

## Lecture 28

Trig functions in right triangles, Identities(Cofunction, Reciprocal, Quotient, Pythagorean), Reference angles, Angles of elevation/depression

1.

The reference angle of 227 degrees is \_\_\_\_\_ degrees.

The reference angle of 327 degrees is \_\_\_\_\_ degrees.

The reference angle of -143 degrees is \_\_\_\_\_ degrees.

2.

The reference angle of  $\frac{12}{5}\pi$  radians is \_\_\_\_\_ radians.

The reference angle of  $-\frac{3}{10}$  radians is \_\_\_\_\_ radians.

The reference angle of  $\frac{7}{5}\pi$  radians is \_\_\_\_\_ radians.

3.

If  $\cos(38^\circ) = \sin(\theta)$  and  $0^\circ < \theta < 90^\circ$ , then

$\theta =$  \_\_\_\_\_ degrees

4.

If  $\sin\left(\frac{\pi}{6}\right) = \cos(\theta)$  and  $0^\circ < \theta < \frac{\pi}{2}$ , then

$\theta =$  \_\_\_\_\_

5.

Find the **exact value** of each of the following.

$$\sin(30^\circ) = \underline{\hspace{2cm}}$$

$$\cos(30^\circ) = \underline{\hspace{2cm}}$$

$$\tan(30^\circ) = \underline{\hspace{2cm}}$$

$$\csc(30^\circ) = \underline{\hspace{2cm}}$$

$$\sec(30^\circ) = \underline{\hspace{2cm}}$$

$$\cot(30^\circ) = \underline{\hspace{2cm}}$$

6.

Find the **exact value** of each of the following.

$$\tan(60^\circ) = \underline{\hspace{2cm}}$$

$$\csc(30^\circ) = \underline{\hspace{2cm}}$$

$$\cos(45^\circ) = \underline{\hspace{2cm}}$$

7.

For  $0 < \theta < \frac{\pi}{2}$ , find the values of the trigonometric functions based on the given one

(give your answers with THREE DECIMAL PLACES or as expressions, e.g. you can enter 3/5).

If  $\sin(\theta) = \frac{8}{9}$  then

$$\cos(\theta) = \underline{\hspace{2cm}}$$

$$\sec(\theta) = \underline{\hspace{2cm}}$$

$$\csc(\theta) = \underline{\hspace{2cm}}$$

$$\tan(\theta) = \underline{\hspace{2cm}}$$

$$\cot(\theta) = \underline{\hspace{2cm}}$$

8.

Suppose that  $\theta$  is an angle in quadrant I and  $\sin(\theta) = \frac{2}{13}$ . Find the values of the other five trigonometric functions for  $\theta$ . Give exact answers, but do not rationalize denominators.

$$\cos(\theta) = \underline{\hspace{2cm}}$$

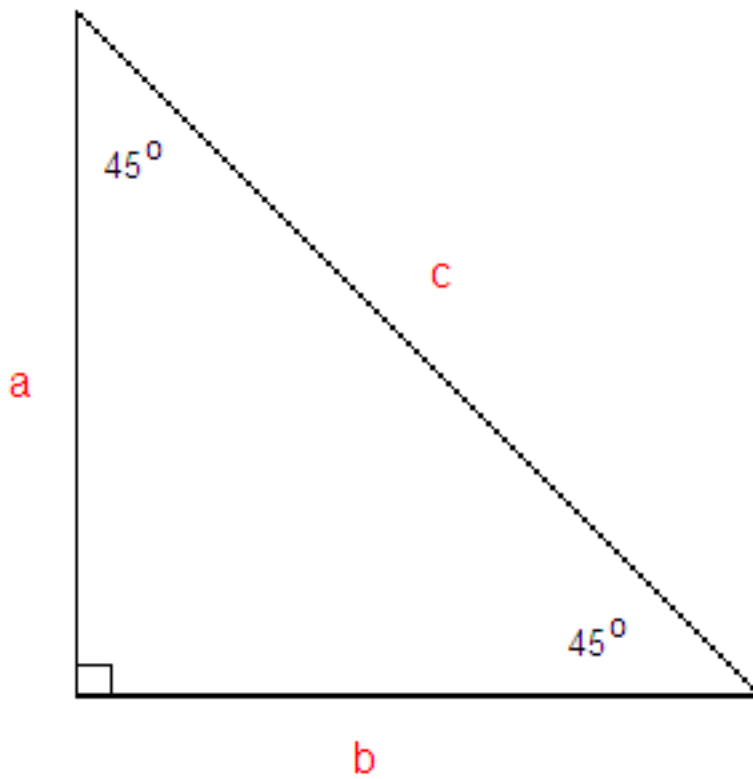
$$\tan(\theta) = \underline{\hspace{2cm}}$$

$$\csc(\theta) = \underline{\hspace{2cm}}$$

$$\sec(\theta) = \underline{\hspace{2cm}}$$

$$\cot(\theta) = \underline{\hspace{2cm}}$$

9.



Note: Triangle may not be drawn to scale.

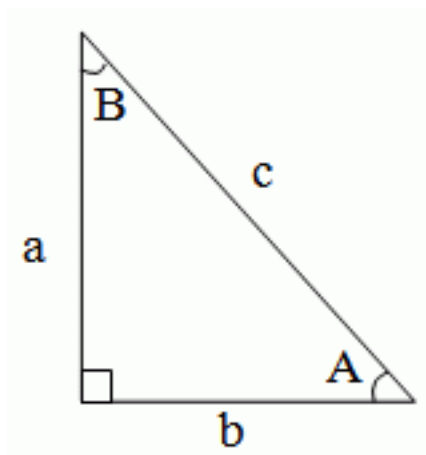
Suppose  $b = 4$

Find exact values for the other sides.

$a =$  \_\_\_\_\_

$c =$  \_\_\_\_\_

10.



Note: Triangle may not be drawn to scale.

Suppose  $a = 8$  and  $b = 10$ .

Find an exact value or give at least two decimal places:

$$\sin(A) = \underline{\hspace{2cm}}$$

$$\cos(A) = \underline{\hspace{2cm}}$$

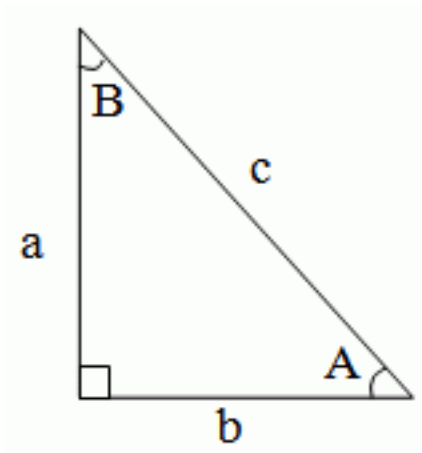
$$\tan(A) = \underline{\hspace{2cm}}$$

$$\sec(A) = \underline{\hspace{2cm}}$$

$$\csc(A) = \underline{\hspace{2cm}}$$

$$\cot(A) = \underline{\hspace{2cm}}$$

11.



Note: Triangle may not be drawn to scale.

Suppose  $a = 6$  and  $A = 30$  degrees.

Find:

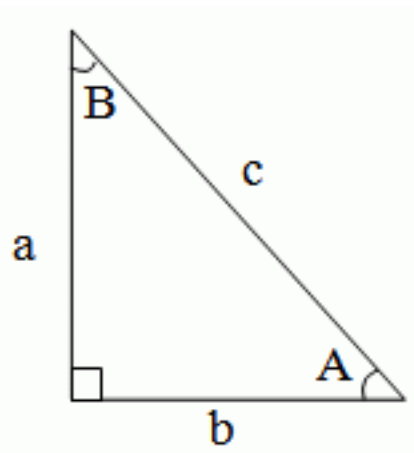
$$b = \underline{\hspace{2cm}}$$

$$c = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}} \text{ degrees}$$

Give all answers to at least one decimal place. Give angles in **degrees**

12.



Note: Triangle may not be drawn to scale.

Suppose  $c = 10$  and  $A = 15$  degrees.

Find:

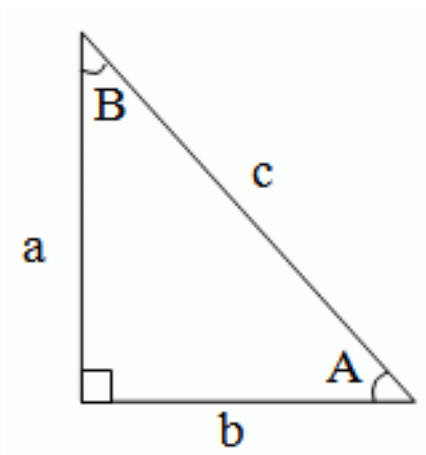
$a =$  \_\_\_\_\_

$b =$  \_\_\_\_\_

$B =$  \_\_\_\_\_ degrees

Give all answers to at least one decimal place. Give angles in **degrees**

13.



Note: Triangle may not be drawn to scale.

Suppose  $a = 120$  and  $b = 119$  and  $c = 169$ .

Find an exact value (report answer as a fraction):

$$\sin(A) = \underline{\hspace{2cm}}$$

$$\cos(A) = \underline{\hspace{2cm}}$$

$$\tan(A) = \underline{\hspace{2cm}}$$

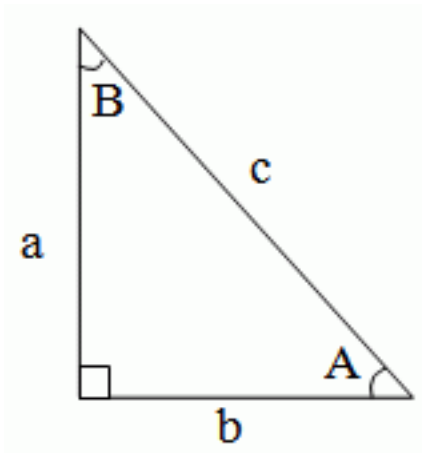
$$\sec(A) = \underline{\hspace{2cm}}$$

$$\csc(A) = \underline{\hspace{2cm}}$$

$$\cot(A) = \underline{\hspace{2cm}}$$



14.



Note: Triangle may not be drawn to scale.

Suppose  $a = 7$  and  $b = 10$ .

Find an exact value for each of the following trig functions.

$$\sin(A) = \underline{\hspace{2cm}}$$

$$\cos(A) = \underline{\hspace{2cm}}$$

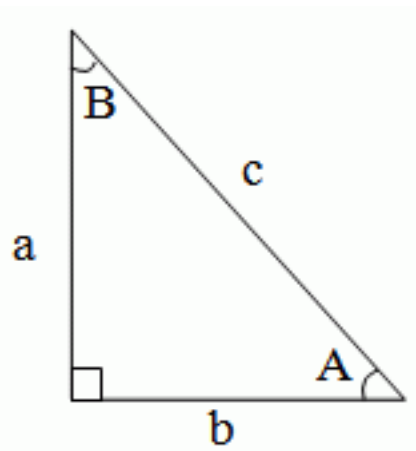
$$\tan(A) = \underline{\hspace{2cm}}$$

$$\sec(A) = \underline{\hspace{2cm}}$$

$$\csc(A) = \underline{\hspace{2cm}}$$

$$\cot(A) = \underline{\hspace{2cm}}$$

15.



Note: Triangle may not be drawn to scale.

Suppose  $a = 105$  and  $b = 88$ .

