MAC 1147 — Fall 2019 — Final Exam A

Questions 1–21 are worth 5 points each.

1. The two expressions below are equivalent if which restrictions are made on x?

$$\frac{\overbrace{x^2 - 11x + 28}^{(\cancel{x-7})(x-4)}}{\underbrace{2x^2 - 11x + 15}_{(2x-5)(\cancel{x-3})}} \cdot \frac{\cancel{x-3}}{\cancel{x-7}} \qquad\qquad \frac{x-4}{2x-5}$$

A. $x \neq 0$

B. $x \neq 3$

C. $x \neq 3, 7$

D. $x \neq \frac{5}{2}$

E. The expressions are equivalent for all x.

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2. The points $(7, 4)$ and $(13, 12)$ are the endpoints of a diameter of a circle. What is the equation of the circle?

$\text{center} = \left(\frac{13+7}{2}, \frac{12+4}{2}\right) = (10, 8)$

$\sqrt{(10-7)^2 + (8-4)^2} = \sqrt{3^2 + 4^2}$

$= \sqrt{25} = 5$

A. $(x-7)^2 + (y-4)^2 = 5$

B. $(x-7)^2 + (y-4)^2 = 25$

C. $(x-10)^2 + (y-8)^2 = 5$

D. $(x-10)^2 + (y-8)^2 = 25$

E. $x^2 + y^2 = 1$

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3. What is the y-intercept of the line that passes through the points $(10, 25)$ and $(20, 43)$?
Hint: First find the equation of the line.

A. 9　　B. $\frac{5}{7}$　　C. $\frac{1}{9}$　　D. 8　　E. 7

$$\frac{43-25}{20-10} = \frac{18}{10} = \frac{9}{5}$$

$$y = \frac{9}{5}(x-10)+25$$
$$= \frac{9}{5}x - 18 + 25$$
$$= \frac{9}{5}x + 7$$

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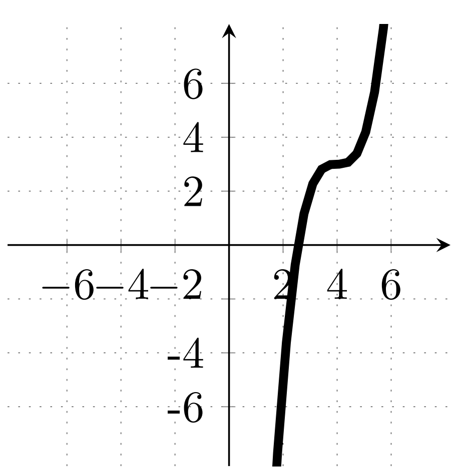
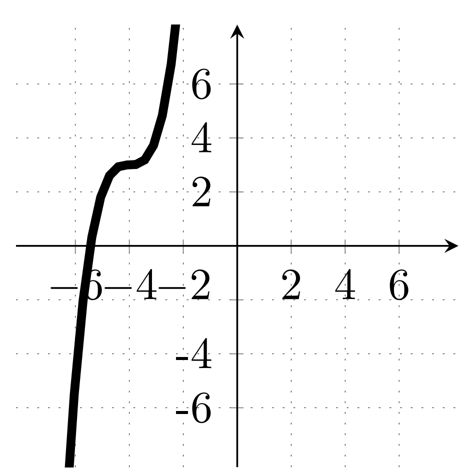
4. Which picture shows the graph of $f(x) = (x-4)^3 + 3$?



A.



