MAC 2311 Online: Final Exam Review

This review is based on the material from Module 5. You also need to review the material from Module 1–4 to prepare for the final exam.

1. Use the Right Endpoint Approximation to estimate the value of

$$\int_1^5 \sqrt{2x-1}\,dx$$

with the Riemann Sum using four subintervals of equal width and letting x_i be the right endpoint of the subinterval $[x_{i-1}, x_i]$.

2. Express the following limit as an integral: $\lim_{n\to\infty} \sum_{i=1}^{n} \left(1+\frac{2i}{n}\right)^3 \frac{2}{n}$

3. Write the given limit of Riemann sums as a definite integral:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\sqrt{1+\frac{9i^2}{n^2}}$$

4. Consider the area under the graph of $f(x) = 6x^2 - x^3$ on $[0, 2]$.

a) Find the Riemann sum which approximates the area using n subintervals of equal width with x_i = right endpoint of the subinterval $[x_{i-1}, x_i]$.

b) Find the exact area under the graph of $f(x) = 6x^2 - x^3$ on $[0, 2]$ by taking the limit of the Riemann sum as $n \to \infty$.

5. A particle moving along a line has acceleration function $a(t) = e^{2t}$ m/sec^2. The initial velocity is 3 m/sec and the initial displacement is 2 meters. Find its position function $s(t)$.

6. Evaluate each integral:

a) $\displaystyle\int \frac{\csc\theta}{\csc\theta - \sin\theta}\,d\theta$ (Hint: multiply each term in the fraction by $\sin\theta$.)

b) $\displaystyle\int_{-1/2}^{0} 3 - \frac{4}{\sqrt{1-x^2}}\,dx$

c) $\displaystyle\int_0^{1/\sqrt{3}} \frac{x^2-1}{x^4-1}\,dx$ (Hint: simplify the integrand.)

7. Evaluate each integral:

a) $\displaystyle\int \frac{3x}{x^2-4}\,dx$ b) $\displaystyle\int \frac{3x+3}{\sqrt{x^2+2x+5}}\,dx$ c) $\displaystyle\int_0^{\pi/6} \sec(2x)[3\tan(2x)+\sec(2x)]\,dx$

d) $\displaystyle\int_1^{e^2} \frac{(1+2\ln x)^2}{x}\,dx$ e) $\displaystyle\int \frac{\sin(2x)}{1+\cos^2 x}\,dx$ f) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}\,dx$

g) $\displaystyle\int_0^{\ln 2} \frac{e^{4x}-e^x}{e^{2x}}\,dx$ h) $\displaystyle\int \cot x\,dx$ i) $\displaystyle\int \frac{e^{\frac{1}{2x-1}}}{(2x-1)^2}\,dx$

j) $\displaystyle\int_{-2}^{0} x\sqrt{2x+4}\,dx$ k) $\displaystyle\int \frac{e^{2x}}{e^x-1}\,dx$ l) $\displaystyle\int_0^{\pi/2} \sin(\sin x)\cos x\,dx$

8. Find the maximum and minimum values of $f(x) = x\sqrt{2x+4}$ on $[-2,0]$ and use them to find upper and lower bounds for the definite integral $\displaystyle\int_{-2}^{0} x\sqrt{2x+4}\,dx$.

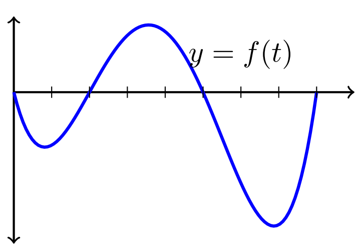
9. If $g(x) = \displaystyle\int_1^x [(\ln t)^2 + 2t]\,dt$, find $g'(e)$. Determine the intervals on which $g(x)$ is increasing and decreasing.

10. If $h(x) = \displaystyle\int_x^0 (3t^5 - 5t^3)\,dt$, find the intervals on which $h(x)$ is concave down.

11. Evaluate $\displaystyle\frac{d}{dx}\int_{\sqrt{x}}^{0} \frac{t^2}{t^2+2}\,dt$.

12. Evaluate $\displaystyle\lim_{n\to\infty}\sum_{i=0}^{n} \frac{i}{n^2}\, e^{(\frac{i}{n})^2}$ by first expressing it as a definite integral.