Find an exact value (report answer as a fraction):

$$\sin(B) = \underline{\hspace{2cm}}$$

$$\cos(B) = \underline{\hspace{2cm}}$$

$$\tan(B) = \underline{\hspace{2cm}}$$

$$\sec(B) = \underline{\hspace{2cm}}$$

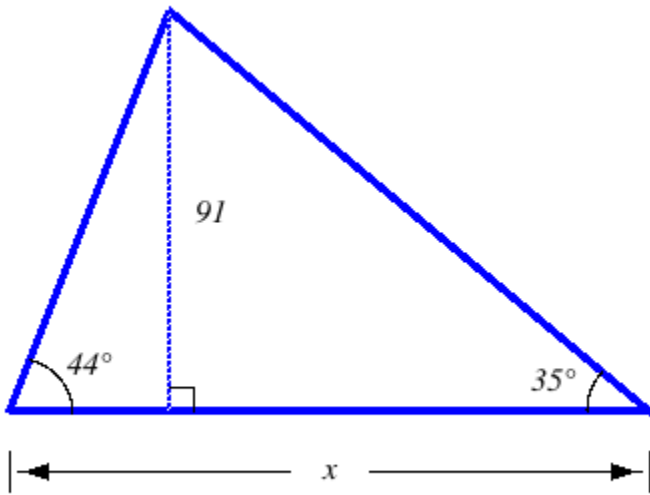
$$\csc(B) = \underline{\hspace{2cm}}$$

$$\cot(B) = \underline{\hspace{2cm}}$$

16.

Find  $x$  correct to 2 decimal places.

*NOTE: The triangle is NOT drawn to scale.*

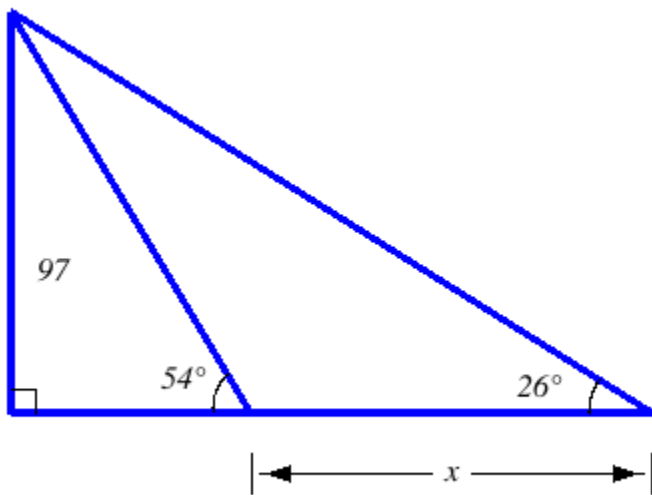


$x =$  \_\_\_\_\_

17.

Find  $x$  correct to 2 decimal places.

*NOTE: The triangle is NOT drawn to scale.*



$x =$  \_\_\_\_\_

18.

A 32 -ft ladder leans against a building so that the angle between the ground and the ladder is  $81^\circ$  .

How high does the ladder reach on the building? \_\_\_\_\_ ft

19.

From the top of a 153-ft lighthouse, the angle of depression to a ship in the ocean is  $28^\circ$  . How far is the ship from the base of the lighthouse?

distance = \_\_\_\_\_ feet

*Report answer accurate to 2 decimal places.*

20.

A survey team is trying to estimate the height of a mountain above a level plain. From one point on the plain, they observe that the angle of elevation to the top of the mountain is  $26^\circ$  . From a point 2000 feet closer to the mountain along the plain, they find that the angle of elevation is  $31^\circ$  .

How high (in feet) is the mountain?

21.

The angle of elevation to the top of a Building in New York is found to be 3 degrees from the ground at a distance of 2 miles from the base of the building. Using this information, find the height of the building. Round to the tenths. Hint: 1 mile = 5280 feet

Your answer is \_\_\_\_\_ feet.

22.

A radio tower is located 275 feet from a building. From a window in the building, a person determines that the angle of elevation to the top of the tower is  $32^\circ$  and that the angle of depression to the bottom of the tower is  $28^\circ$ . How tall is the tower?

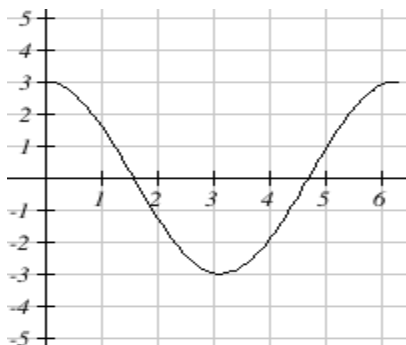
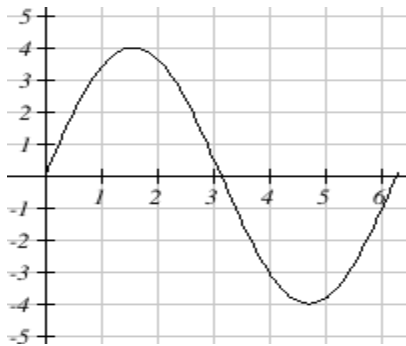
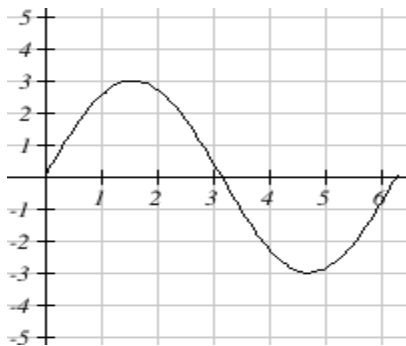
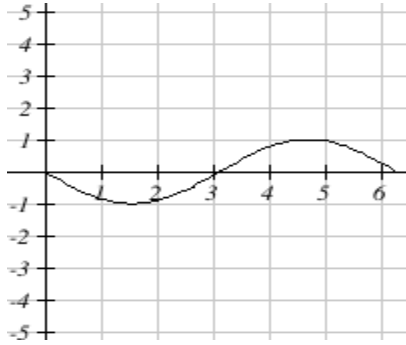
\_\_\_\_\_ feet

## Lecture 29

Period, Amplitude, Phase shift, Vertical translation

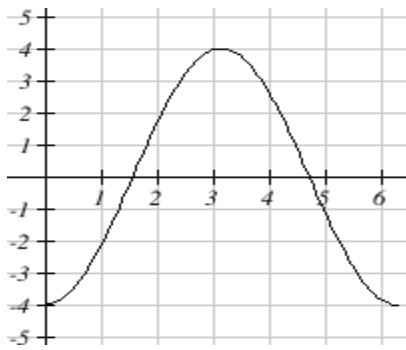
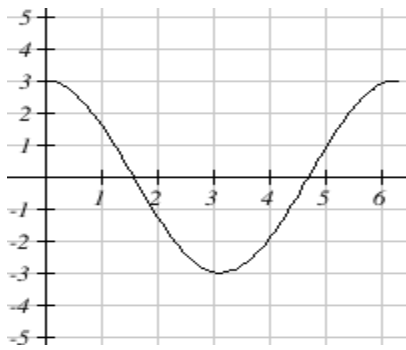
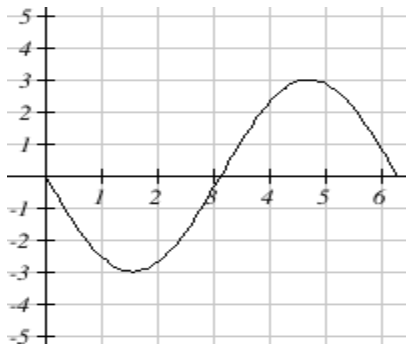
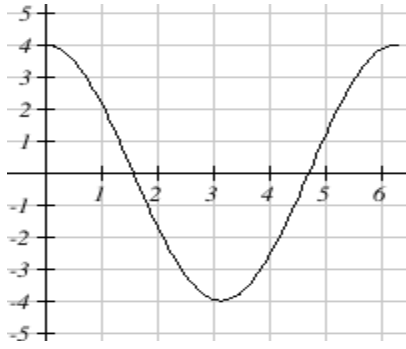
1.

Which of the following graphs is the correct plot of  $y = 3\sin(x)$  ?



2.

Which of the following graphs is the correct plot of  $y = 4\cos(x)$ ?



3.

Find the equation of a sine wave that is obtained by shifting the graph of  $y = \sin(x)$  to the right 5 units and downward 8 units and is vertically stretched by a factor of 8 when compared to  $y = \sin(x)$ .

$f(x) =$  \_\_\_\_\_

4.

For  $y = 6\sin 6x$ ,

its amplitude is \_\_\_\_\_

its period is \_\_\_\_\_

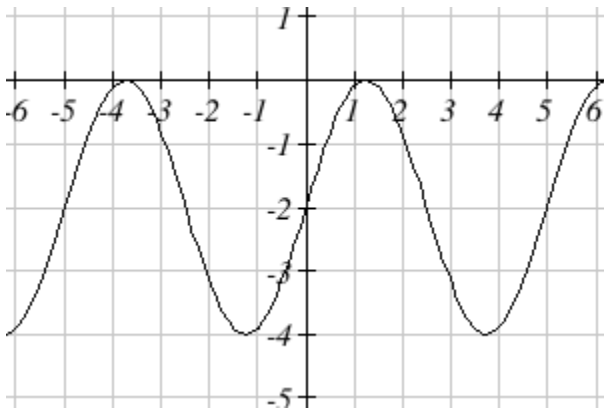
5.

For  $y = -2\cos \frac{1}{8}x$ ,

its amplitude is \_\_\_\_\_

its period is \_\_\_\_\_

6.



Based on the graph above, determine the amplitude, midline, and period of the function

Amplitude: \_\_\_\_\_

Period: \_\_\_\_\_

Midline:  $y =$  \_\_\_\_\_



7.

Given the equation  $y = 5\sin(8(x - 3)) + 4$

The amplitude is: \_\_\_\_\_

The period is: \_\_\_\_\_

The horizontal shift is: \_\_\_\_\_ units to the (Right/ Left)

The midline is:  $y =$  \_\_\_\_\_

8.

Given the equation  $y = 5\sin(6x - 42) + 3$

The amplitude is: \_\_\_\_\_

The period is: \_\_\_\_\_

The horizontal shift is: \_\_\_\_\_ units to the (Right/ Left)

The midline is:  $y =$

9.

Given the equation  $y = 5\sin\left(\frac{\pi}{6}x + \frac{\pi}{3}\right) + 3$

The amplitude is: \_\_\_\_\_

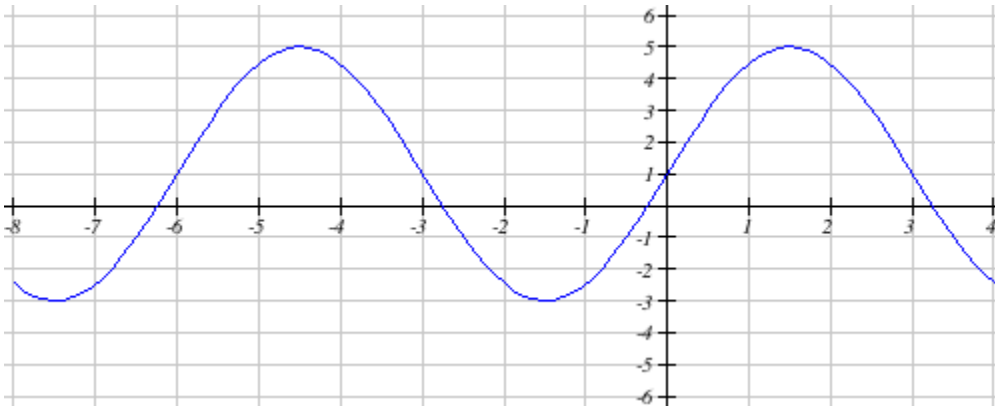
The period is: \_\_\_\_\_

The horizontal shift is: \_\_\_\_\_ units to the (Right/ Left)

The midline is:  $y =$  \_\_\_\_\_

10.

Find a function of the form  $y = A\sin(kx) + C$  or  $y = A\cos(kx) + C$  whose graph matches the function shown below:

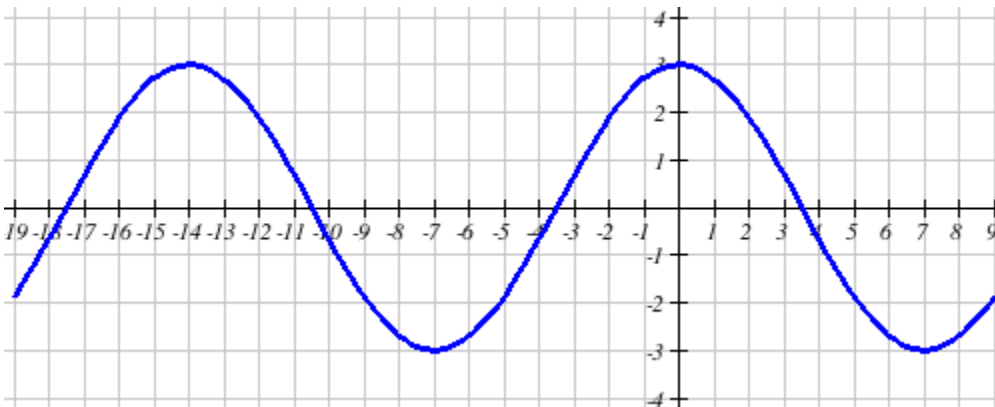


Leave your answer in exact form.

