi}{A} = \frac{B}{2}.$$

$$A = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}}$$

8.

Use an addition or subtraction formula to write the expression as a trigonometric function of one angle:

$$\cos \frac{3\pi}{7} \cos \frac{2\pi}{21} + \sin \frac{3\pi}{7} \sin \frac{2\pi}{21} = \underline{\hspace{2cm}}$$

The exact value of this function is $\underline{\hspace{2cm}}$.

9.

Use an addition or subtraction formula to write the expression as a trigonometric function of one number:

$$\frac{\tan 72^\circ - \tan 12^\circ}{1 + \tan 72^\circ \tan 12^\circ} = \tan A^\circ = \sqrt{B}.$$

$$A = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}}$$

10.

If $\cos \alpha = 0.665$ and $\cos \beta = 0.587$ with both angles' terminal rays in Quadrant-I, find the values of

(a)
 $\sin(\alpha + \beta) = \underline{\hspace{2cm}}$

(b)
 $\sin(\alpha - \beta) = \underline{\hspace{2cm}}$

Round your answer to 4 decimal places

11.

If $\cos\alpha = 0.766$ and $\sin\beta = 0.198$ with both angles' terminal rays in Quadrant-I, find the values of

(a)

$$\cos(\alpha + \beta) = \underline{\hspace{2cm}}$$

(b)

$$\sin(\beta - \alpha) = \underline{\hspace{2cm}}$$

Round your answer to 4 decimal places

12.

If $\sin\alpha = 0.363$ and $\sin\beta = 0.147$ with both angles' terminal rays in Quadrant-I, find the values of

(a)

$$\sin(\alpha + \beta) = \underline{\hspace{2cm}}$$

(b)

$$\cos(\alpha - \beta) = \underline{\hspace{2cm}}$$

Round your answer to 4 decimal places

13.

If $\sin\alpha = 0.417$ and $\sin\beta = 0.151$ with both angles' terminal rays in Quadrant-I, find the values of

$$\tan(\alpha + \beta) = \underline{\hspace{2cm}}$$

Round your answer to 4 decimal places

14.

If $\cos\alpha = 0.392$ and $\sin\beta = 0.592$ with both angles' terminal rays in Quadrant-I, find the values of

$$\tan(\beta - \alpha) = \underline{\hspace{2cm}}$$

Round your answer to 4 decimal places

15.

Suppose $\tan(\alpha) = -6$, where $\frac{3\pi}{2} < \alpha < 2\pi$ and β is a Quadrant II angle with $\tan(\beta) = -\frac{1}{2}$.

$$\tan(\alpha + \beta) = \underline{\hspace{2cm}}$$

16.

If $\tan\theta = \frac{14}{5}$ and $\cot\phi = \frac{11}{7}$, find the exact value of $\sin(\theta + \phi)$.

$\sin(\theta + \phi) =$ _____

17.

Rewrite $\sin\left(x + \frac{\pi}{3}\right)$ in terms of $\sin(x)$ and $\cos(x)$

18.

If $\sin(x + y) - \sin(x - y) = 2f(x)\sin y$, then
 $f(x) =$ _____.

Lecture 37

Double angle, half angle, and power reducing formulas, using $2x$ angle for higher multiples

1.

Complete the double angle formulas for sine, cosine, and tangent:

$$\sin(2x) = \underline{\hspace{2cm}}$$

$$\cos(2x) = \underline{\hspace{2cm}}$$

$$\tan(2x) = \underline{\hspace{2cm}}$$

2.

Simplify without a calculator:

$$2\cos^2(15^\circ) - 1$$

$$= \cos(\underline{\hspace{2cm}})$$

$$= \underline{\hspace{2cm}}$$

3.

Using a double-angle or half-angle formula to simplify the given expressions.

(a) If $\cos^2(29^\circ) - \sin^2(29^\circ) = \cos(A^\circ)$, then

$$A = \underline{\hspace{2cm}} \text{ degrees}$$

(b) If $\cos^2(3x) - \sin^2(3x) = \cos(B)$, then

$$B = \underline{\hspace{2cm}} .$$

4.

Use a half angle formula to find the **exact value**. You may have need of the quotient, reciprocal or even/odd identities as well.

$$\cos(255^\circ) = \underline{\hspace{2cm}}$$

5.

Use half angle formulas to find the exact value. You may have need of the quotient, reciprocal or even even/odd identities as well.

$$\cos\left(\frac{19\pi}{12}\right) = \underline{\hspace{2cm}}$$

6.

Suppose A is an acute angle, and $\sin A = \frac{144}{145}$, $\cos A = \frac{17}{145}$.

Find $\sin 2A$ and $\cos 2A$.

$$\sin 2A = \underline{\hspace{2cm}}$$

$$\cos 2A = \underline{\hspace{2cm}}$$

7.

$\tan(\theta) = \frac{5}{12}$ where $0 < \theta < \frac{\pi}{2}$.

