MAC 2311 Online: Review 4 Answer

1. $\sqrt{3}+\sqrt{5}+\sqrt{7}+3$

2. $\displaystyle\int_1^3 x^3\,dx$ or $\displaystyle\int_0^2 (1+x)^3\,dx$

3. $\displaystyle\int_0^3 \sqrt{1+x^2}\,dx$

4. a) $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{\infty}\left(\frac{24i^2}{n^2}-\frac{8i^3}{n^3}\right)\frac{2}{n}$ b) 12

5. $s(t)=\frac{1}{4}e^{2t}+\frac{5}{2}t+\frac{7}{4}$ in meters

6. a) $\tan\theta + C$ b) $\frac{3}{2}-\frac{2}{3}\pi$ c) $\frac{\pi}{6}$

7. a) $\frac{3}{2}\ln|x^2-4|+C$ b) $3\sqrt{x^2+2x+5}+C$ or $3\sqrt{(x+1)^2+4}+C$

   c) $\frac{3+\sqrt{3}}{2}$ d) $\frac{62}{3}$ e) $\ln|\frac{1}{1+\cos^2 x}|+C$

   f) $-\arctan(\cos x)+C$ g) 1 h) $\ln|\sin x|+C$ i) $-\frac{1}{2}e^{\frac{1}{2x-1}}+C$

   j) $-\frac{32}{15}$; $\displaystyle\int x\sqrt{2x+4}\,dx=\frac{(2x+4)^{5/2}}{10}-\frac{2}{3}(2x+4)^{3/2}+C$

   k) $(e^x-1)-\ln|e^x-1|+C$ or $e^x-\ln|e^x-1|+C$ l) $1-\cos(1)$

8. minimum value $=\frac{-8}{3\sqrt{3}}$; maximum value $=0$;

$$-\frac{16}{3\sqrt{3}}\le\int_{-2}^{0}x\sqrt{2x+4}\,dx\le 0$$

9. $g'(x)=(\ln x)^2+2x$; $g'(e)=1+2e$; g is increasing on $(0,\infty)$, never decreasing.

10. $(-\infty,-1)\cup(1,\infty)$

11. $\dfrac{-\sqrt{x}}{2(x+2)}$

12. $\dfrac{e-1}{2}$

13. a) g is increasing on $(2,5)$ and decreasing on $(0,2)\cup(5,8)$.

    b) local maximum at $x=5$ and local minimum at $x=2$

14. $f(x) = -\dfrac{4}{3}\cos^3\left(\dfrac{x}{2}\right) + \dfrac{7}{3}$

15. $\dfrac{35}{3}$

16. a) 36 (b) 20 units^2

17. a) $\dfrac{2}{3}\ln\left(\dfrac{5}{2}\right)$ b) $\dfrac{1}{5}$

18. $5\ln 9$ ml. of drink

19. displacement: $\displaystyle\int_0^4 \left(3 - \frac{18}{1+2t}\right) dt = 12 - 9\ln 9$ in

    distance $\displaystyle\int_0^{5/2} \left(\frac{18}{1+2t} - 3\right) dt + \int_{5/2}^4 \left(3 - \frac{18}{1+2t}\right) dt = 18\ln 2 - 3$ in

20. decrease of 880

21. (a) True (b) False (c) False

MAC 2311 Online: Review 3 Answer

1. 4.041

2. a) $c = \dfrac{3\pi}{2}$ b) not possible; f is not differentiable at $x = 0$

   c) not possible; f is not continuous (or differentiable) at $x = \dfrac{\pi}{2}$

3. a) $c = 5 - \sqrt{5}$ b) $c = e - 1$

4. $12 \leq f(7) - f(1) \leq 30$

5. critical numbers: $x = 4$ only; local maximum $g(4) = \dfrac{3}{16}$ and no local minimum

6. max: $f(1) = 1$, min: $f(2) = 4 - 8\ln(2)$

7. Increasing and concave down on $(-3, -1)$; Inflection points: $\left(-3, \dfrac{10}{e^3}\right), \left(-1, \dfrac{2}{e}\right)$

8. On $[2, 4]$: absolute max $= f(2) = 8$, absolute min $= f(3) = 27/4$

   On $(1, \infty)$: absolute min $= f(3) = 27/4$; no absolute max

9. Use the Second Derivative Test: $f(-2)$ and $f(5)$ are relative minima, $f(1)$ is a relative maximum.

10. relative minimum value: $f\left(\dfrac{7\pi}{6}\right) = -\dfrac{\sqrt{3}}{3}$;

    relative maximum value: $f\left(\dfrac{11\pi}{6}\right) = \dfrac{\sqrt{3}}{3}$;

    inflection points $\left(\dfrac{\pi}{2}, 0\right)$ and $\left(\dfrac{3\pi}{2}, 0\right)$

11. a) 2 b) 1 c) $-\infty$ d) $\dfrac{1}{2}$ e) $e^{2/\pi}$ f) 0 g) 1

12. a) $\frac{2}{3}\sqrt{x^3} + 2x + 2\sqrt{x} + C$ b) $-\cos x + \sec x + C$

    c) $\dfrac{x^3}{3} - 2\arctan x + C$ d) $\dfrac{5}{4}x^4 - \cot x + \ln|x| + \dfrac{2}{3}x^{3/2} + C$