$y =$  \_\_\_\_\_

11.

Find a function of the form  $y = A\sin(kx) + C$  or  $y = A\cos(kx) + C$  whose graph matches this one:

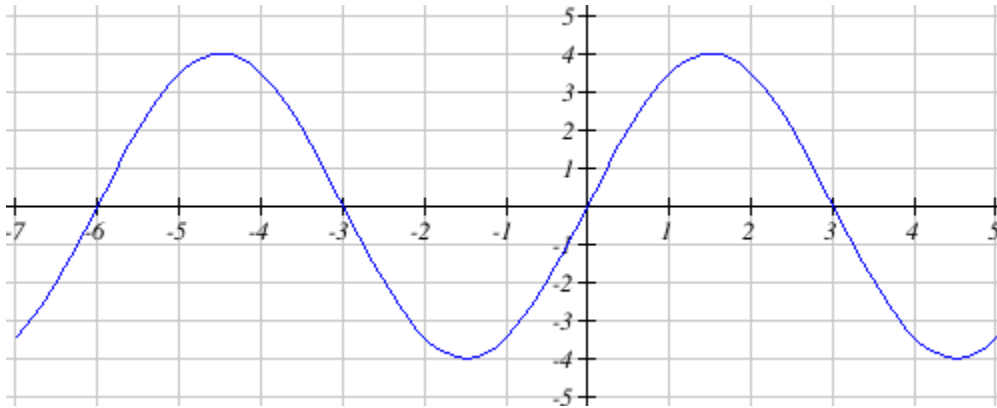


Leave your answer in exact form.

$y =$  \_\_\_\_\_

12.

Find a function of the form  $y = A\sin(kx)$  or  $y = A\cos(kx)$  whose graph matches the function shown below:



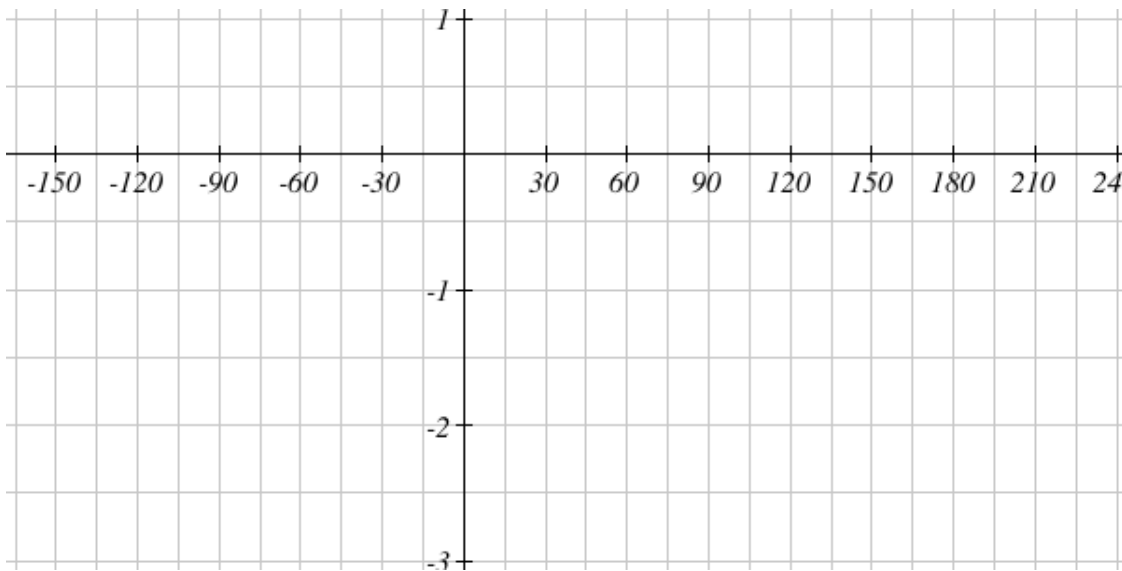
Leave your answer in exact form.

$y =$  \_\_\_\_\_

13.

Draw the following graph on the interval  $-165^\circ < x < 240^\circ$  :

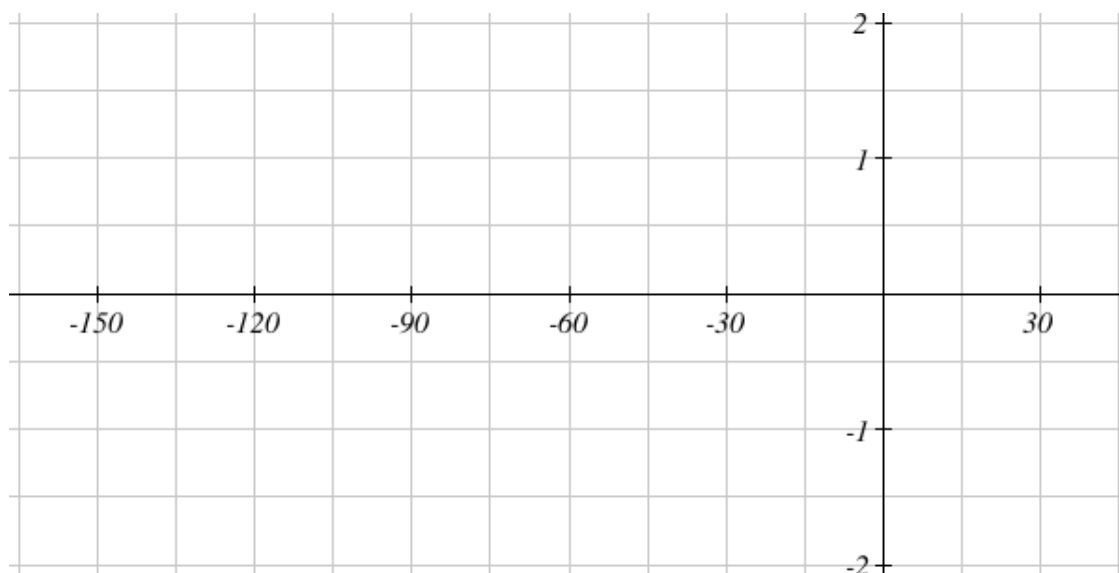
$y = -\sin(x) - 1$



14.

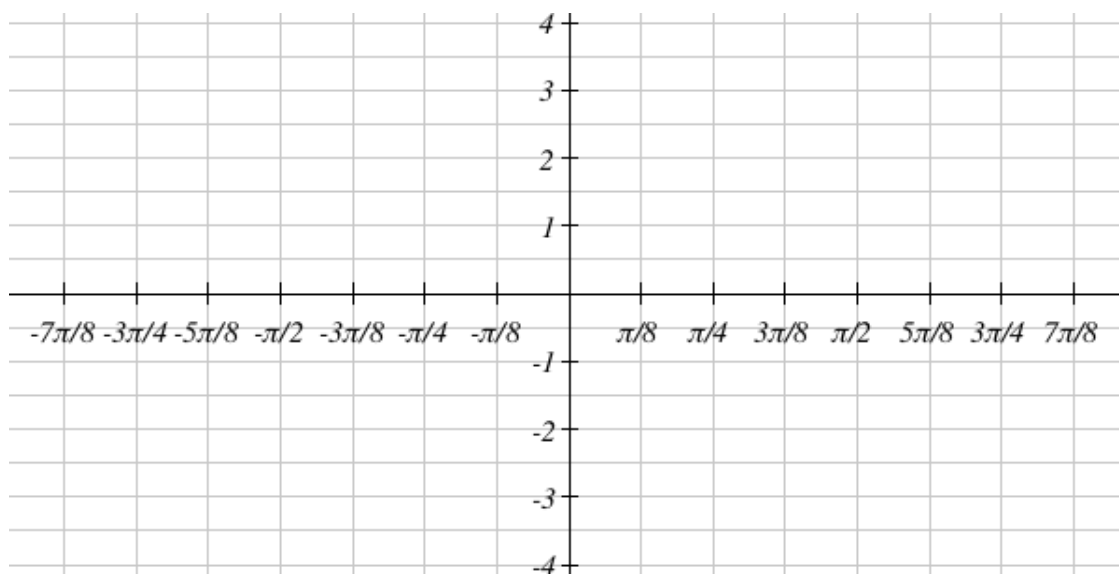
Draw the following graph on the interval  $-165^\circ < x < 45^\circ$  :

$$y = \sin(2x)$$



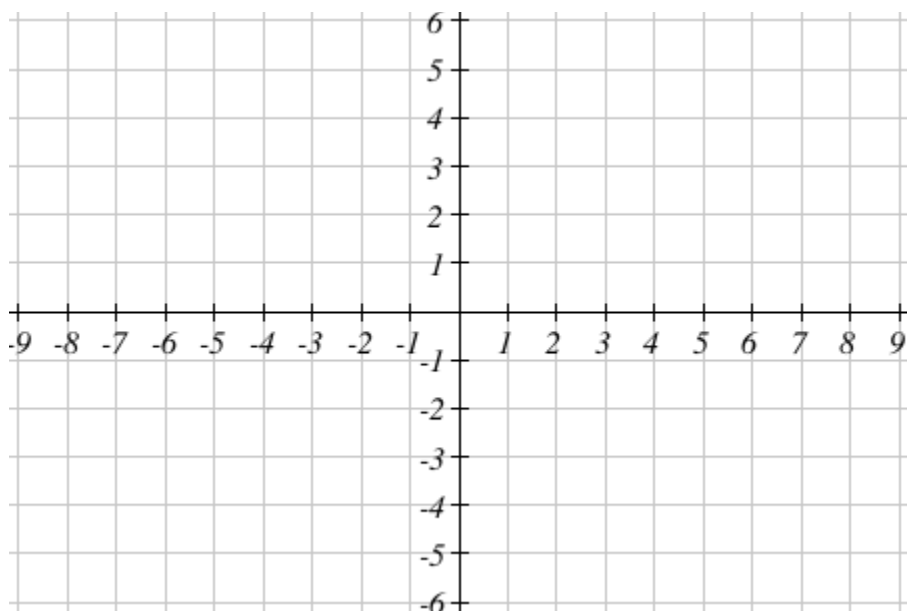
15.

Sketch a graph of the function  $f(x) = -2\sin(2x)$  .



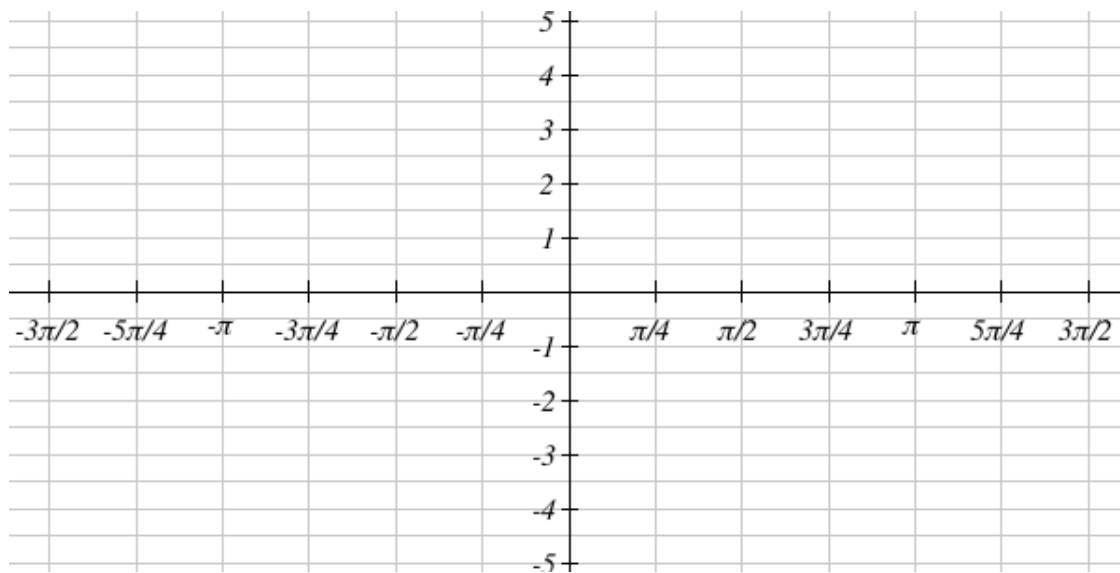
16.

