This is the Answer Key for Module 9L Version A.

41. Subtract the following functions, then choose the domain of the resulting function from the list below.

$$f(x) = 7*x**4 + 5*x**3 + 5*x**2 + 4*x + 1 \text{ and } g(x) = \sqrt{-4x - 14}$$

The solution is The domain is all Real numbers less than or equal to$x = -3.5$.

A. The domain is all Real numbers except $x = a,$ where $a \in [4, 12]$

B. The domain is all Real numbers less than or equal to $x = a,$ where $a \in [-8, 1]$

C. The domain is all Real numbers greater than or equal to $x = a,$ where $a \in [3, 11]$

D. The domain is all Real numbers except $x = a$ and $x = b,$ where $a \in [-12, -5]$ and $b \in [5, 7]$

E. The domain is all Real numbers.

General Comments: The new domain is the intersection of the previous domains.

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42. Choose the interval below that f composed with g at $x = 1$ is in.

$$f(x) = -x**3 - 2*x**2 + x + 3 \text{ and } g(x) = 4*x**3 - 4*x**2 + 4*x - 2$$

The solution is -11.0

A. $(f \circ g)(1) \in [-14.7, -10.9]$

* This is the correct solution

B. $(f \circ g)(1) \in [-8.1, -2.3]$

Distractor 3: Corresponds to being slightly off from the solution.

C. $(f \circ g)(1) \in [-17.2, -16.9]$

Distractor 2: Corresponds to being slightly off from the solution.

D. $(f \circ g)(1) \in [1.3, 5.4]$

Distractor 1: Corresponds to reversing the composition.

E. It is not possible to compose the two functions.

General Comments: f composed with g at x means $f(g(x))$. The order matters!

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43. Determine whether the function below is 1-1.

$$f(x) = -64x^3 + 1056x^2 - 5808x + 10648$$

The solution is yes

A. Yes, the function is 1-1.

* This is the solution.

B. No, because the range of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the range is all Real numbers.

C. No, because the domain of the function is not $(-\infty, \infty)$.

Corresponds to believing 1-1 means the domain is all Real numbers.

D. No, because there is a y-value that goes to 2 different x-values.

Corresponds to the Horizontal Line test, which this function passes.

E. No, because there is an x-value that goes to 2 different y-values.

Corresponds to the Vertical Line test, which checks if an expression is a function.

General Comments: There are only two valid options: The function is 1-1 OR No because there is a y-value that goes to 2 different x-values.

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44. Find the inverse of the function below (if it exists). Then, evaluate the inverse at $x = -10$ and choose the interval that $f^{-1}(-10)$ belongs to.

$$5 * x * *2 + 3$$

The solution is The function is not invertible for all Real numbers.

A. $f^{-1}(-10) \in [1.32, 2.23]$

Distractor 1: This corresponds to trying to find the inverse even though the function is not 1-1.

B. $f^{-1}(-10) \in [3.41, 3.73]$

Distractor 3: This corresponds to finding the (nonexistent) inverse and dividing by a negative.

C. $f^{-1}(-10) \in [1.04, 1.22]$

Distractor 2: This corresponds to finding the (nonexistent) inverse and not subtracting by the vertical shift.

D. $f^{-1}(-10) \in [6.23, 7.19]$

Distractor 4: This corresponds to both distractors 2 and 3.

E. The function is not invertible for all Real numbers.

* This is the correct option.

General Comments: Be sure you check that the function is 1-1 before trying to find the inverse!

---

45. Find the inverse of the function below. Then, evaluate the inverse at $x = 9$ and choose the interval that $f^{-1}(9)$ belongs to.

$$e^{x+3} + 5$$

The solution is $f^{-1}(9) = -1.614$

A. $f^{-1}(9) \in [7.57, 7.7]$

This solution corresponds to distractor 2.

B. $f^{-1}(9) \in [7.44, 7.54]$

This solution corresponds to distractor 4.

C. $f^{-1}(9) \in [6.73, 6.92]$

This solution corresponds to distractor 3.

D. $f^{-1}(9) \in [4.31, 4.5]$

This solution corresponds to distractor 1.

E. $f^{-1}(9) \in [-1.65, -1.37]$

This is the solution.

Natural log and exponential functions always have an inverse. Once you switch the x and y, use the conversion $e^y = x \leftrightarrow y = \ln(x)$.

This is the Answer Key for Module 10L Version A.

46. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6 * x * *3 - 40 * x * *2 + 86 * x - 57}{x - 3}$$

The solution is $6x^2 + -22x + 20 + \frac{3}{x-3}$

A. $a \in [3, 10], \; b \in [-27, -16], \; c \in [18, 22], \;$ and $r \in [-7, 8]$.

   * This is the solution!