13. 27

14. ± 0.48 inches

15. a) marginal revenue: \$15 so revenue is increasing by \$15 per item /item

    b) production increases by 30 items/week

16. $x_1 = 2$ and $x_2 = \dfrac{5}{3}$

17. $\sqrt[3]{2}$

18. Water level is rising by $\dfrac{2}{3\pi}$ cm/sec.

19. Minimize Cost $= 6x^2 + 8xy$ with constraint Volume $x^2y = 12$:

    $x = 2$, $y = 3$ and minimum cost is \$72

20. $x = 1$, $y = \sqrt{3}$, $A = \dfrac{3\sqrt{3}}{2}$

21. Dimensions: $x = 550$ ft, $y = \dfrac{2200}{3}$ ft

22. relative maximum: $f(-2) = 0$, relative minimum: $f(0) = -2^{\frac{2}{3}}$,

    vertical tangent line and inflection point at $(1, 0)$

23. local minimum at $x = -1$, inflection points at $x = 0$, $x = 1$

    local maximum and inflection point at $x = 4$

MAC 2311 Online: Review 2 Answer

1. 1

2. $\frac{2}{5}$

3. $f'(x) = \frac{3}{2}\sqrt{x} + 1$

4. $a = 15$

5. $\frac{8x}{(1-4x^2)^{3/2}}$

6. a) 0 b) ∞

7. $\lim_{h\to 0} \frac{\cos(2x+2h) - \cos(2x)}{h} = -2\sin(2x)$

   (Note: Need to apply the formula for $\cos(A+B)$.)

8. a) $\lim_{h\to 0} \frac{\sqrt{2x+2h-1} - \sqrt{2x-1}}{h} = \frac{1}{\sqrt{2x-1}}$

   b) $\left(\frac{1}{2}, \infty\right)$ c) $y = \frac{1}{3}x + \frac{4}{3}$

9. a) $f'(x) = \frac{4x-2}{(3x-2)^{2/3}}$

   horizontal tangent line: $y = -\frac{1}{2\sqrt[3]{2}}$, vertical tangent line: $x = \frac{2}{3}$

   b) $f'(x) = \frac{6(x+1)}{(x^2+3)^{3/2}}$

   horizontal tangent line: $y = -4$, no vertical tangent line.

10. a) $v(t) = 3t^2 - 12t + 9$ b) $t = 1$ and $t = 3$

    c) $(0, 1)$ and $t > 3$ d) 1 cm/sec

11. a) -14 b) $\frac{17}{4}$

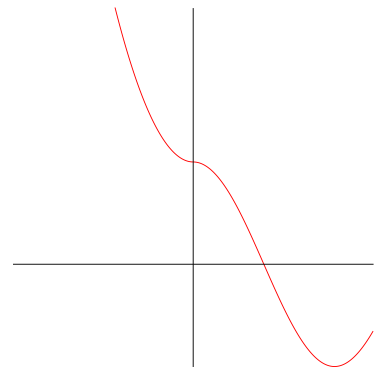
12. $x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

13. $f'(0) = e$; Yes, f is continuous at $x = 0$

14. a) $f(x)$ is continuous at $x = 0$ since $\lim_{x\to 0} f(x) = 2 = f(0)$.

b) $f'(0) = 0$ since $\lim_{x\to 0} \dfrac{f(x) - f(0)}{x - 0} = \dfrac{f(x) - 2}{x} = 0.$

c) $f'(x) = \begin{cases} 2x & x < 0 \\ 0 & x = 0 \\ -2\sin x & x > 0 \end{cases}$

d) 

15. $\dfrac{dy}{dx} = \dfrac{y}{2y - x}$; $\dfrac{d^2y}{dx^2} = \dfrac{2y^2 - 2xy}{(2y - x)^3}$

16. $y = x + \dfrac{\pi}{4}$

17. a) $x^{x^2+4x}[x + 4 + (2x + 4)\ln x]$

b) $(1 + \sin(x))^x \left[\dfrac{x\cos(x)}{1 + \sin(x)} + \ln(1 + \sin(x))\right]$; $m = -\pi$

18. $\dfrac{\ln 2}{(\ln 2)^2 + 1}$

19. a) decreasing at 0.48 parts per million per mile

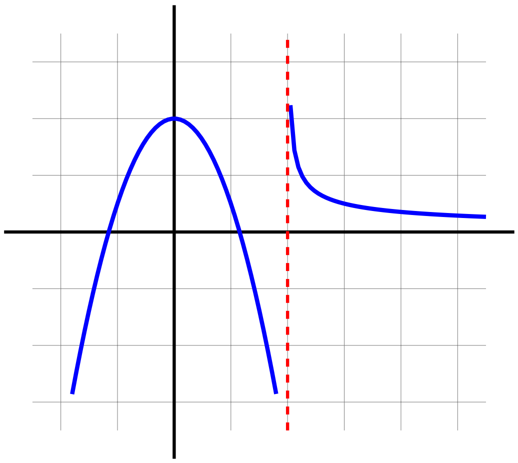
b) decreasing at 0.3 parts per million per mile

c) decreasing at $\dfrac{3}{32}$ parts per million per hour

20. a) 80000 gallons b) running out at 8000 gallons/min

c) approximately 72,000 gallons

d) draining at 10,000 gallons/min

21. 