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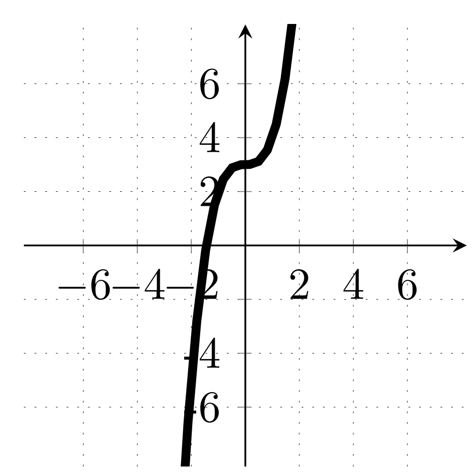

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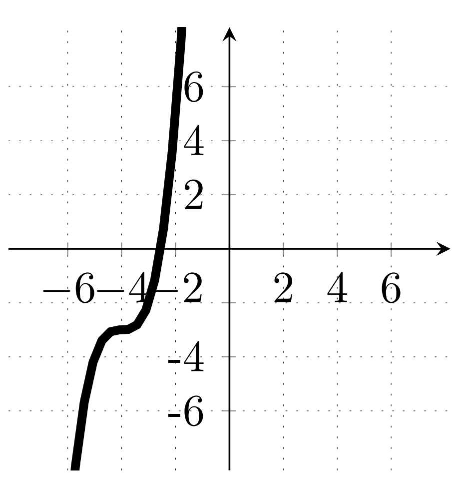
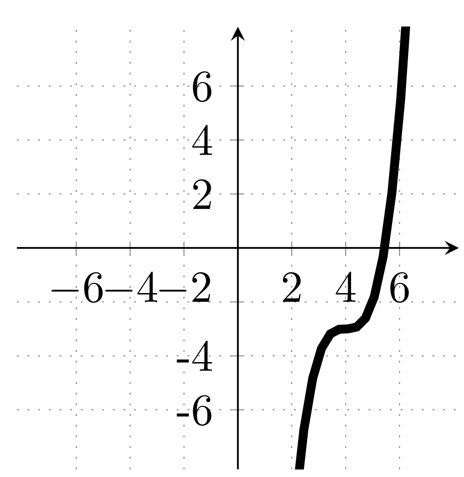


D.