13. If $g(x) = \displaystyle\int_0^x f(t)\,dt$, where the graph of f on $[0,8]$ is shown below.



a) Determine the intervals on which $g(x)$ is increasing and decreasing.

b) Determine the x-value(s) where g has local extrema.

14. The slope of the tangent line to the curve $y = f(x)$ at any point is given by $2\sin\left(\frac{x}{2}\right)\cos^2\left(\frac{x}{2}\right)$. If $f(0) = 1$, find the function $f(x)$.

15. Find the area under the graph of $f(x) = \begin{cases} x^2 & x < 0 \\ e^{\frac{x}{3}} & x \geq 0 \end{cases}$ on $[-2, 3\ln 4]$.

16. Suppose that f is an even function and g is odd, both are continuous, and $g(x) \geq 0$ on $[0, 10]$. Be sure to make a sketch.

If $\displaystyle\int_5^{10} g(x)\,dx = 10$, $\displaystyle\int_0^5 g(x)\,dx = 5$, $\displaystyle\int_{-5}^5 f(x)\,dx = 12$ and $\displaystyle\int_0^{10} f(x)\,dx = 20$, find:

a) $\displaystyle\int_{-5}^{10} [f(x) + g(x)]\,dx$ b) the area of region bounded by the x-axis and $g(x)$ on $[-5, 10]$

17. Evaluate $\displaystyle\int_1^e f(x)\,dx$ if a) $f(x) = \dfrac{2}{x(3\ln x + 2)}$ and b) $f(x) = \dfrac{2}{x(3\ln x + 2)^2}$.

18. A soft drink dispenser pours a soft drink at the rate of $f(t) = \dfrac{20t}{1 + 2t^2}$ ml/sec, where t is the time elapsed in seconds. How much of the drink is dispensed in the first 2 seconds?

19. A particle moves along a straight line with velocity $v(t) = 3 - \dfrac{18}{1 + 2t}$ inches per second. Find the displacement and total distance traveled by the particle on the time interval $[0, 4]$.

20. A population is changing at the rate $\dfrac{dP}{dt} = 50t - 100t^{3/2}$ where P is the population in thousands after t years. Find the net change in population in the first 4 years ($t = 0$ to $t = 4$).

21. True or false:

(a) $\displaystyle\frac{d}{dx}\int_1^2 f(x)\,dx = 0$. Assume f is continuous on $[1, 2]$.

(b) $\displaystyle\int_1^x g'(t)\,dt = \frac{d}{dx}\int_1^x g(t)\,dt$

(c) $\displaystyle\int_{-1}^1 \frac{1}{x}\,dx = 0$

MAC 2311 Online: Exam 3 Review

1. Use differentials to approximate the value of $\sqrt{16.328}$.

2. Find each value of c that satisfies Rolle's Theorem on the following intervals. If not possible, state why.

a) $f(x) = \sin x + \cos^2 x$ on $[\pi, 2\pi]$

b) $f(x) = 2 - x^{2/3}$ on $[-1, 1]$

c) $f(x) = \tan x$ on $[0, \pi]$

3. Find each value of c implied by the MVT for

a) $f(x) = \dfrac{x}{x-5}$ on $[0, 4]$

b) $f(x) = x + \ln x$ on $[1, e]$

4. Suppose that $f(x)$ is a function so that $2 \leq f'(x) \leq 5$ for all x. Use the Mean Value Theorem to complete the inequality:

$$\_\_\_\_\_\_\_\_\_\_ \leq f(7) - f(1) \leq \_\_\_\_\_\_\_\_\_\_$$

5. Find all critical numbers and local extrema of $g(x) = \dfrac{1}{2\sqrt{x}} - \dfrac{1}{x^2}$.

6. Find the absolute maximum and minimum values of $f(x) = x^2 - 8\ln x$ on $[1, e]$.

7. Find each interval on which the graph of $f(x) = (x^2+1)e^x$ is both increasing and concave down. Find each inflection point.