Sketch a graph of the function  $f(x) = 3\sin\left(\frac{\pi}{2}x\right) - 2$



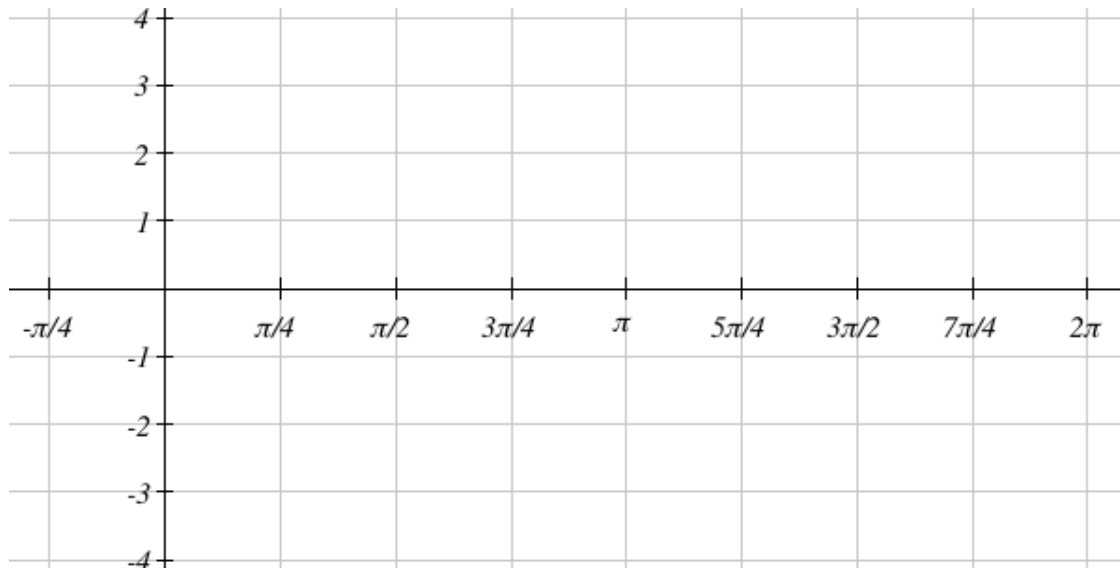
17.

Sketch a graph of the function  $f(x) = -2\sin\left(x - \frac{3\pi}{4}\right) + 1$ .



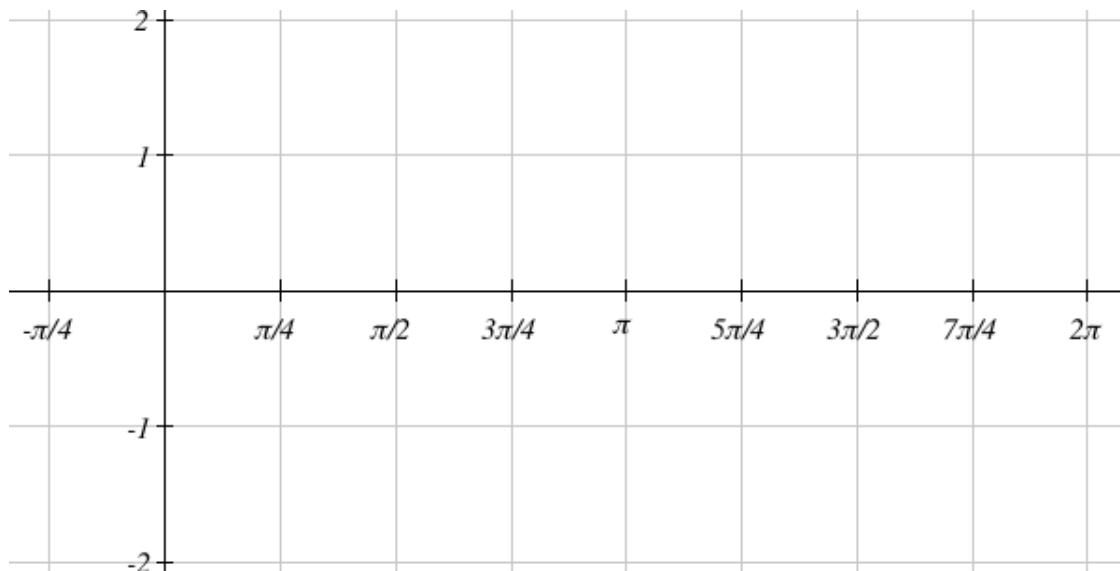
18.

Sketch a graph of the function  $f(x) = \sin(x) + 1$ .



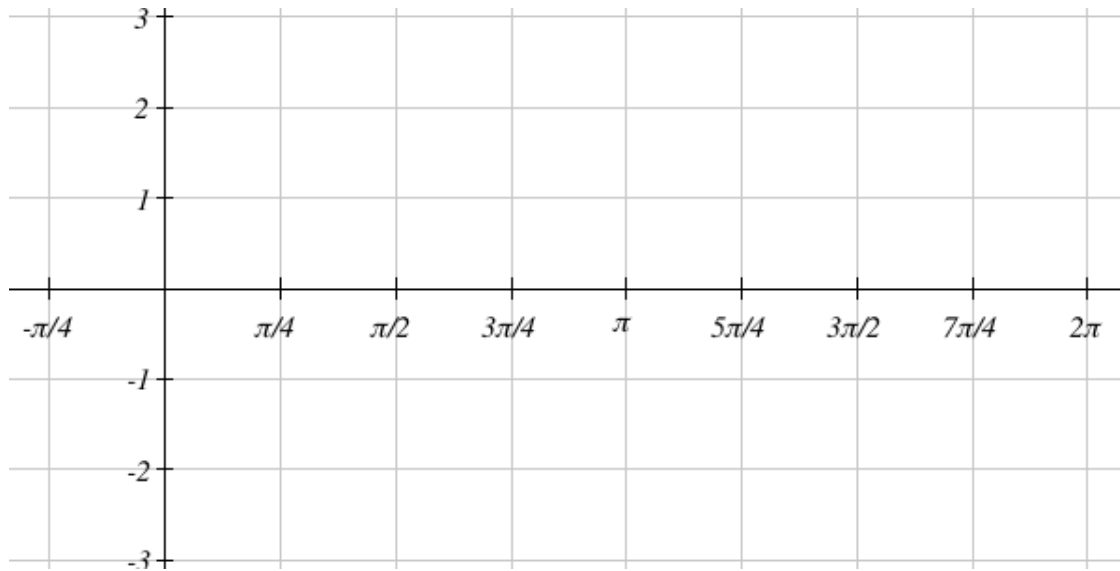
19.

Sketch a graph of the function  $f(x) = \sin(x)$ .



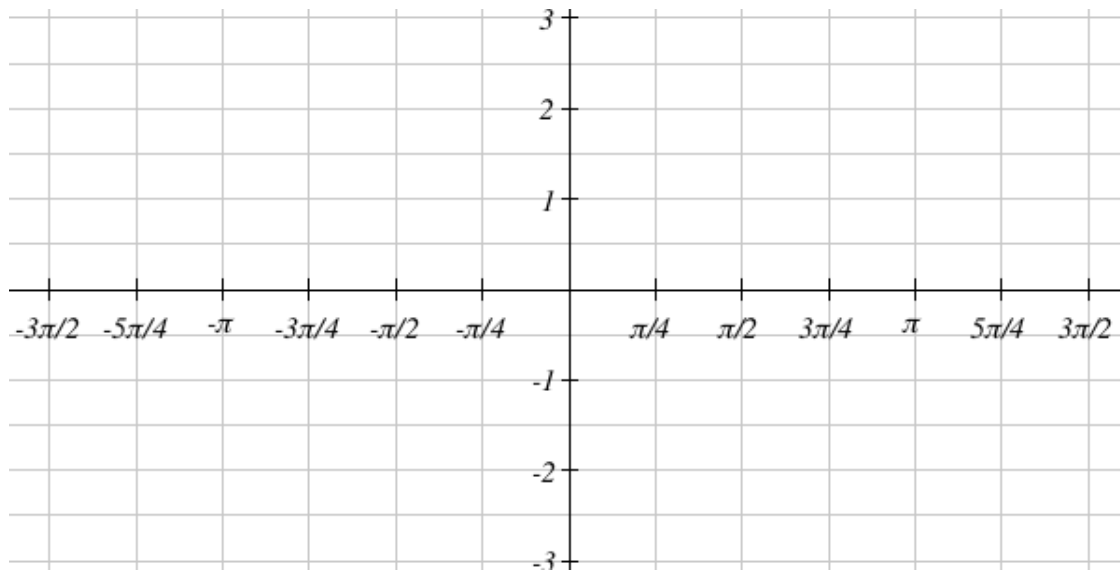
20.

Sketch a graph of the function  $f(x) = -2\sin(x)$ .



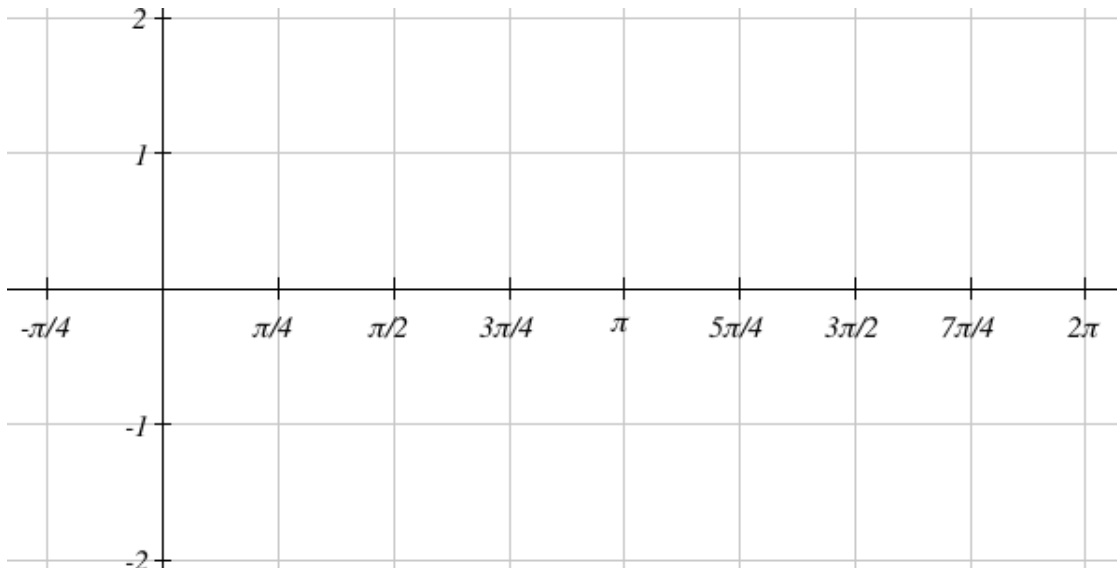
21.

Sketch a graph of the function  $f(x) = \sin\left(x + \frac{3\pi}{4}\right)$ .