B. $a \in [13, 21], \; b \in [-99, -85], \; c \in [363, 374], \;$ and $r \in [-1164, -1153]$.

   You divided by the opposite of the factor AND multiplied the first factor rather than just bringing it down.

C. $a \in [3, 10], \; b \in [-64, -55], \; c \in [259, 262], \;$ and $r \in [-838, -833]$.

   You divided by the opposite of the factor.

D. $a \in [13, 21], \; b \in [9, 16], \; c \in [126, 133], \;$ and $r \in [321, 334]$.

   You multiplied by the synthetic number rather than bringing the first factor down.

E. $a \in [3, 10], \; b \in [-32, -23], \; c \in [26, 31], \;$ and $r \in [-7, 8]$.

   You multiplied by the synthetic number and subtracted rather than adding during synthetic division.

General Comments: Be sure to synthetically divide by the zero of the denominator!

---

47. Perform the division below. Then, find the intervals that correspond to the quotient in the form $ax^2 + bx + c$ and remainder r.

$$\frac{6 * x * *3 - 18 * x + 8}{x + 2}$$

The solution is $6x^2 + -12x + 6 + \frac{-4}{x+2}$

A. $a \in [-18, -5], b \in [-24.5, -23.3], c \in [-67, -62], \;$ and $r \in [-125, -123]$.

   You divided by the opposite of the factor AND multipled the first factor rather than just bringing it down.

B. $a \in [5, 11], b \in [-20.4, -17.7], c \in [31, 39], \;$ and $r \in [-107, -98]$.

   You multipled by the synthetic number and subtracted rather than adding during synthetic division.

C. $a \in [5, 11], b \in [-15.6, -10.7], c \in [5, 13], \;$ and $r \in [-5, -3]$.

   * This is the solution!

D. $a \in [-18, -5], b \in [21.6, 25.7], c \in [-67, -62], \;$ and $r \in [136, 142]$.

   You multipled by the synthetic number rather than bringing the first factor down.

E. $a \in [5, 11], b \in [10.5, 13], c \in [5, 13], \;$ and $r \in [17, 24]$.

   You divided by the opposite of the factor.

General Comments: Be sure to synthetically divide by the zero of the denominator! Also, make sure to include 0 placeholders for missing terms.

---

48. What are the *possible* Integer roots of the polynomial below?

$$5 * x ** 4 - 6 * x ** 3 + 4 * x ** 2 + 3 * x - 7$$

The solution is $\pm 1, \pm 7$

A. All combinations of: $\dfrac{\pm 1, \pm 5}{\pm 1, \pm 7}$

Distractor 3: Corresponds to the plus or minus of the inverse quotient (an/a0) of the factors.

B. All combinations of: $\dfrac{\pm 1, \pm 7}{\pm 1, \pm 5}$

Distractor 2: Corresponds to the plus or minus of the quotient of the factors.

C. $\pm 1, \pm 7$

* This is the solution!

D. $\pm 1, \pm 5$

Distractor 1: Corresponds to the plus or minus factors of a1 only.

E. There is no formula or theorem that tells us all possible Integer roots.

Distractor 4: Corresponds to not recognizing Integers as a subset of Rationals.

General Comments: We have a way to find the possible Rational roots. The possible Integer roots are the Integers in this list.

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49. Factor the polynomial below completely. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3$.

$$6 * x ** 3 - 19 * x ** 2 + 19 * x - 6$$

The solution is $[0.6666666666666666, 1, 1.5]$

A. $z_1 \in [-3.7, -2.2], \; z_2 \in [-3, 0], \text{ and } z_3 \in [-0.44, 0.12]$

Distractor 4: Corresponds to moving factors from one rational to another.

B. $z_1 \in [-0.1, 1.7], \; z_2 \in [0, 5], \text{ and } z_3 \in [1.43, 1.66]$

* This is the solution!

C. $z_1 \in [-0.1, 1.7], \; z_2 \in [0, 5], \text{ and } z_3 \in [1.43, 1.66]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-1.8, -0.1], \; z_2 \in [-3, 0], \text{ and } z_3 \in [-1.25, -0.62]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-1.8, -0.1], \; z_2 \in [-3, 0], \text{ and } z_3 \in [-1.25, -0.62]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

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50. Factor the polynomial below completely, knowing that $x - 3$ is a factor. Then, choose the intervals the zeros of the polynomial belong to, where $z_1 \leq z_2 \leq z_3 \leq z_4$.

$$3 * x ** 4 - 19 * x ** 3 + 39 * x ** 2 - 29 * x + 6$$

The solution is $[0.3333333333333333, 1, 2.0, 3]$

A. $z_1 \in [-3.7, -2.4], \; z_2 \in [-3.6, -2.79], z_3 \in [-1.65, -0.91], \text{ and } z_4 \in [-0.54, -0.49]$

Distractor 3: Corresponds to negatives of all zeros AND inversing rational roots.