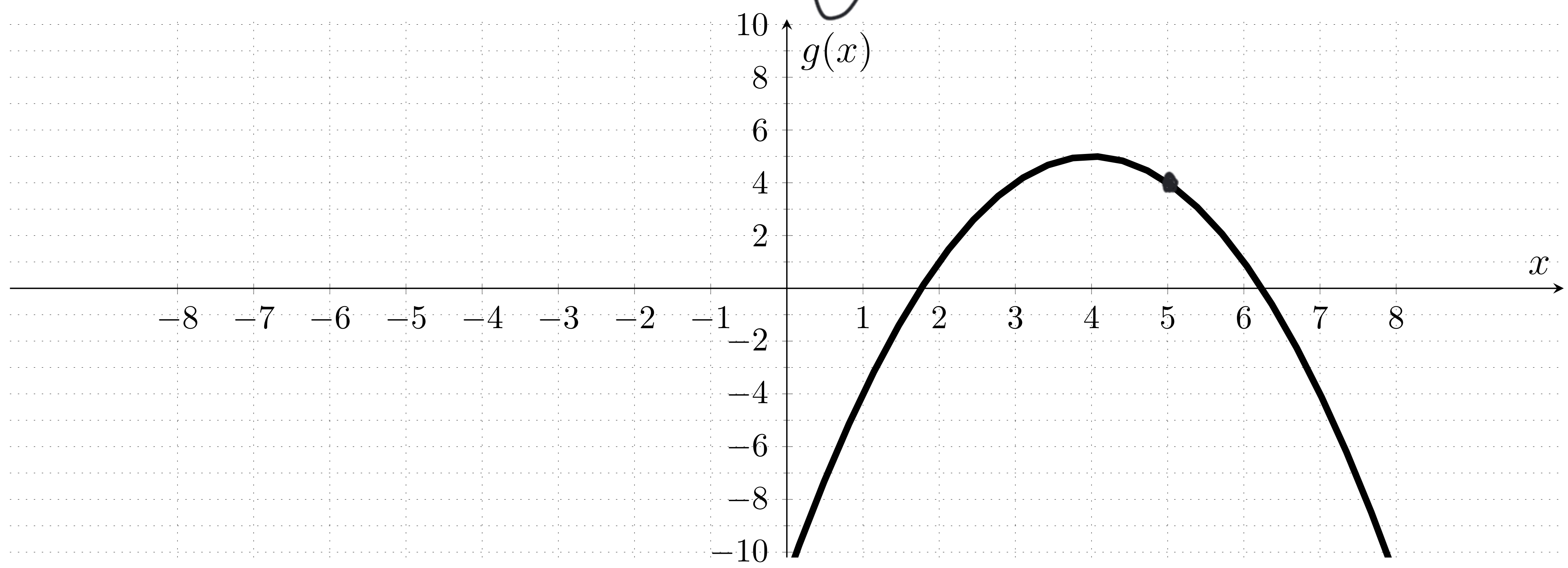


E.

5. Use the table of $f(x)$ and graph of $g(x)$ below to evaluate $(g \circ f)(3)$. $f(3)=5$

$g(5)=4$

x	4	0	3	2	5
$f(x)$	3	8	5	2	9



A. 3 B. 4 C. 0 D. 5 E. 9

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6. The function $f(x)$ below is one-to-one. What is its inverse?

$x=\sqrt[5]{4y+7}$

$x^5=4y+7$

$x^5-7=4y$

$$f(x) = \sqrt[5]{4x+7}$$

A. $f^{-1}(x) = \dfrac{1}{\sqrt[5]{4x+7}}$

B. $f^{-1}(x) = \left(\dfrac{1}{4}x - 7\right)^5$

C. $f^{-1}(x) = \dfrac{x^5-7}{4}$

D. $f^{-1}(x) = \sqrt[5]{4x+7}$

E. $f^{-1}(x) = \left(\dfrac{x}{4}\right)^5 - 7$

$\dfrac{x^5-7}{4}=y$

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7. For which value of c does the function $f(x) = -9x^2 + 12x + c$ have two x-intercepts?
Hint: Use the discriminant.

$D = 12^2 - 4(-9)(-3) = 144 - 108$
$= 32 \quad 70$

A. -3

B. -4

C. -6

D. -8

E. There are no values of c for which $f(x)$ has two x-intercepts.

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8. Suppose that $f(x)$ is a polynomial with zeros at $x = 3$ and $x = 4i$. Which polynomial is a factor of $f(x)$?

and $x = -4i$

$(x-3)(x-4i)(x-(-4i)) = (x-3)(x^2+16)$
$=$

A. $\cancel{x^2 + 7x + 12}$ B. $x + 3$ C. $x^2 - 16$

D. $x^3 - 3x^2 + 16x - 48$ E. $x - 4$

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9. Which statements correctly describe the behavior of the function below?

$$g(x) = \frac{-7x^5 + 11x^2 + 9}{3x^8 + 2x^3 + 10}$$

deg 5

deg 8

Hint: Find the horizontal asymptote.

A. As $x \to \infty, g(x) \to \infty$. As $x \to -\infty, g(x) \to \infty$.

B. As $x \to \infty, g(x) \to -\infty$. As $x \to -\infty, g(x) \to -\infty$.

C. As $x \to \infty, g(x) \to 0$. As $x \to -\infty, g(x) \to 0$.

D. As $x \to \infty, g(x) \to \dfrac{-7}{3}$. As $x \to -\infty, g(x) \to \dfrac{-7}{3}$.

E. As $x \to \infty, g(x) \to -\infty$. As $x \to -\infty, g(x) \to \infty$.

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10. Find the solution to the inequality.