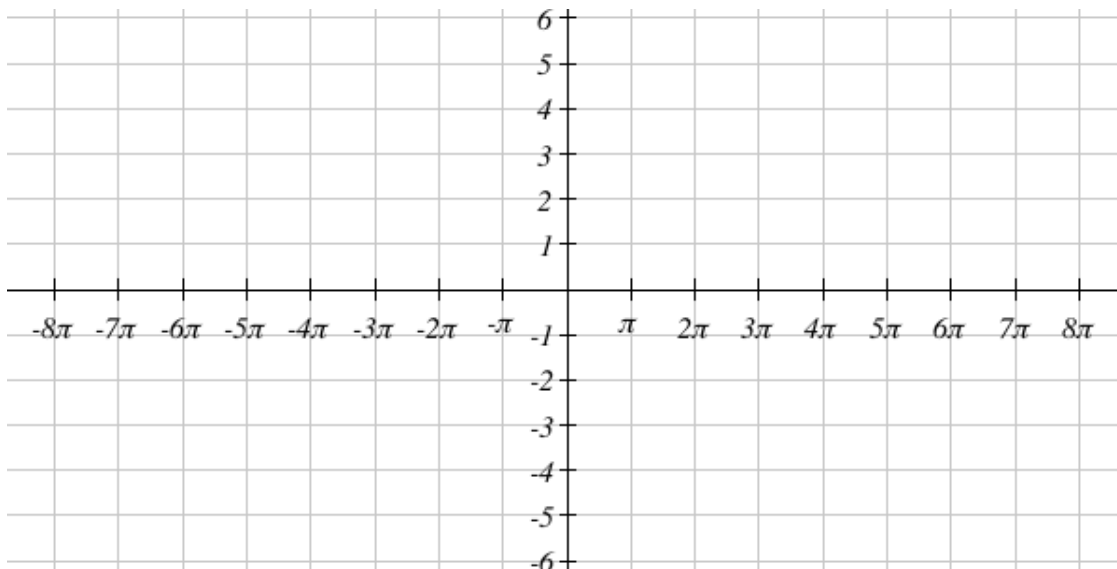
22.

Sketch a graph of the function  $f(x) = \cos(x)$ .



23.

Sketch a graph of the function  $f(x) = -4\cos\left(\frac{2}{5}x\right)$ .



24.

Outside temperature over a day can be modeled as a sinusoidal function. Suppose you know the temperature is 70 degrees at midnight and the high and low temperature during the day are 81 and 59 degrees, respectively. Assuming  $t$  is the number of hours since midnight, find an equation for the temperature,  $D$ , in terms of  $t$ .

$D(t) =$  \_\_\_\_\_



25.

A ferris wheel is 15 meters in diameter and boarded from a platform that is 4 meters above the ground. The six o'clock position on the ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 4 minutes. The function  $h = f(t)$  gives your height in meters above the ground  $t$  minutes after the wheel begins to turn.

What is the Amplitude? \_\_\_\_\_ meters

What is the Midline?  $y =$  \_\_\_\_\_ meters

What is the Period? \_\_\_\_\_ minutes

How High are you off of the ground after 2 minutes? \_\_\_\_\_ meters

26.

A ferris wheel is 30 meters in diameter and boarded from a platform that is 4 meters above the ground. The six o'clock position on the ferris wheel is level with the loading platform. The wheel completes 1 full revolution in 4 minutes. The function  $h = f(t)$  gives your height in meters above the ground  $t$  minutes after the wheel begins to turn. Write an equation for  $h = f(t)$ .

$f(t) =$  \_\_\_\_\_

## Lecture 30

Sec/csc, tan/cot, periods, asymptotes, transformations

1.

On the interval  $[0, 2\pi)$  determine which angles are not in the domain of the tangent function,  $f(\theta) = \tan(\theta)$

What angles are NOT in the domain of the tangent function on the given interval? \_\_\_\_\_

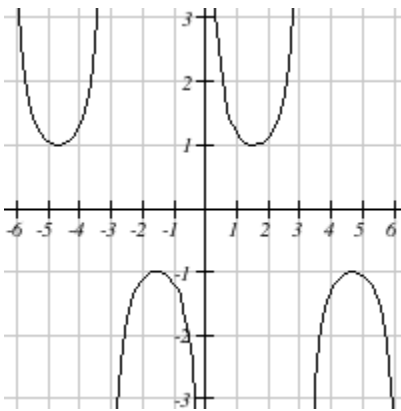
2.

On the interval  $[0, 2\pi)$  determine which angles are not in the domain of the given functions.

What angles are NOT in the domain of the secant function on the given interval? \_\_\_\_\_

What angles are NOT in the domain of the cosecant function on the given interval? \_\_\_\_\_

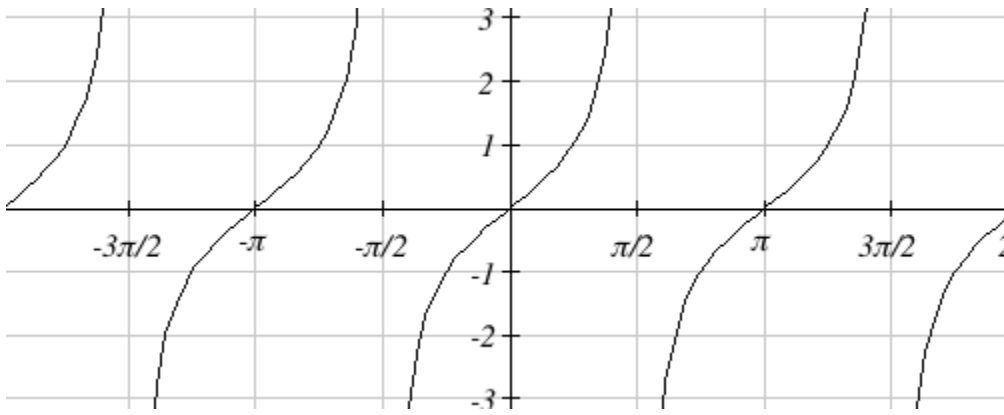
3.



The graph above is a graph of what function?

- $y = \sin(x)$
- $y = \csc(x)$
- $y = \cos(x)$
- $y = \sec(x)$
- $y = \tan(x)$
- $y = \cot(x)$

4.

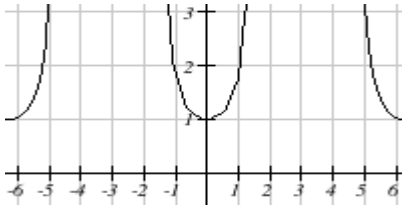


The graph above is a graph of what function?

- $y = \sin(x)$
- $y = \sec(x)$
- $y = \cot(x)$
- $y = \cos(x)$
- $y = \csc(x)$
- $y = \tan(x)$

5.

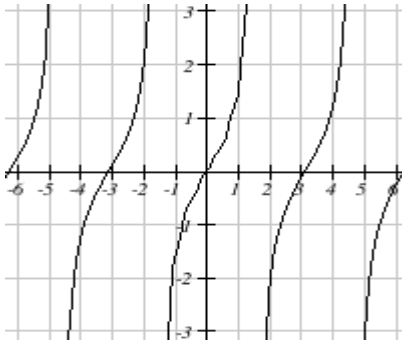
Match each graph with its equation. Not all equations will be used.



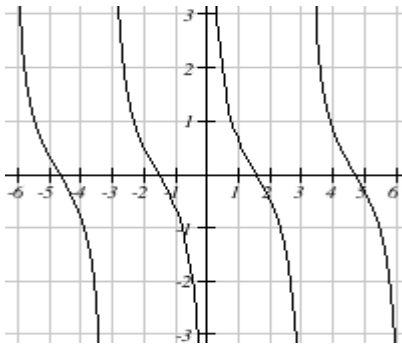
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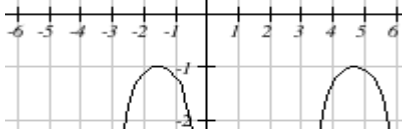
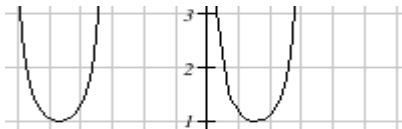
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- a.  $y = \cot(x)$
- b.  $y = \csc(x)$
- c.  $y = \tan(x)$
- d.  $y = \cos(x)$
- e.  $y = \sec(x)$
- f.  $y = \sin(x)$

6.

What is the period of the graph of the function  $y = \tan\left(\frac{7\pi}{5}x\right)$ ?

period = \_\_\_\_\_

7.

What is the period of the graph of the function  $y = \csc\left(\frac{5x}{8}\right)$ ?

period = \_\_\_\_\_

8.

What is the period of the graph of the function  $y = \sec\left(\frac{9\pi}{2}x - 7\right)$ ?

period = \_\_\_\_\_

9.

Given the equation  $y = 6\tan(3x - 6)$

The exact period (in terms of  $\pi$ ) is: \_\_\_\_\_

The phase shift is: \_\_\_\_\_ units to the (Left/Right)

10.

Given the equation  $y = 2\sec(3x + 24)$

The exact period (in terms of  $\pi$ ) is: \_\_\_\_\_

The phase shift is: \_\_\_\_\_ units to the (Left/Right)

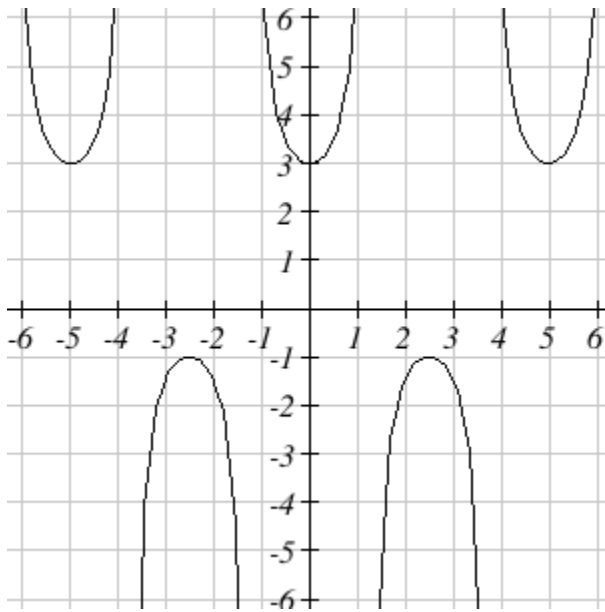
11.

Given the equation  $y = 7\csc\left(\frac{2\pi}{3}x + \frac{10\pi}{3}\right)$

The exact period (give as an integer or fraction) is: \_\_\_\_\_

The horizontal shift is: \_\_\_\_\_ units to the (Left/Right)

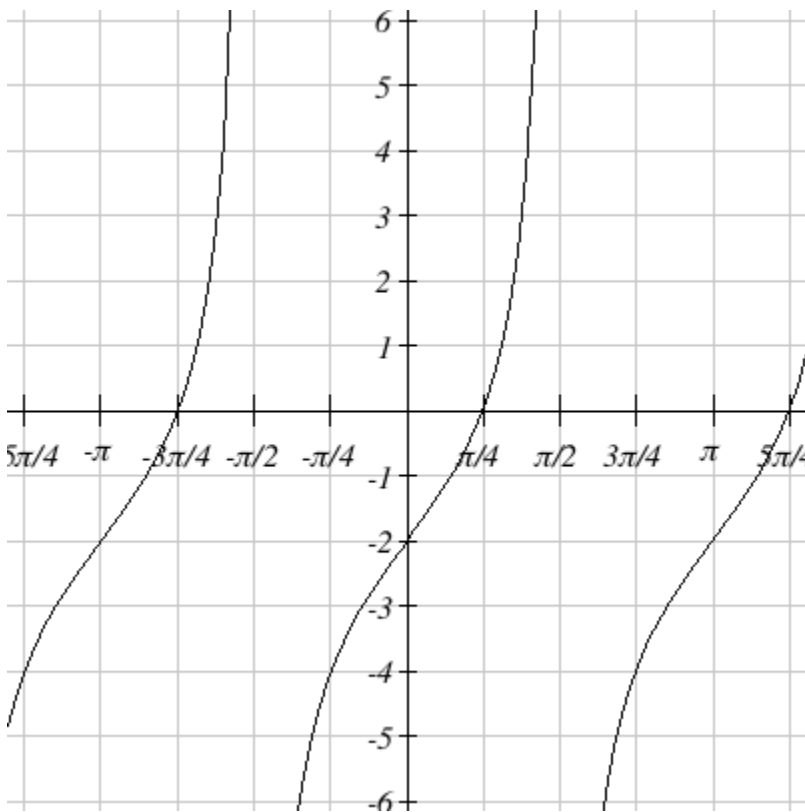
12.