8. If $f(x) = \dfrac{x^3}{(x-1)^2}$, show that $f'(x) = \dfrac{x^3 - 3x^2}{(x-1)^3}$ and find the absolute extreme values of $f(x)$ on the intervals $[2, 4]$ and $(1, \infty)$.

9. Suppose that $f(x)$ has horizontal tangent lines at $x = -2$, $x = 1$ and $x = 5$. If $f''(x) > 0$ on intervals $(-\infty, 0)$ and $(2, \infty)$ and $f''(x) < 0$ on the interval $(0, 2)$, find the x- values at which $f(x)$ has relative extrema. Assume that f and f' are continuous on $(-\infty, \infty)$.

10. Find the relative extrema and inflection points of $f(x) = \dfrac{\cos x}{\sin x + 2}$ on $[0, 2\pi]$. Note that $f'(x) = \dfrac{-2\sin x - 1}{(\sin x + 2)^2}$ and $f''(x) = \dfrac{2\cos x(\sin x - 1)}{(\sin x + 2)^3}$.

11. Find the following limits, using L'Hospital's rule if it applies. If it does, indicate the indeterminate form of your limit.

a) $\displaystyle\lim_{x\to\frac{\pi}{4}} \frac{\tan(3x) - 1}{1 - \sec^2(x)}$ b) $\displaystyle\lim_{x\to 0} \left(1 + \frac{2}{x}\right)^{4x}$ c) $\displaystyle\lim_{x\to 0^+} \left(\csc(x) - \frac{3}{x}\right)$

d) $\displaystyle\lim_{x\to 1^-} \frac{\ln(1 - x^2)}{\ln(1 - x)^2}$ e) $\displaystyle\lim_{x\to 1} (2 - x)^{\tan(\pi x/2)}$ f) $\displaystyle\lim_{x\to 0} \frac{\sin^2(x)}{e^{x/3} + 1}$

g) $\displaystyle\lim_{x\to\frac{\pi}{2}^-} (\tan x)^{\cos x}$

12. Find the most general antiderivative of the following:

a) $f(x) = \dfrac{(\sqrt{x} + 1)^2}{\sqrt{x}}$ on $(0, \infty)$

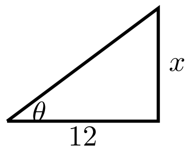
b) $g(x) = \dfrac{\cos x + \sec x}{\cot x}$ on $\left(0, \dfrac{\pi}{2}\right)$

c) $f(x) = \dfrac{x^4 + x^2 - 2}{x^2 + 1}$ Hint: use long division to rewrite the function.

d) $f(x) = 5x^3 + \csc^2 x + \dfrac{1}{x} + \sqrt{x}$

13. The slope of a curve $y = f(x)$ at any point is given by $f'(x) = \dfrac{6x - 1}{\sqrt{x}}$. If the curve passes through the point $(1, 1)$, find $f(4)$.

14. One side of a right triangle is known to be exactly twelve inches. The angle θ is measured to be $\frac{\pi}{4}$ radians with a maximum possible error of ± 0.02 radians. Use differentials to estimate the maximum error in calculating the length of the opposite side x.



15. a) The demand function for a product is $p(x) = 45 - \frac{\sqrt{x}}{2}$ where p is the price at which x items will sell. If the total revenue $R(x) = xp$, find the marginal revenue when 1600 items are produced.

b) Now suppose that when 1600 items are produced in a week, revenue increases by \$450 per week. At what rate is the production level x changing at that production level?

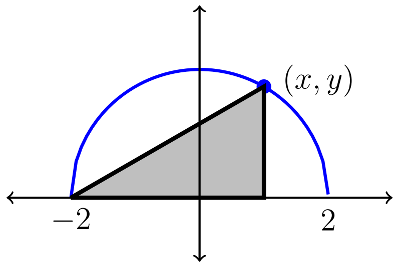
16. Use Newton's Method to find x_1 and x_2 with an initial guess $x_0 = 1$ when approximating $\sqrt[3]{4}$.