$(x-7)(x-2)$

$$\frac{x^2 - 9x + 14}{x^2 - 13x + 36} > 0$$

$(x-9)(x-4)$

+ − + − +

2 4 7 9

A. $(-\infty, \infty)$

B. $(2, 4) \cup (7, 9)$

C. $(-\infty, 2) \cup (7, 9)$

D. $(4, 7)$

E. $(-\infty, 2) \cup (4, 7) \cup (9, \infty)$

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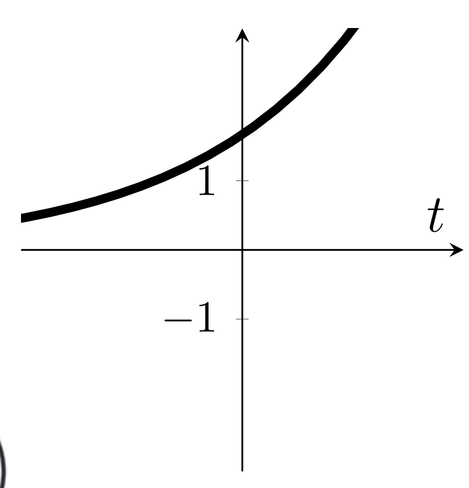
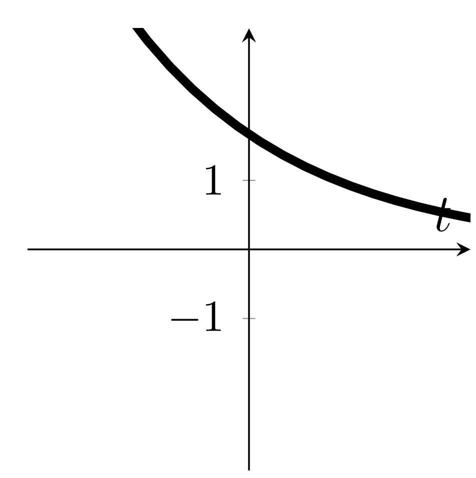
11. Given the function $f(t) = Pe^{rt}$ with $r > 0$, and $P > 1$, which graph below could be the graph of $f(t)$?

inc

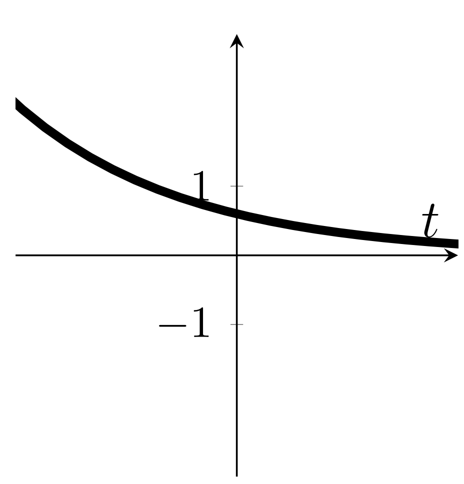
y-int > 1



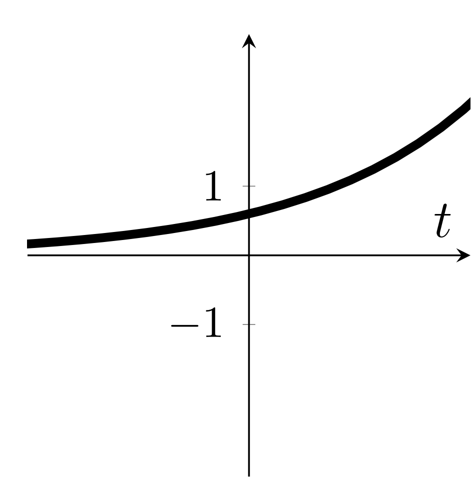
A.



C.



B.



D.

12. Given that $x > 0$, simplify the expresison using the properties of logarithms and exponents.

$$\ln\left(9\sqrt[3]{x^4}\right) - \ln\left(7\sqrt[5]{x^3}\right) = \ln(9x^{4/3}) - \ln(7x^{3/5})$$

$$= \ln\left(\frac{9x^{4/3}}{7x^{3/5}}\right) = \ln\left(\frac{9}{7}x^{11/15}\right) =$$

A. $\ln\left(\frac{9}{7}\sqrt[15]{x^{11}}\right)$

B. $\ln(2\sqrt{x})$

C. $\ln\left(2\sqrt[5]{x^4}\right)$

D. $\ln\left(16\sqrt[5]{x^3}\right)$

E. $\ln\left(\frac{9}{7}\sqrt[15]{x^{29}}\right)$

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13. A population's growth is modeled using the function $f(t) = ae^{bt}$. The population grows from 150 to 450 over the course of 8 years. What is the value of b?