Write an equation for the function graphed above. (There are multiple correct answers)

$y =$  \_\_\_\_\_

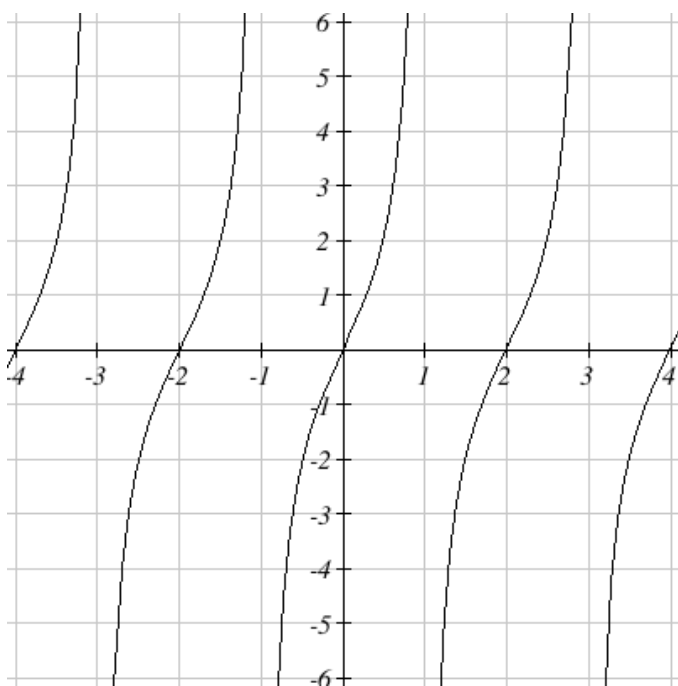
13.



Identify the function whose graph appears above. (There are multiple correct answers.)

$f(x) =$  \_\_\_\_\_

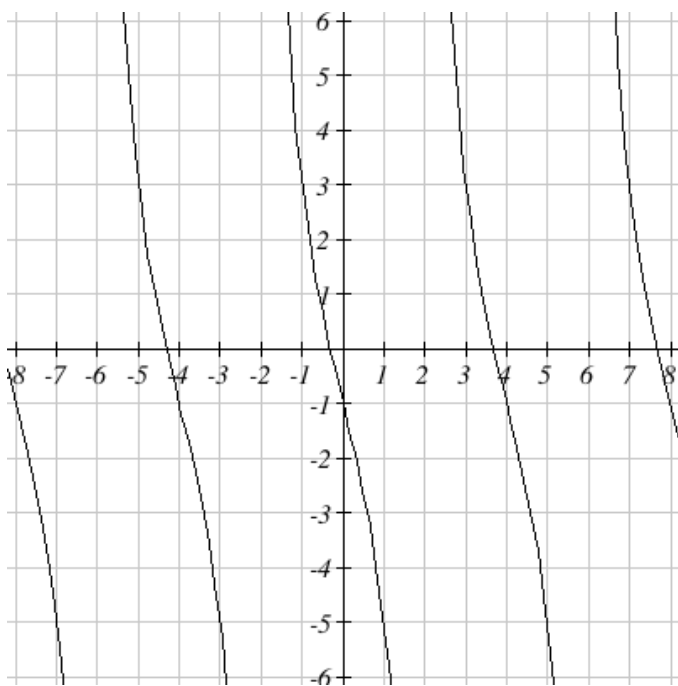
14.



Identify the function whose graph appears above. (There are multiple correct answers.)

$f(x) = \underline{\hspace{2cm}}$

15.



Give the equation for the function whose graph appears above. (There are multiple correct answers.)

$f(x) = \underline{\hspace{2cm}}$

## Lecture 31

Inverse sin/cos/tan, restrictions, properties, composition, examples

1.

Evaluate the following expressions. Your answer must be an angle in radians.

(a)  $\sin^{-1}(-1) = \underline{\hspace{2cm}}$

(b)  $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$

(c)  $\sin^{-1}(1) = \underline{\hspace{2cm}}$

2.

Evaluate the following expressions. Your answer must be an exact angle in radians.

(a)  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$

(b)  $\cos^{-1}\left(\frac{1}{2}\right) = \underline{\hspace{2cm}}$

(c)  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \underline{\hspace{2cm}}$

3.

Evaluate the following expressions. Your answer must be an exact angle in radians.

(a)  $\tan^{-1}(1) = \underline{\hspace{2cm}}$

(b)  $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right) = \underline{\hspace{2cm}}$

(c)  $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \underline{\hspace{2cm}}$

4.

Use your calculator to evaluate  $\cos^{-1}(0.58)$  to at least 3 decimal places. Give the answer in radians.



5.

Suppose  $\sin\theta = -\frac{1}{10}$ , and  $\theta$  is an angle in standard position.

Then the terminal side of  $\theta$  could be in (choose all that apply):

- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

$\arcsin\left(-\frac{1}{10}\right)$  is an angle whose terminal side is in (choose all that apply):

- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

6.

Suppose  $\cos\theta = -\frac{9}{10}$ , and  $\theta$  is an angle in standard position.

Then the terminal side of  $\theta$  could be in (choose all that apply):

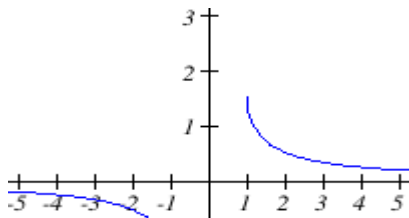
- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

$\arccos\left(-\frac{9}{10}\right)$  is an angle whose terminal side is in (choose all that apply):

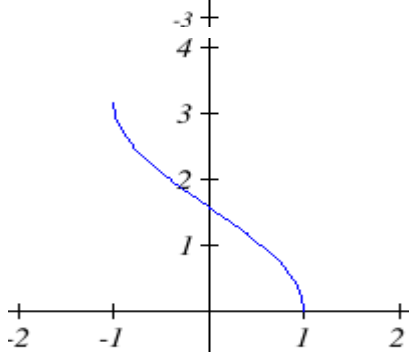
- Quadrant 1
- Quadrant 2
- Quadrant 3
- Quadrant 4

7.

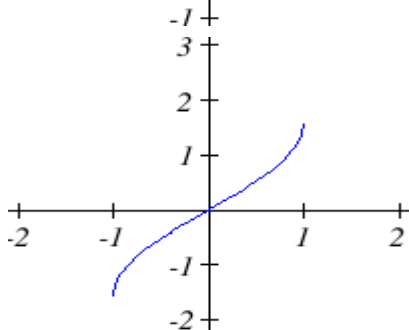
Match each graph with its equation. Not all equations will be used.



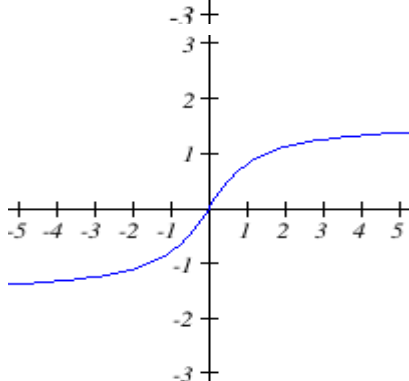
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- a.  $y = \text{arccot}(x)$
- b.  $y = \text{arccsc}(x)$
- c.  $y = \text{arcsec}(x)$
- d.  $y = \text{arcsin}(x)$
- e.  $y = \text{arccos}(x)$
- f.  $y = \text{arctan}(x)$

8.

Evaluate the following expression.

$$\sin(\arcsin(0.6)) = \underline{\hspace{2cm}}$$

9.

Evaluate the following expression.

$$\cos(\arccos(2.3)) = \underline{\hspace{2cm}}$$

10.

Evaluate the following expression.

$$\arcsin\left(\sin\left(\frac{-3\pi}{7}\right)\right) = \underline{\hspace{2cm}}$$

11.

Evaluate the following expression.

$$\arcsin\left(\sin\left(\frac{19\pi}{12}\right)\right) = \underline{\hspace{2cm}}$$

12.

Evaluate the following expression.

$$\sin^{-1}\left(\sin\left(\frac{-7\pi}{4}\right)\right) = \underline{\hspace{2cm}}$$

13.

Find the exact value or state that it is undefined. In the latter case, enter "DNE".

$$\sin^{-1}\left(\sin\left(\frac{3\pi}{4}\right)\right) = \underline{\hspace{2cm}}$$

14.

Find  $\arcsin(\sin 230^\circ)$ .

$\arcsin(\sin 230^\circ) = \underline{\hspace{2cm}}$  degrees.

15.

Evaluate the following expression.

$\tan(\arctan(1.3)) = \underline{\hspace{2cm}}$

16.

Evaluate the following expression.

$\arctan\left(\tan\left(\frac{32\pi}{9}\right)\right) = \underline{\hspace{2cm}}$

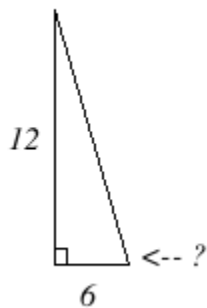
17.

Find the exact value or state that it is undefined. In the latter case, enter "DNE".

$\tan(\tan^{-1}(1)) = \underline{\hspace{2cm}}$

18.

For the right triangle below, find the measure of the angle.  
*Figure is not to scale.*



$\underline{\hspace{2cm}}$  degrees

19.

Evaluate the expression:  $\cos^{-1}\left(\sin\left(\frac{\pi}{6}\right)\right) = \underline{\hspace{2cm}}$

20.

Evaluate the expression:  $\sin^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right) = \underline{\hspace{2cm}}$

21.

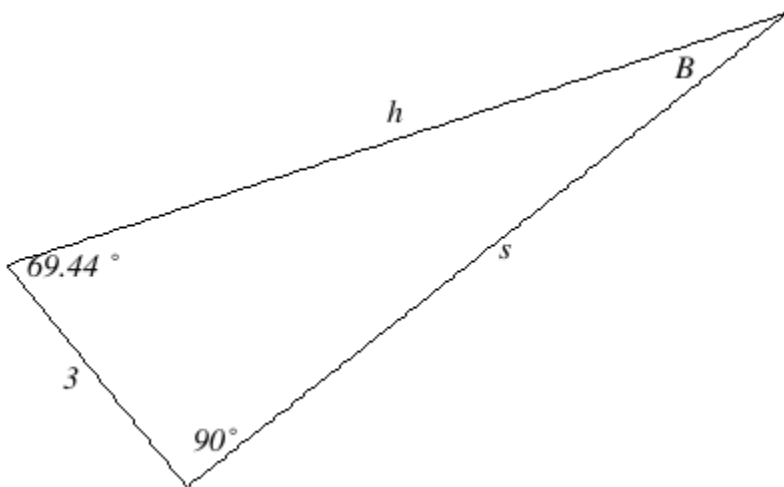
Evaluate:  $\sin\left(\cos^{-1}\left(\frac{1}{3}\right)\right) = \underline{\hspace{2cm}}$

22.

Find an algebraic expression for  $\cos\left(\tan^{-1}\left(\frac{a}{3}\right)\right)$

23.

Find the unknowns in the graph below:



$B = \underline{\hspace{2cm}}$

$h = \underline{\hspace{2cm}}$

$s = \underline{\hspace{2cm}}$

24.

Write the given expression in algebraic form.

$$\cos(\tan^{-1}(y)) = \underline{\hspace{2cm}}$$

25.

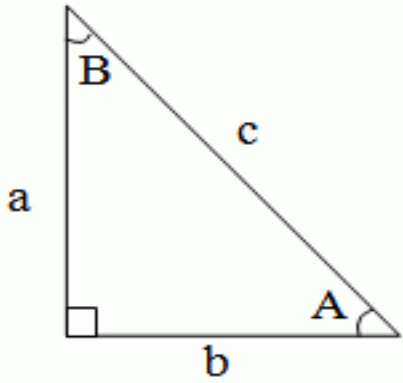
Write the given expression in algebraic form.

$$\cot(\sin^{-1}(x)) = \underline{\hspace{2cm}}$$

## Lecture 32

Solving triangles, bearing, other applications

1.