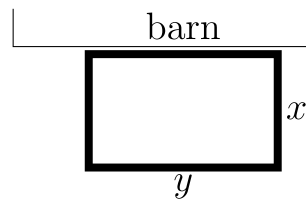
17. Find the x-coordinate of the point on the curve $y = \frac{x^2}{2}$ located at the shortest distance from the point $(1, 1)$.

18. A paper cup has the shape of an inverted cone with height 10 cm and radius 3 cm. If water is poured into the cup at a rate of 1.5 cm^3/sec, how fast is the water level rising when it is 5 cm deep?

19. A rectangular shipping crate is to be constructed with a square base. The material for the two square ends costs \$3 per square foot and the material for the sides costs \$2 per square foot. What dimensions will minimize the cost of constructing the crate if it must have a volume of 12 cubic feet? What is the minimum cost? Let x be the length of the side of a square end, and y be the height of the crate. Be sure to check your answer.

20. For what values of x and y will the right triangle inscribed in a semicircle of radius 2 have maximum area? What is the maximum area?



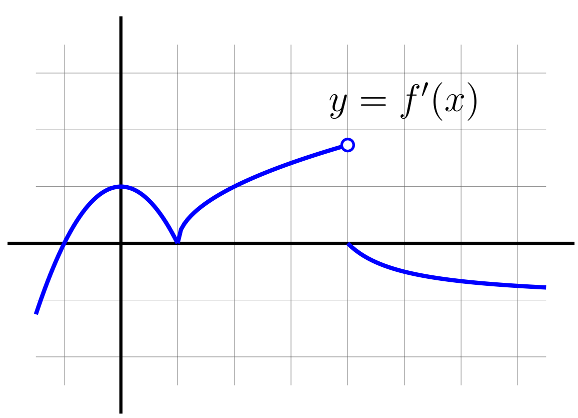


21. A farmer wishes to fence an area in front of his barn. He needs a wire fence that costs \$1 per linear foot in front of the barn and and wooden fencing that costs \$2 per foot on the other three sides. Find the lengths x (sides perpendicular to the barn) and y (side across from the barn) so that he can enclose the maximum area if his budget for materials is \$4400.

22. Sketch the graph of $f(x) = (x-1)^{\frac{1}{3}}(x+2)^{\frac{2}{3}}$.

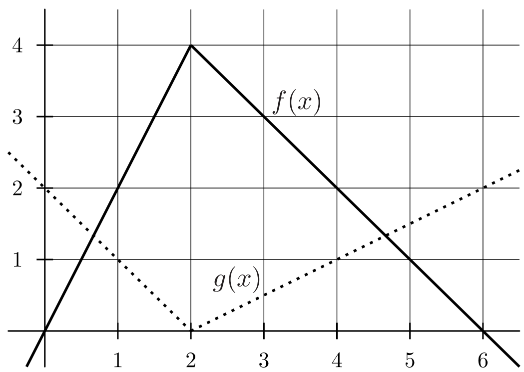
Note that $f'(x) = \dfrac{x}{(x-1)^{\frac{2}{3}}(x+2)^{\frac{1}{3}}}$ and $f''(x) = \dfrac{-2}{(x-1)^{\frac{5}{3}}(x+2)^{\frac{4}{3}}}$.

23. Given the graph of the derivative $f'(x)$, find a possible graph of the function $f(x)$ for $x \geq 0$. Assume that $f(-1) = -2$, $f(0) = 0$, $f(1) = 2$, $f(4) = 4$ and $f(6) = 0$, and that $f(x)$ is a continuous function.



MAC 2311 Online: Exam 2 Review

1. $f(x)$ and $g(x)$ have the graphs shown below. If $h(x) = f(g(x))$, find $h'(4)$.