$$450 = 150e^{b(8)} \rightarrow 3 = e^{8b} \rightarrow \ln 3 = 8b$$

A. $\frac{\ln(2)}{8}$

B. $\frac{\ln(8)}{2}$

C. $\frac{\ln(3)}{8}$

D. $\frac{\ln(8)}{3}$

E. $\frac{\ln(3)}{\ln(2)}$

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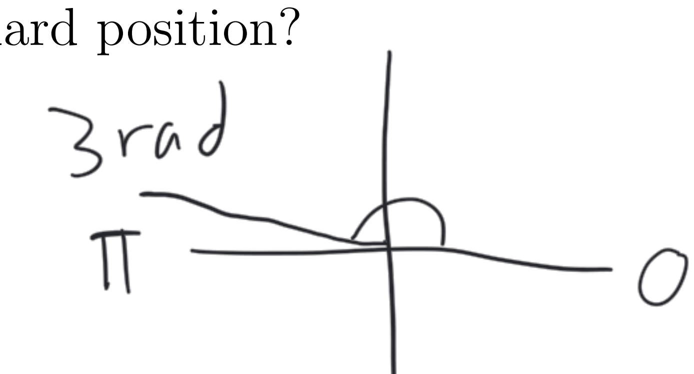
14. Which angle's terminal side lies in Quadrant II when in standard position?



A. $\theta = 0$ radians

B. $\theta = 1$ radian

C. $\theta = 3$ radians

D. $\theta = 5$ radians

E. $\theta = 7$ radians

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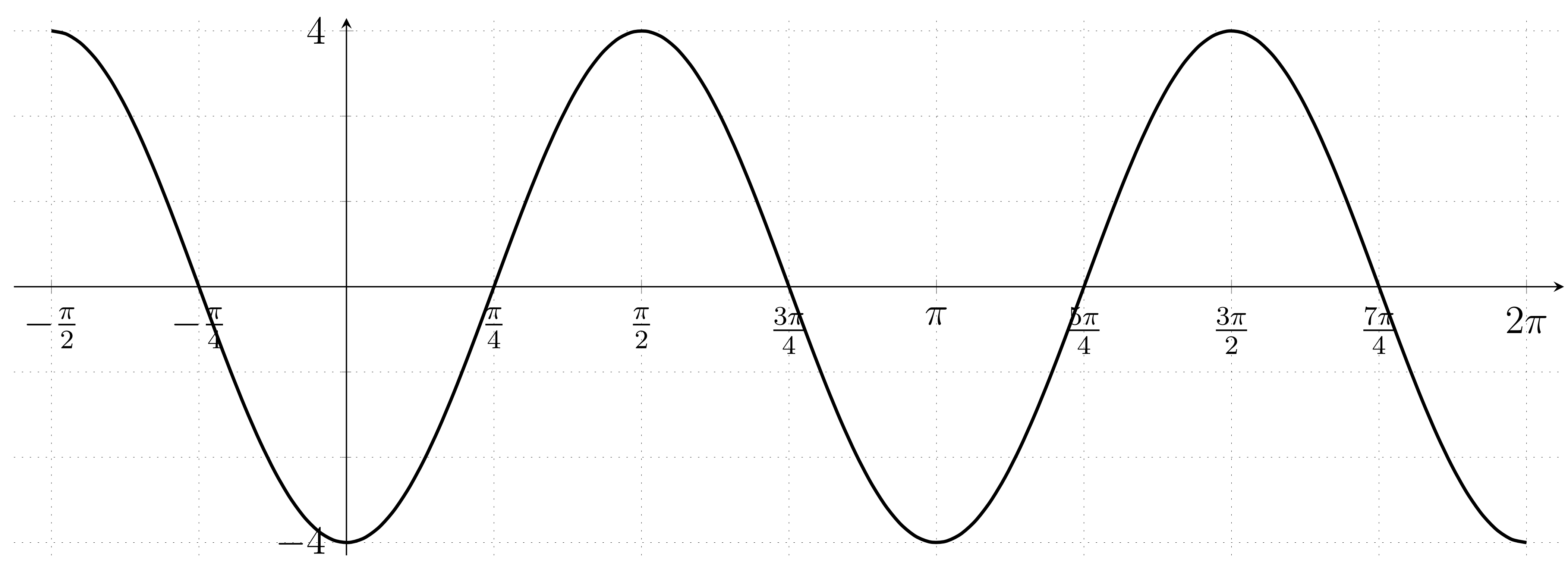
15. Choose the value that is different from the others.