Suppose  $a = 8$  and  $c = 13$ .

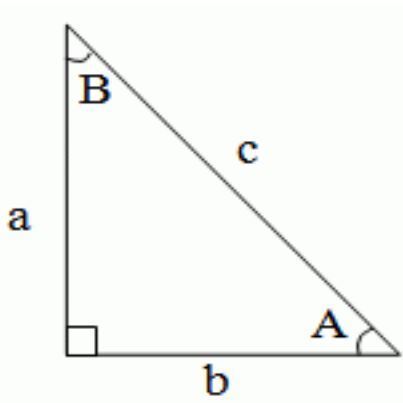
Find:

$b =$  \_\_\_\_\_

$A =$  \_\_\_\_\_ degrees

$B =$  \_\_\_\_\_ degrees

2.



Suppose  $a = 7$  and  $b = 2$ .

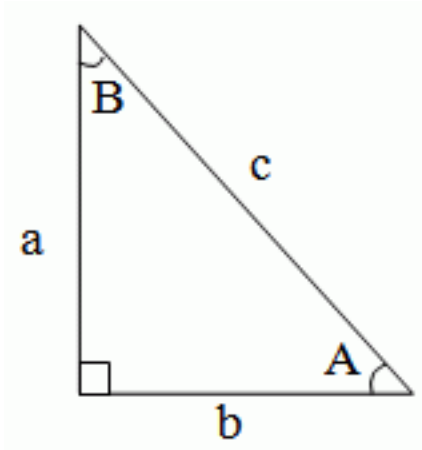
Find:

$c =$  \_\_\_\_\_

$A =$  \_\_\_\_\_ degrees

$B =$  \_\_\_\_\_ degrees

3.



Suppose  $a = 5$  and  $A = 70$  degrees.

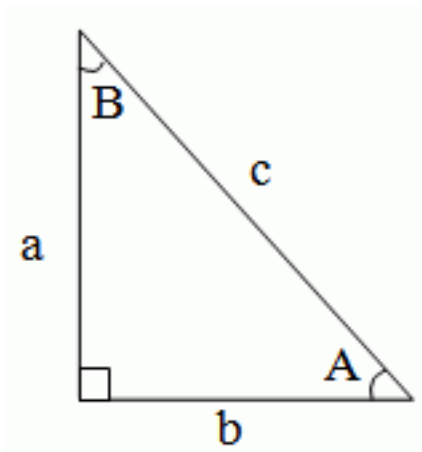
Find:

$$b = \underline{\hspace{2cm}}$$

$$c = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}} \text{ degrees}$$

4.



Suppose  $c = 7$  and  $A = 10$  degrees. Find:

$$a = \underline{\hspace{2cm}}$$

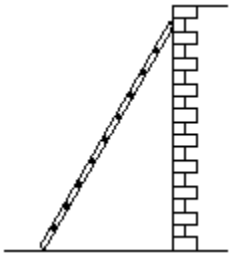
$$b = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}} \text{ degrees}$$



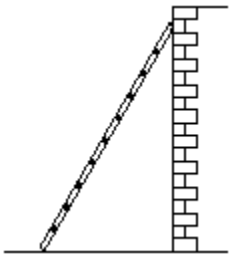
5.

The proper angle for a ladder is about  $75^\circ$  from the ground. Suppose you have a 12 foot ladder. How far from the house should you place the base of the ladder? \_\_\_\_\_ feet

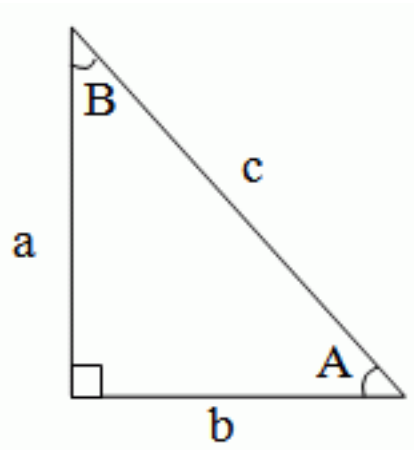


6.

The proper angle for a ladder is about  $75^\circ$  from the ground. Suppose you have a 13 foot ladder. How high can it reach? \_\_\_\_\_ feet



7.



Suppose  $\angle A = 30^\circ$  and  $a = 22$ .

$\angle B = \quad^\circ$

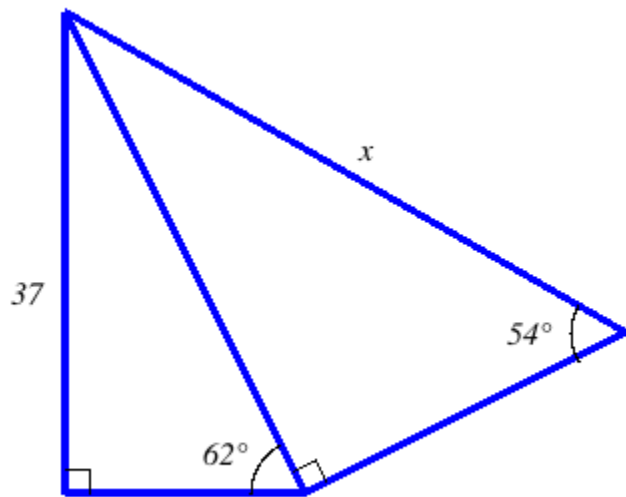
Find an exact value (report answer as a fraction, use sqrt if necessary):

$c =$  \_\_\_\_\_ feet

8.

Find  $x$  correct to 2 decimal places.

*NOTE: The triangle is NOT drawn to scale.*



$x =$  \_\_\_\_\_ feet

9.

To measure the height of the cloud cover at an airport, a worker shines a spotlight upward at an angle of  $65^\circ$  from the horizontal. An observer 682 m away measures the angle of elevation to the spot of light to be  $41^\circ$ . Find the height of the cloud cover.

height = \_\_\_\_\_ m

10.

From the top of a 183-ft lighthouse, the angle of depression to a ship in the ocean is  $27^\circ$ . How far is the ship from the base of the lighthouse?

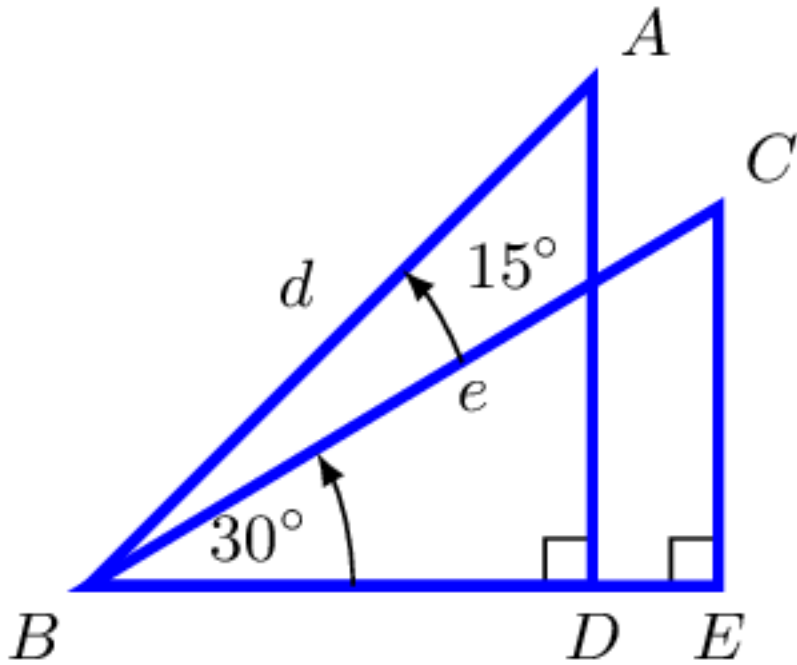
distance = \_\_\_\_\_ feet

11.

A smokestack is 160 feet high. A guy wire must be fastened to the stack 20 feet from the top. The guy wire makes an angle of  $40^\circ$  with the ground. Find the length of the guy wire rounded to the nearest foot.

\_\_\_\_\_ feet

12.



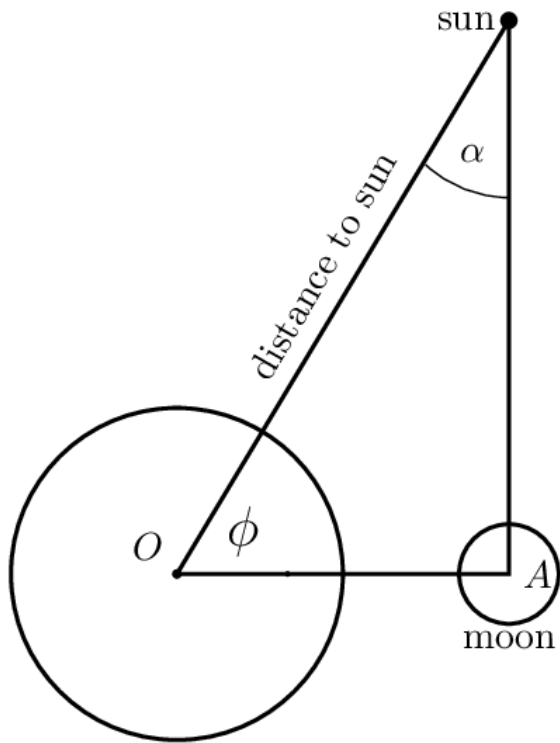
Using the special triangles, determine the exact value of segment DE. Segments  $d = BA$  and  $e = BC$  have length 2. Express your answer in simplified radical form.

DE = \_\_\_\_\_

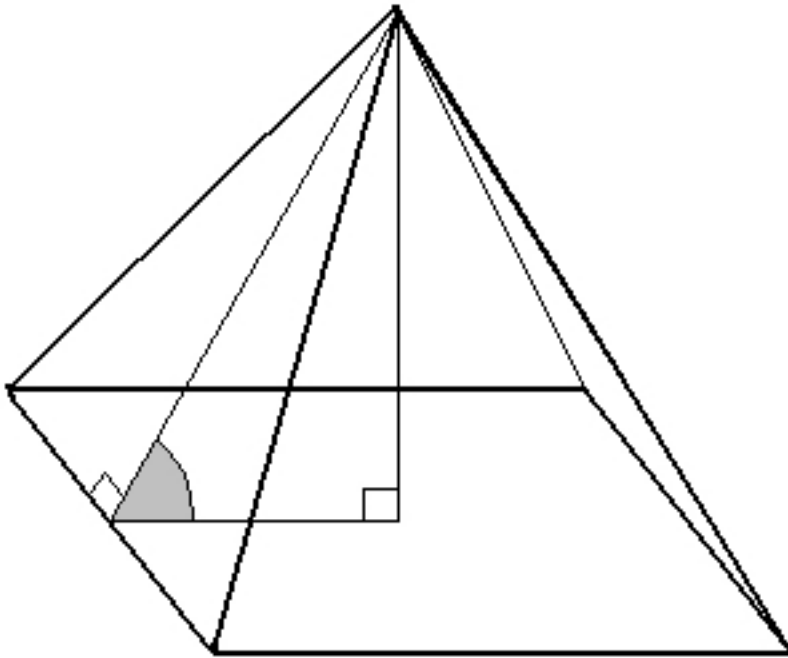
13.

An alien on a distant planet realizes that using trigonometry and the distance to one of its moons it is possible to calculate the distance to the nearby sun. Let  $O$  be the center of the planet and let  $A$  be the center of the moon. The alien begins with the premise that, during a half moon, the moon forms a right triangle with the Sun and the planet. By observing the angle between the Sun and Moon,  $\phi = 89.43$  degrees and knowing the distance to the moon is about 179000 km estimate the distance from the planet to the sun using these values. Round to the nearest 1000 km

distance =



14.



