Review 4 – Answers

1. (a) $\nabla\varphi = \dfrac{\langle -y, x\rangle}{x^2+y^2} = \dfrac{\langle -y, x\rangle}{\|\mathbf{r}\|^2}$; (b) $\nabla\varphi = \left\langle \sqrt{y^2+z^2}, \dfrac{xy}{\sqrt{y^2+z^2}}, \dfrac{xz}{\sqrt{y^2+z^2}} \right\rangle$

2. (a) $\mathbf{F} = k\dfrac{\mathbf{r}}{\|\mathbf{r}\|}$; (b) radial; (c) outward the origin; (d) $\|\mathbf{F}\| = k$.

3. (a) $\varphi = \dfrac{x}{y} + C,\ C \in \mathbb{R}$; (b) $\varphi = x^2y + y^2z^3 + z + C,\quad C \in \mathbb{R}$; (c) not conservative

4. $2b\pi^2\sqrt{a^2+b^2}$ 5. 18 6. 4 7. $4\pi^2$ 8. (a) $\varphi = \dfrac{x^2}{2}\left(1+y^2\right) + C,\ C \in \mathbb{R}$, (b) -1

9. (a) Yes; (b) $\varphi = x^2y + y^2z + C,\ C \in \mathbb{R}$ (c) 0 10. (b) 2π; (c) No; (d) 0

11. $\dfrac{\sqrt{6}}{6}$ 12. $\dfrac{\pi}{6}\left(5\sqrt{5}-1\right)$

13. (a) $S : \mathbf{r}(u,v) = \langle 2\cos u, 2\sin u, v\rangle$, where $D = \{(u,v) \mid 0 \le u \le 2\pi, 0 \le v \le 1\}$
(b) $dS = 2\,dudv$; (c) 8π

14. 8π 15. $\dfrac{ab}{6}$ 16. $\dfrac{625}{2}\pi$ 17. $\dfrac{\pi}{3}$ 18. 2π 19. $\dfrac{5\pi}{6}$ 20. $-\pi$

21. (a) -2π (b) -2π (c) -2π; (d) Stokes' 22. (a), (b), and (d) 23. (c) and (d)
24. True: (a), (d), (e); Not true in general: (c), (f), (h); Do not make sense: (b), (g)