B. $z_1 \in [-3.7, -2.4], \; z_2 \in [-1.05, -0.87], z_3 \in [-1.65, -0.91], \text{ and } z_4 \in [-0.67, -0.55]$

Distractor 4: Corresponds to moving factors from one rational to another.

C. $z_1 \in [0.4, 0.6], \; z_2 \in [0.15, 1.1], z_3 \in [2.88, 3.75], \text{ and } z_4 \in [2.98, 3.08]$

Distractor 2: Corresponds to inversing rational roots.

D. $z_1 \in [-3.7, -2.4], \; z_2 \in [-2.32, -1.45], z_3 \in [-1.65, -0.91], \text{ and } z_4 \in [-0.38, -0.15]$

Distractor 1: Corresponds to negatives of all zeros.

E. $z_1 \in [-0.6, 0.4], \; z_2 \in [0.15, 1.1], z_3 \in [1.04, 2.42], \text{ and } z_4 \in [2.98, 3.08]$

* This is the solution!

General Comments: Remember to try the middle-most integers first as these normally are the zeros. Also, once you get it to a quadratic, you can use your other factoring techniques to finish factoring.

This is the Answer Key for Module 11 Limits all Versions.

71. This is a "translate the phrase" questioned pulled directly from the homework. As x approaches -3, our function $f(x) = \frac{x+3}{x^2-9}$ approaches $-\frac{1}{6}$. Answer A.

72. Notice it says the right-hand limit ($x \to 0^+$). This means as we approach numbers just barely bigger than 0, what happens to the function? It goes toward $-\infty$. You can graph the function or plug in some values to confirm.

73. Plugging in 9 we get $\frac{0}{0}$, so no help there. We also haven't learned how to factor out the hole, so this is a question where you had to plug in values close to 9 and see what happened.

x	y
8.9	0.252
8.99	0.250
9.01	0.250
9.1	0.248

On either side, it appears to be approaching $0.25 = \frac{1}{4}$.

74. Approaching 1 from the left brings us to the value $y = 6$.

75. Notice how it does not specify left or right for this limit - that means both one-sided limits need to agree! In this case, approaching from the left or right lead to $-\infty$, so the limit is $-\infty$.

This is the Answer Key for Module 12 Limits all Versions.

76. Holes are values the numerator AND denominator are 0 at. Factoring, we get:

$$\frac{(x+2)(x+4)}{(x+2)(x^2+10)}$$

Therefore, there is a hole at $x = -2$.

77. Vertical asymptotes are the values that make the denominator 0 (and are not holes). Factoring, we get:

$$\frac{(x+4)(x+5)}{(x+5)(x^2+18)}$$

There is a hole at $x = -5$, which is **not** a vertical asymptote. The other values that make the denominator 0 are complex, so there are no vertical asymptotes.

78. We can easily spot horizontal asymptotes by taking the limit of the leading terms:

$$\lim_{x\to\infty} \frac{x}{-x^3} = \lim_{x\to\infty} -\frac{1}{x^2} = 0$$

$$\lim_{x\to-\infty} \frac{x}{-x^3} = \lim_{x\to-\infty} -\frac{1}{x^2} = 0$$

Therefore, we have a single horizontal asymptote: the line $y = 0$.

79. We can easily spot oblique asymptotes by taking the limit of the leading terms:

$$\lim_{x\to\infty} \frac{27x^3}{x} = \lim_{x\to\infty} 27x^2 = \infty$$

That means we have an oblique asymptote that looks like $27x^2$. We'll need to synthetically divide to figure out the exact oblique asymptote, which gives us $y = 27x^2 + 54x + 51$.

80. This question is asking us to put all of our knowledge about asymptotes together. We see 3 vertical asymptotes: at $x = 2, x = 4$, and some fraction between -2 and -1. Based on our options, this must be associated to the factor $(2x + 3)$ and so the value is $x = -3/2$. We've narrowed it down to options A and B, so how can we know which is correct? We need these values to be asymptotes (not holes), so let's factor the numerators of each.

$6x^2 + 11x + 3 = (3x + 1)(2x + 3)$

$3x^2 + 10x + 3 = (3x + 1)(x + 3)$

Notice how we have a hole at $x = -3/2$ for option A? That means options B **must** be the correct answer.