Consider a square-based straight pyramid. Suppose that the base is a square with sides 6 cm long, and all other edges are 7 cm long. Find an approximate value of the angle formed between the base and a triangular face. Present your answer in degrees, accurate up to four or more decimal places.

$\alpha = \text{_____}^\circ$  (degrees)

15.

From a fire tower 200 feet above level ground in the Sasquatch National Forest, a ranger spots a fire off in the distance. The angle of depression to the fire is  $2.7^\circ$ . How far away from the base of the tower is the fire? Round to the nearest foot.

\_\_\_\_\_ft

16.

From the observation deck of the lighthouse at Sasquatch Point 48 feet above the surface of Lake Ippizuti, a lifeguard spots a boat out on the lake sailing directly toward the light house. The first sighting had a angle of depression of  $8.2^\circ$  and the second sighting had an angle of depression of  $26^\circ$ . How far had the boat traveled between the sightings?

\_\_\_\_\_ft

## Lecture 33

Identities: Reciprocal, Quotient, Pythagorean, Even/odd, Cofunction

Simplifying, Factoring expressions, Combining using identities, Trig substitution, Simplifying log expressions

1.

Simplify  $\sin(t)\sec(t)$  to a single trig function or constant.

2.

Simplify  $\frac{\csc(t)}{\sec(t)}$  to a single trig function.

3.

Simplify  $\frac{\cot(t)}{\csc(t)-\sin(t)}$  to a single trig function.

4.

Simplify  $\frac{1+\csc(t)}{1+\sin(t)}$  to a single trig function.

5.

Simplify  $\frac{\cos^2(t)}{1-\cos^2(t)}$  to an expression involving a single trig function with no fractions.

6.

Fill in the blanks:

1. If  $\tan x = -3$  then  $\tan(-x) =$  \_\_\_\_\_

2. If  $\sin x = 0.1$  then  $\sin(-x) =$  \_\_\_\_\_

3. If  $\cos x = 0.7$  then  $\cos(-x) =$  \_\_\_\_\_

4. If  $\tan x = -3.5$  then  $\tan(\pi + x) =$  \_\_\_\_\_

7.  
Simplify to an expression involving a single trigonometric function with no fractions.

$$\cot(-x)\cos(-x) + \sin(-x) = \underline{\hspace{2cm}}$$

8.  
Simplify and write the trigonometric expression in terms of sine and cosine:  
 $\tan^2 x - \sec^2 x = \underline{\hspace{2cm}}$ .

9.  
Determine the value of  $\sin^2 x + \cos^2 x$  for  $x = 50$  degrees.

10.  
Simplify and write the trigonometric expression in terms of sine and cosine:

$$\cot(-x)\cos(-x) + \sin(-x) = -\frac{1}{f(x)}$$
$$f(x) = \underline{\hspace{2cm}}.$$

11.  
If  $\tan^2 t - \sin^2 t = \frac{\sin^a t}{\cos^b t}$ , then  
the positive power  $a = \underline{\hspace{2cm}}$ ,  
the positive power  $b = \underline{\hspace{2cm}}$ .

12.  
Simplify and write the trigonometric expression in terms of sine and cosine:

$$\frac{2 + \tan^2 x}{\sec^2 x} - 1 = g(x)$$
$$g(x) = \underline{\hspace{2cm}}.$$

13.

Simplify  $\frac{1+\csc(t)}{1+\sin(t)}$  to a single trig function.

14.

Simplify and write the trigonometric expression without any fractions:

$$\tan u + \cot u = \underline{\hspace{2cm}}$$

15.

$$\text{Factor: } 2\sin^2(x) - 3\sin(x) + 1 = \underline{\hspace{2cm}}$$

16.

$$\text{Factor: } 2\sin^2(x) - \sin(x) - 1 = \underline{\hspace{2cm}}$$

17.

Suppose that  $\alpha$  is an acute angle with  $\tan \alpha = \frac{11}{10}$ . Compute the exact value of  $\sec \alpha$ . You do not have to rationalize the denominator.

$$\sec \alpha = \underline{\hspace{2cm}}$$



18.

Use a substitution  $x = f(t)$  to re-express  $\sqrt{49 - x^2}$  as a trigonometric expression in terms of  $t$ . State the function  $f(t)$  used for substitution and the new expression.

$$f(t) = \underline{\hspace{2cm}}$$

$$\sqrt{49 - x^2} \text{ can be rewritten as } \underline{\hspace{2cm}}$$

19.

Use a substitution  $x = f(t)$  to re-express  $\sqrt{x^2 + 4}$  as a trigonometric expression in terms of  $t$ . State the function  $f(t)$  used for substitution and the new expression.

$$f(t) = \underline{\hspace{2cm}}$$

$$\sqrt{x^2 + 4} \text{ can be rewritten as } \underline{\hspace{2cm}}$$

20.

Use a substitution  $x = f(t)$  to re-express  $\sqrt{x^2 - 25}$  as a trigonometric expression in terms of  $t$ . State the function  $f(t)$  used for substitution and the new expression.

$$f(t) = \underline{\hspace{2cm}}$$

$$\sqrt{x^2 - 25} \text{ can be rewritten as } \underline{\hspace{2cm}}$$

### Lecture 34

Linear, quadratic, multiple angle, and using inverse trig functions

1.

Find all solutions to  $2\sin(\theta) = 1$  on the interval  $0 \leq \theta < 2\pi$

$\theta =$  \_\_\_\_\_

Give your answers as exact values, as a list separated by commas.

2.

Find all solutions to  $2\sin(\theta) = \sqrt{2}$  on the interval  $0 \leq \theta < 2\pi$

$\theta =$  \_\_\_\_\_

Give your solutions as exact values, separating multiple solutions by commas.

3.

Find all solutions to  $2\sin(\theta) = -\sqrt{2}$  on the interval  $0 \leq \theta < 2\pi$ .

$\theta =$  \_\_\_\_\_

Give your answers as exact values in a list separated by commas.

4.

Find all solutions to  $2\cos(\theta) = \sqrt{3}$  on the interval  $0 \leq \theta < 2\pi$ .

$\theta =$  \_\_\_\_\_

Give your answers as exact values in a list separated by commas.

5.

Solve  $\sin(x) = 0.42$  on  $0 \leq x < 2\pi$ .

There are two solutions, A and B, with  $A < B$ .

A = \_\_\_\_\_

B = \_\_\_\_\_

Give your answers accurate to 3 decimal places.

6.

Solve  $\cos(x) = 0.31$  on  $0 \leq x < 2\pi$ .

There are two solutions, A and B, with  $A < B$ .

A = \_\_\_\_\_

B = \_\_\_\_\_

Give your answers accurate to 3 decimal places.

7.

Find all solutions of the equation  $2\cos x - 1 = 0$ .

\_\_\_\_\_ +  $2k\pi$  where  $k$  is any integer

8.

Solve  $5\cos(w) = 0$  for all solutions.

$w =$  \_\_\_\_\_ where  $k$  is any integer

9.

Solve  $2\sin(x) = 2$  for all solutions.

$x =$  \_\_\_\_\_ where  $k$  is any integer

10.

Without using a calculator, find all the solutions of

$$\tan(t) = 1$$

$$t = \underline{\hspace{2cm}} \text{ where } -\pi < t \leq \pi .$$

11.

Find the exact solutions to  $\sin(x) = \cos(x)$  in the interval  $[0, 2\pi)$ . If the equation has no solutions, answer DNE.

12.

Solve  $2\sin^2(t) + 3\sin(t) + 1 = 0$  for all solutions  $0 \leq t < 2\pi$ .

$$t = \underline{\hspace{2cm}}$$

Give your answers as exact values in a list separated by commas.

13.

Solve  $2\cos^2(w) + 3\cos(w) + 1 = 0$  for all solutions.

$$w = \underline{\hspace{2cm}} + 2k\pi \text{ where } k \text{ is any integer}$$

Give your answers as exact values in a list separated by commas.

14.

Solve  $2\cos^2(x) - 7\cos(x) + 5 = 0$  for all solutions.

$$x = \underline{\hspace{2cm}} \text{ where } k \text{ is any integer}$$

15.

Find all solutions of  $\sin^2(x) - 4\cos(x) = 4$ .

$$x = \underline{\hspace{2cm}} \text{ where } n \text{ is any integer}$$

16.

Find all solutions of  $\sin^2(x) - 8\cos(x) = -8$ .

$x = \underline{\hspace{2cm}}$  where  $n$  is any integer.

17.

Find all solutions on the interval  $[0, 2\pi)$ . Give exact answers.

$$\sin^2(x) - \cos^2(x) + \sin(x) = 0$$

$x = \underline{\hspace{2cm}}$

18.

Solve for the exact solutions in the interval  $[0, 2\pi)$ . If the equation has no solutions, respond with DNE.

$$\sec(x) = 2\csc(x)$$

$x = \underline{\hspace{2cm}}$

19.

REMOVED

20.

$$\text{Suppose } \sin 3x = -\frac{\sqrt{3}}{2}.$$

Find all solutions  $0 \leq x \leq 2\pi$ . Give exact values in radians.

$x = \underline{\hspace{2cm}}$

21.

Find all solutions in the interval  $[0, 360^\circ)$ . List your answers in degrees. If there is no real solution, answer DNE.

$$\tan 3x = 0$$

$$x = \underline{\hspace{2cm}}$$

22.

Find all solutions in the interval  $[0, 360^\circ)$ . List your answers in degrees. If there is no real solution, answer DNE.

$$\sin 2x = -\frac{1}{2}$$

$$x = \underline{\hspace{2cm}}$$

## Lecture 35

Conditional vs identity equations, guidelines for verifying

1.

If  $(\tan x + \sec x)^2 = \frac{A + \sin x}{B - \sin x}$ , then

$$A = \underline{\hspace{2cm}},$$

$$B = \underline{\hspace{2cm}}.$$

2.

The expression  $3\tan(x)\sin(x) + 5\sec(x)$  simplifies to  $A\sec(x) - B\cos(x)$ , determine  $A$  and  $B$ .

$$A = \underline{\hspace{2cm}}$$

$$B = \underline{\hspace{2cm}}$$

3.

$\tan(x) + \sec(x)$  simplifies to  $\frac{f(x)}{\cos(x)}$  where

$$f(x) = \underline{\hspace{2cm}}$$

4.

Simplify  $\frac{1 + \cos(t)}{1 + \sec(t)}$  to a single trig function.

5.

Simplify  $\frac{\sin^2(t) + \cos^2(t)}{\sin^2(t)}$  to an expression involving a single trig function with no fractions.

6.

Simplify and write the trigonometric expression in terms of sine and cosine:

$$\frac{1 + \cos y}{1 + \sec y} = \underline{\hspace{2cm}}.$$

7.

Simplify the lefthandside so that  $LHS = RHS$  :

$$-\frac{\cos(x)}{1 - \sin(x)} + \frac{\cos(x)}{1 + \sin(x)} = -2\tan(x)$$

\_\_\_\_\_ =

\_\_\_\_\_ =

\_\_\_\_\_ =

\_\_\_\_\_ =

\_\_\_\_\_ =  $-2\tan(x)$

8.

Simplify the lefthandside so that  $LHS = RHS$  :

$$\left(\frac{1}{\tan(y)} + \frac{1}{\sin(y)}\right)^2 = \frac{1 + \cos(y)}{1 - \cos(y)}$$

\_\_\_\_\_ =

\_\_\_\_\_ =

\_\_\_\_\_ =

\_\_\_\_\_ =

\_\_\_\_\_ =  $\frac{1 + \cos(y)}{1 - \cos(y)}$



9.

Prove the given identity.

$$\frac{1}{1 - \sin x} - \frac{1}{1 + \sin x} = 2 \tan x \sec x$$

10.

Prove the given identity.

$$\frac{1 + \tan^2 x}{1 - \tan^2 x} = \frac{1}{\cos^2 x - \sin^2 x}$$

11.

Prove the given identity.

$$\frac{\tan x + \tan y}{\cot x + \cot y} = \tan x \tan y$$

12.

Prove the given identity.

$$\tan^2 x - \sin^2 x = \tan^2 x \sin^2 x$$