2. Use the definition of derivative to evaluate: $\displaystyle\lim_{h\to 0} \frac{(32+h)^{4/5} - 16}{h}$

3. Given $f(x) = \dfrac{x^2 + x\sqrt{x}}{\sqrt{x}}$, find $f'(x)$.

4. Find the value of a so that the tangent line to $y = x^2 - 2\sqrt{x} + 1$ is perpendicular to the line $ay + 2x = 2$ when $x = 4$.

5. If $f(x) = \arcsin(2x)$, find an expression for $f''(x)$.

6. Evaluate the limits:

a) $\displaystyle\lim_{x\to 0} \frac{\sin x - x}{2x}$

b) $\displaystyle\lim_{\theta\to 0^-} \frac{\theta}{\cos\theta - 1}$

7. Use the definition of derivative to find: $\dfrac{d}{dx}\cos(2x)$

8. If $f(x) = \sqrt{2x-1}$, find:

a) $f'(x)$ using the **definition of derivative**,

b) each interval over which $f(x)$ is differentiable, and

c) the equation of the tangent line to $f(x)$ at $x = 5$.

9. Find the equation of all horizontal and vertical tangent lines of

a) $f(x) = x\sqrt[3]{3x-2}$ and b) $f(x) = \dfrac{2x-6}{\sqrt{x^2+3}}$.

Be sure to write $f'(x)$ as a single fraction.

10. The position (in centimeters) of a particle moving in a straight line at time t (in seconds) is given by $s(t) = t^3 - 6t^2 + 9t$ for $0 \le t \le 6$.

a) Find the velocity function $v(t)$.

b) At what time(s) is the particle at rest?

c) For what time interval(s) over the first six seconds is the particle traveling in a positive direction?

d) Find the average velocity from $t = 0$ to $t = 4$ seconds.

11. Suppose that $f(4) = 7$, $g(4) = 2$, $f(-4) = 1$, $g(-4) = 3$, $f'(4) = 10$, $g'(4) = 12$, $f'(-4) = 6$, and $g'(-5) = -2$. Find:

a) $h'(4)$ if $h(x) = g(f(x) - 3x)$ and b) $H'(4)$ if $H(x) = \sqrt{xf(x) + \dfrac{x^2}{2}}$.

12. Find each x-value on $[0, 2\pi)$ at which $f(x) = \dfrac{\cos x}{2 + \sin x}$ has a horizontal tangent line.

13. Let $f(x) = \begin{cases} xe^{\frac{\tan x}{x}} & x \neq 0 \\ 0 & x = 0 \end{cases}$.

Find $f'(0)$ if possible. Is f continuous at $x = 0$?

14. Let $f(x) = \begin{cases} 2 - x|x| & x < 0 \\ 2\cos x & x \geq 0 \end{cases}$.

a) Use the limit definition of continuity to show that $f(x)$ is continuous at $x = 0$.

b) Find $f'(0)$ if possible using the **limit definition** of derivative at a point.

c) Find an expression for $f'(x)$.

d) Sketch the graph of $f(x)$. Does it confirm your conclusions in part a and b?

15. Find $\dfrac{dy}{dx}$ if $xy - 2 = y^2$. Then find $\dfrac{d^2y}{dx^2}$ in terms of x and y.

16. Find the equation of the tangent line to $y = \tan^{-1}(e^{2x})$ at $x = 0$.

17. Use logarithmic differentiation to differentiate the following:

a) $y = x^{x^2+4x}$

b) $f(x) = (1 + \sin x)^x$. Then find the slope of the tangent line to $f(x)$ at $x = \pi$.

18. If $g(x)$ is the inverse of $f(x) = 2^x + \log_2(x + 1)$, find $g'(1)$.

19. A pollutant from a factory is carried away by wind currents. Its concentration in the air x miles from the factory is given by $P(x) = \dfrac{1.6}{3x - 2}$ where $P(x)$ is measured in parts per million.

a) What is the average rate at which the concentration is changing when the pollutant has moved from 1 to 4 miles away from the factory? Include units in your answer.

b) At what rate is the concentration changing when the pollutant is two miles from the factory? Include units.

c) Now suppose that the distance of the pollutant from the factory is given by $x(t) = t^2 + t$ where t is measured in hours. Find the rate at which the concentration of the pollutant is changing with respect to time after 2 hours.