This is the Answer Key for Module 9 Modeling all Versions.

41. In each version, a linear model was appropriate. Regardless of the context, they stated "grows at a constant rate", "X amount per mile", and "steadily decreased". These all point to **constant** growth, which is what we are looking for with a linear model.

42. For many students, they saw money and immediately went for "Proper Subset of the Real numbers". However, the variable here is weeks. "every week he/she saves an additional X amount of money" tells us we count the weeks as 1, 2, 3, 4, etc... This means our restricted domain is a subset of the Natural numbers.

43. This question was a new context for the UFPD/Bicyclist question. We have a jet flying against the jet stream goes $\frac{1880\text{miles}}{4\text{hours}} = 470\text{mph}$. If we model this, we have the jet J, going **against** the wind, $-W$, and that total being 470. So our model is $J - W = 470$. Solving for J, we get $J = W + 470$.

44. This question was a new context for the tickets/chemistry question. We have a show with two types of film: b black and white and c color. Their combined total worth was \$4000, so $b + c = 4000$. Profits were 12% on black and white and 21% on color, so the profit equation would be $0.12b + 0.21c = \text{Profits}$. The question wanted this in terms of b, so we solve the first equation in terms of $c = 4000 - b$ and plug that in to our second equation. This gives us

$$0.12b + 0.21(4000 - b) = 840 - 0.9b$$

Notice this is **exactly** the same as the chemistry model!

45. This is **exactly** a question from the homework. It asks for part C - a model of the amount of acid in one container in terms of the volume of the other container.

$$v_{10} + v_{30} = 7 \to v_{30} = 7 - v_{10}$$
$$A_{30} = 0.3 * v_{30}$$
$$A_{30} = 0.3(7 - v_{10}) = 2.1 - 0.3v_{10}$$

This is the Answer Key for Module 10 Modeling all Versions.

46. "A penny doubled every day for the next 30 days". Let's try to model this: 0.01, 0.02, 0.04, 0.08, ... What we see is we are multiplying by 2 at every step, which means our equation looks something like 2^t. This is an exponential model and a power model would **not** be appropriate.

47. This is an indirect variation: as the number of workers **increases**, the number of days required to build **decreases**.

48. Here is our Ideal Gas Laws problem from the homework: $PV = nRT$. If we use the description provided, we would have

$$T = k\frac{PV}{n}$$

We want to know the number of moles, n, so we solve for n.

$$n = k\frac{PV}{T}$$

We can plug in some of the values and find k, which is 0.12. So our model is A.

49. This works exactly the same as the Ideal Gas Laws problem *(and every other problem in this module)*. Building the model from the words, we get

$$C = k\frac{P_1P_2}{d^2}$$

Then we can solve for d:

$$d = \sqrt{k\frac{P_1P_2}{C}}$$

Now we can just plug in the values given and find that $\sqrt{k} = 0.134$ and the answer is B.

50. The area of a triangle is a direct variation you know! $A = \frac{1}{2}bh$. If we modify this by increasing the base by 40% and decreasing the height by 10%, we get

$$A_{new} = \frac{1}{2}(1.4b)(0.9h) = 0.63bh$$

Why does increasing make it 1.4? Increasing by 40% means **the new value is 140% the old value**. You could try some values to test this, such as $b = 10$. If we tried $10 * 0.4$, we would see the base is shrinking as it is now 4!

This is the Answer Key for Module 11 Modeling all Versions.

51. We see a slow initial growth, then a rapid increased growth late. This is what we are looking for in an Exponential model! *(and yes, this data was from a real study to support the legal BAC level when driving)*

52. This is an exact example we saw from the homework. While this may look like direct variation ($k = 0.2$), the growth amount is affected by initial balance.

53. Here is our bacteria population question. Many students chose $P(t) = 200 * 4^t$, which is a correct model of the current bacteria population, P, based on the amount of time t in hours. **Reading the instructions, it asks for the reverse relation.** That is, solve for t.

$$\begin{aligned} P &= 200 * 4^t \\ \frac{P}{200} &= 4^t \\ \ln(P/200) &= \ln(4^t) \\ \frac{\ln(P/200)}{\ln(4)} &= t \end{aligned}$$

54. For this question, the formula was given:

$$T(t) = Ae^{kt} + T_s$$

After plugging in the information given, we have:

$$T(t) = Ae^{kt} + 35$$

We have multiple constants, so let's try to solve for some. We know that the cheesecake started at 165, so let's use $t = 0$ and $T(0) = 165$.