$-\frac{\sqrt{2}}{2}$ $\quad \frac{\sqrt{2}}{2}$ $\quad \frac{\sqrt{2}}{2}$ $\quad \frac{\sqrt{2}}{2}$ $\quad \frac{\sqrt{2}}{2}$

A. $\sin\left(\frac{5\pi}{4}\right)$ $\qquad$ B. $\sin\left(\frac{3\pi}{4}\right)$ $\qquad$ C. $\sin\left(\frac{\pi}{4}\right)$ $\qquad$ D. $\sin\left(\frac{11\pi}{4}\right)$ $\qquad$ E. $\cos\left(\frac{-\pi}{4}\right)$

16. Identify the function $f(x)$ graphed below.



A. $f(x) = 4\cos\left(2x - \frac{\pi}{2}\right)$ $\qquad$ B. $f(x) = 4\sin\left(2x - \frac{\pi}{2}\right)$ $\qquad$ C. $f(x) = 4\cos\left(x - \frac{\pi}{4}\right)$

D. $f(x) = 4\sin\left(x - \frac{\pi}{4}\right)$ $\qquad$ E. $f(x) = 4\sin(2x)$

Per $= \pi \to b = 2$

shift $= \pi/4 \to c = \frac{\pi}{2}$

17. Rewrite the trigonometric expression below in algebraic form.

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$$\cos\left(\arctan\frac{k}{5}\right)$$

A. $\dfrac{\sqrt{25-k^2}}{5}$ B. $\dfrac{k}{\sqrt{k^2+25}}$ C. $\dfrac{\sqrt{25-k^2}}{k}$ D. $\dfrac{k}{5}$ E. $\dfrac{5}{\sqrt{k^2+25}}$

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18. Given $a > 0$ and $a \neq 1$, $y = \log_a x$ if and only if...

$\Downarrow$

$a^y = x$

A. $x^y = a$ B. $y^x = a$ C. $a^x = y$ D. $a^y = x$ E. $x^a = y$

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19. Find the solutions to the equation $6\cos^2(x) + 5 = 8$ on the interval $[0, 2\pi)$. What is the sum of the solutions?

$6\cos^2 x = 3 \rightarrow \cos^2 x = \frac{1}{2}$

$\cos x = \pm\frac{\sqrt{2}}{2} \rightarrow x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

A. 0 B. π C. 2π D. 3π E. 4π

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20. Evaluate: $\cos(15°)$ $= \cos(45-30) = \cos 45 \cdot \cos 30 + \sin 45 \sin 30$

$\frac{\sqrt{2}}{2} \quad \frac{\sqrt{3}}{2} \quad + \quad \frac{\sqrt{2}}{2} \quad \frac{1}{2}$

A. $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ B. $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ C. $\dfrac{\sqrt{2}-\sqrt{6}}{4}$ D. $\dfrac{\sqrt{6}-\sqrt{3}}{4}$ E. $\dfrac{\sqrt{6}+\sqrt{3}}{4}$

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21. Which of these expresses $\sin(160°)$ in terms of $\sin(80°)$ and $\cos(80°)$?

$\sin(160) = \sin(2 \cdot 80) = 2 \sin 80 \cos 80$

A. $\pm\sqrt{\dfrac{1+\cos(80°)}{2}}$ B. $2\sin(80°)\cos(80°)$ C. $\dfrac{\sin((80°))}{1+\cos(80°)}$

D. $\cos^2(80°) - \sin^2(80°)$ E. $\dfrac{1-\cos(80°)}{\sin(80°)}$