Use the given information about θ to find the exact value of $\cos(2\theta)$.

8.

$\cos(\theta) = \frac{4}{5}$ where $\frac{3\pi}{2} < \theta < 2\pi$.

Use the given information about θ to find the exact value of $\sin(2\theta)$.

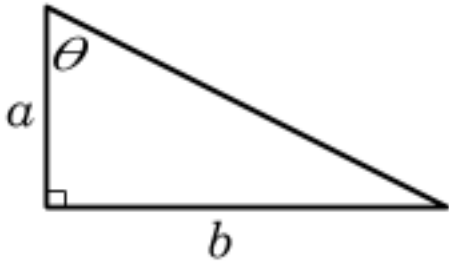
9.

$\tan(\theta) = 9$ where $\pi < \theta < \frac{3\pi}{2}$.

Use the given information about θ to find the exact value of $\tan(2\theta)$.

10.

Let θ be the acute angle described in the figure



where $a = 2$ and $b = 3$. Find the exact values of the following:

$$\cos\left(\frac{\theta}{2}\right) = \underline{\hspace{2cm}}$$

$$\sin\left(\frac{\theta}{2}\right) = \underline{\hspace{2cm}}$$

$$\tan\left(\frac{\theta}{2}\right) = \underline{\hspace{2cm}}$$

11.

Suppose that $\frac{\alpha}{2}$ is an angle in quadrant 3 and that $\cos\alpha = -\frac{161}{289}$. Compute the exact value of $\sin\left(\frac{\alpha}{2}\right)$.

$$\sin\left(\frac{\alpha}{2}\right) = \underline{\hspace{2cm}}$$

12.

Suppose $\cos 2A = \frac{2}{9}$, and $2A$ is an angle in the Fourth Quadrant.

$$\text{Then } \cos A = \underline{\hspace{2cm}}$$

13.

Suppose $\sin 2A = \frac{\sqrt{17}}{9}$, and $2A$ is an angle in the Second Quadrant.

$$\text{Then } \sin A = \underline{\hspace{2cm}}$$

14.

$$\sin(\theta) = \frac{-5}{13}, \text{ where } \pi < \theta < \frac{3\pi}{2}.$$

Use the given information about θ to find the exact value of $\sin\left(\frac{\theta}{2}\right)$

15.

$$\tan(\theta) = -\frac{7}{24}, \text{ where } \frac{3\pi}{2} < \theta < 2\pi.$$

Use the given information about θ to find the exact value of $\cos\left(\frac{\theta}{2}\right)$

16.

Evaluate the following (assume initial angles are in Quadrant-I)

$$\sin\left(2\cot^{-1}\left(\frac{17}{18}\right)\right) = \underline{\hspace{2cm}}$$

17.

Simplify the expression: Give the answer in exact form. Please reduce any fractions, but the radical can be left alone.

$$\sin\left(2\cos^{-1}\left(\frac{2}{7}\right)\right) = \underline{\hspace{2cm}}$$

18.

Solve for the exact solutions in the interval $[0, 2\pi)$. List your answers separated by a comma, if it has no real solutions, answer DNE.

$$2\sin(\theta) = 2\cos(2\theta)$$

19.

Solve for the exact solutions in the interval $[0, 2\pi)$. List your answers separated by a comma, if it has no real solutions, answer DNE.

$$\cos(2x) - \cos(x) = 0$$

20.

Solve for the exact solutions in the interval $[0, 2\pi)$. List your answers separated by a comma, if it has no real solutions, answer DNE.

$$\sin(2\theta) = \cos(\theta)$$

21.

Solve for the exact solutions in the interval $[0, 2\pi)$. If it has no solutions, respond with DNE.

$$8\sec^2\left(\frac{x}{2}\right) = 4$$

22.

Solve for the exact solutions in the interval $[0, 2\pi)$. List your answers separated by a comma, if it has no real solutions, answer DNE.

$$\sin\left(\frac{x}{2}\right) = \sqrt{2} - \sin\left(\frac{x}{2}\right)$$

23.

Solve for the exact solutions in the interval $[0, 2\pi)$. List your answers separated by a comma, if it has no real solutions, answer DNE.

$$\sec\left(\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$$

24.

Solve for the exact solutions in the interval $[0, 2\pi)$. List your answers separated by a comma, if it has no real solutions, answer DNE.

$$\sin\left(\frac{x}{2}\right) = \cos\left(\frac{x}{2}\right)$$

25.

Solve for the exact solutions in the interval $[0, 2\pi)$. List your answers separated by a comma, if it has no real solutions, answer DNE.

$$2\sqrt{3}\sin\left(\frac{x}{2}\right) = 3$$

26.

Simplify $\frac{(\cos(t)-\sin(t))^2 - (\sin(t)+\cos(t))^2}{2\csc(t)\sin(2t)}$ to a single trig function

27.

Prove the following identity

$$\frac{\sin(2b)}{\cos(2b) - 1} = \frac{1}{-\tan(b)}$$