$165 = Ae^0 + 35$

So $A = 130$. To find k, we'll need to use $T(10) = 150$. Plugging that in, we get $k = -0.0123$.

We now have **an** equation: $T(t) = 130e^{-0.0123t} + 35$. The question asks for a model that describes the amount of time, t, in terms of the cheesecake temperature T. We need to solve for t.

$$\begin{aligned} T &= 130e^{-0.0123t} + 35 \\ T - 35 &= 130e^{-0.0123t} \\ \ln(\frac{T-35}{130}) &= \ln(e^{-0.0123t}) \\ \ln(\frac{T-35}{130}) &= -0.0123t\ln(e) \\ \frac{\ln(\frac{T-35}{130})}{-0.0123} &= t \end{aligned}$$

Therefore, the answer is C.

55. For this question, we are told we have an exponential decay: $G = G_0e^{kt}$, where G_0 is the initial amount. Half-life means the time half of the element is left: $\frac{G_0}{2}$. We plug in both to our equation and get:

$$
\begin{aligned}
\frac{G_0}{2} &= G_0 e^{k*80} \\
\frac{1}{2} &= e^{80k} \\
\ln(1/2) &= \ln(e^{80k}) \\
\ln(1/2) &= 80k\ln(e) \\
\ln(1/2)/80 &= k \\
-0.0087 &= k
\end{aligned}
$$

So we have **an** equation: $G = G_0 e^{-0.0087t}$. The question asks for an equation of the time passed, t, in terms of the ratio of Gallium-67 remaining, r. *So how do we find that ratio?* Let's try some specific values. If there was 80 grams initially and there are now 30, what percentage is remaining? We would divide the amount there is now by the initial amount: G/G_0. This is the ratio!

Going back to our initial equation, that means $r = e^{-0.0087t}$. Solving for t, we get $t = \dfrac{\ln(r)}{-0.0087}$ and thus the answer is C.

This is the Answer Key for Module 12 Modeling all Versions.

56. Pulled from the homework. We know that the ball steadily picks up **speed**, but we want to model the ball's height. We do know that as time passes, the ball gets closer and closer to the ground, so a direct variation model would be appropriate.

57. Again pulled from the homework directly (Module 9). First, we think about their distances in terms of minutes. Officer A is walking 3 miles per hour, which means A is walking at $\frac{3}{60}$ miles per minute. The distance of Officer A is thus $\frac{1}{20}m$. Similarly, Officer B is walking at 5 miles per hour and so their distance is $\frac{1}{12}m$. If they are walking directly 90 degrees from one another, their distance apart is the hypotenuse of a right triangle.

$D = \sqrt{(\frac{1}{20}m)^2 + (\frac{1}{12}m)^2} = 0.097m.$

58. This is nearly the same as the homework question from Module 11. Let's say t is going to be in minutes. Our equation would be $P = 204^{t/30}$ *(That way, 30/30 = 1, 60/30=2, etc and we see that every 30 minutes, we increase our t by 1.)* To find what time the population would reach 1,000,000, we need to solve for t.

$$\begin{aligned} 1000000 &= 204^{t/30} \\ 50000 &= 4^{t/30} \\ \ln(50000) &= t/30 \ln(4) \\ \frac{30\ln(50000)}{\ln(4)} &= t \\ 234 &= t \end{aligned}$$

This is in minutes, so we will need to divide by 60 to convert to hours: $t = 3.9$. Therefore, our answer is B, about 4 hours.

Note: you could have set up the equation in terms of hours to start with $P = 204^{0.5t}$, as 30 minutes is 0.5 hours. You would get the same answer.

59. We saw this model in Module 11. $t = \frac{\ln(\frac{T-35}{130})}{-0.0123}$ Plugging in 70, we get $t = 107$. This is in minutes, so we divide by 60 to get the number of hours: 1.77. Therefore, our answer is about 1.75 hours, D.

60. This is a half-life question, similar to question 55. Our equation is $C = C_0 e^{kt}$. We first solve for k.

$$\begin{aligned} 1/2C_0 &= C_0 e^{k*5730} \\ 1/2 &= e^{5730k} \\ \ln(1/2) &= 5730k\ln(e) \\ \frac{\ln(1/2)}{5730} &= k \\ -0.00012 &= k \end{aligned}$$

So we have $C = C_0 e^{-0.00012t}$. They tell us a bone fragment has 20% of its original carbon-14, which means the ratio of carbon-14 now to what it had originally is $0.2 = \frac{C}{C_0}$. Resolving our original equation, we have: $0.2 = e^{-0.00012t}$. Solving for t, we get $t = 13305$, which is about $13,300$ years.