22. a) False (consider $f(x) = |x|$ at $x = 0$ for example)

b) True c) False, $\dfrac{df}{dx}$ is undefined.

MAC 2311 Online: Review 1 Answer

1. $(f \circ g)(x) = \dfrac{x-1}{x+1}$; domain:$(-\infty, -1) \cup (-1, 0) \cup (0, \infty)$

2. $\left[0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{2}, \dfrac{3\pi}{4}\right)$

3. $\sin 2\theta = -\dfrac{12}{13}$ and $\cos 2\theta = \dfrac{5}{13}$

4. $-\dfrac{1}{2}$

5. $k = \dfrac{1-e}{2}$

6. b and d

7. even

8. a) $\dfrac{5}{4}$ b) $-\dfrac{\pi}{6}$ c) 0 d) 1 e) $-\dfrac{\pi}{3}$ f) 12

9. a) 2 b) $\dfrac{1}{2}$ c) $+\infty$

   $x = -1$: removable, $x = 0$: removable, $x = 1$: infinite; VA: $x = 1$

10. a) $\dfrac{\pi}{2}$ b) 3 c) 0 d) $-\dfrac{1}{2}$

11. a) VA: $x = \dfrac{1}{\sqrt{3}}$ ($x = -\dfrac{1}{\sqrt{3}}$ is an extraneous solution);

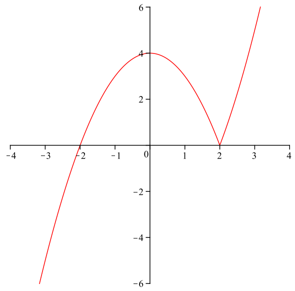
    HA: $y = -4$, $y = -\dfrac{4}{3}$

    b) VA: $x = \ln\left(\dfrac{3}{4}\right)$; HA: $y = \dfrac{1}{2}$, $y = 0$

12. a) domain: $(-\infty, 0) \cup (0, 4) \cup (4, \infty)$;

    VA: $x = 4$; hole at $x = 0$; HA: $y = 0$ (use the Squeeze Theorem to verify)

b) domain: $(-\infty, -1) \cup (0, \infty)$; VA: $x = -1$, $x = 0$; HA: $y = 0$; no holes

13. a) 

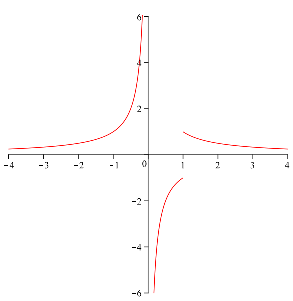
b) graph of $y = \sqrt{x}$ is shifted left three unites, reflected over the y-axis, reflected over the x-axis, and then shifted up 2 units; alternatively, reflect across the y-axis, shift right 3 units, reflect across the x-axis, and shift up 2 units

14. $x = \frac{\pi}{3}, \frac{5\pi}{3}$

15. $f(x) = \frac{2x - x^2}{(2x-1)^{\frac{5}{3}}}$; $x = 0$ and $x = 2$

16. a) does not exist b) 1

$f(x)$ has an infinite discontinuity at $x = 0$ and a jump discontinuity at $x = 1$.



17. $f^{-1}(x) = e^{x^3 - 2}$

domain, range of f : $(0, \infty)$, $(-\infty, \infty)$

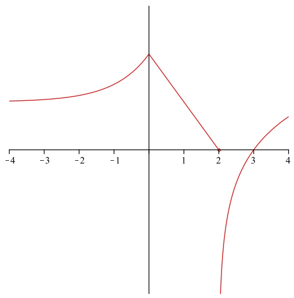
domain, range of f^{-1}: $(-\infty, \infty)$, $(0, \infty)$

18. a) $\frac{3\sqrt{7}}{7}$ b) $\frac{x\sqrt{4 - x^2}}{2}$

19. a) $(-\infty, 0) \cup (2, \infty)$ b) $(0, e^{3/2}) \cup (e^{3/2}, \infty)$

20. $x = \dfrac{5}{2}$ only

21. a)



b) 1, 2, 2, 1, 0, DNE, but $\lim_{x \to 2^+} f(x) = -\infty$

c) $x = 0$: removable, define $f(0) = 2$, $x = 2$: infinite, so a nonremovable discontinuity

22. $A(x) = x\left(1500 - \dfrac{3}{2}x\right)$

If area is 375,000 sq. ft., the dimensions are 500 ft by 750 ft. with $x = 500$.

23. a) 32 ft/sec b) $32 - 16h$ ft/sec c) 32 ft/sec