20. The number of gallons of water in a tank t minutes after the tank has started to drain is given by $Q(t) = 200(30 - t)^2$.

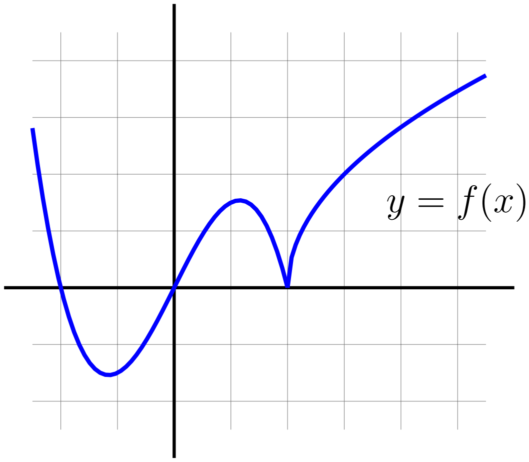
a) How much water is in the tank 10 minutes after it has started to drain?

b) How fast is the water running out of the tank at $t = 10$ minutes?

c) Use part a and b to approximate the amount of water in the tank one minute later($t = 11$).

d) What is the average rate at which the tank is draining during the first ten minutes ($t = 0$ to $t = 10$)?

21. Sketch a possible graph of the derivative of the function sketched below.



22. True or false:

a) If f is continuous at $x = a$, then f is differentiable at $x = a$.

b) If f is not continuous at $x = a$, then f is not differentiable at $x = a$.

c) If f has a vertical tangent line at $x = a$, then $\dfrac{df}{dx} = 0$ at $x = a$.

MAC 2311 Online: Exam 1 Review

1. Let $f(x) = \dfrac{1-x}{1+x}$ and $g(x) = \dfrac{1}{x}$. Find $(f \circ g)(x)$ and its domain.

2. Solve for x in $[0, \pi]$: $2\cos(x) > \sec(x)$

3. If $\tan(\theta) = -\dfrac{2}{3}$ and θ is in quadrant IV, find $\sin(2\theta)$ and $\cos(2\theta)$.

4. Evaluate $\lim\limits_{x \to 2} f(x)$ if $f(x) = \begin{cases} \dfrac{\frac{2}{x} - 1}{x-2} & x \neq 2 \\ 3 & x = 2 \end{cases}$.

5. Find the value of k so that $f(x) = \begin{cases} x^2 - \ln x + 2k & x \leq e \\ x^2 - x & x > e \end{cases}$

is continuous for all positive numbers.

6. The Intermediate Value Theorem guarantees a solution to the equation

$x^3 - \dfrac{1}{x} + 3 = 5x$ on which of the following intervals?

a. $[-1, 1]$ b. $[1, 3]$ c. $[3, 5]$ d. $[-3, -2]$

7. Determine if the function $f(x) = \dfrac{x - \tan x}{x}$ is even, odd, or neither.

8. Find the limits:

a) $\lim_{x\to 2} \frac{\sqrt{x^2+6x}-4}{x-2}$

b) $\lim_{x\to 0} \sin^{-1}\left(x - \frac{e^x}{2}\right)$

c) $\lim_{x\to 0^-} e^{\frac{2}{x}}$

d) $\lim_{x\to -\infty} e^{\frac{2}{x}}$

e) $\lim_{x\to 3} \tan^{-1}\left(\frac{3-x}{2\sqrt{x}-2\sqrt{3}}\right)$

f) $\lim_{x\to 2^+} \frac{x^2+8x-20}{|2-x|}$

9. If $f(x) = \frac{x^3+3x^2+2x}{x-x^3}$, find

a) $\lim_{x\to 0^+} f(x)$ b) $\lim_{x\to -1^+} f(x)$ and c) $\lim_{x\to 1^-} f(x)$.

List all discontinuities and describe as infinite, jump, or removable and find each vertical asymptote of $f(x)$.

10. Evaluate the limits:

a) $\lim_{x\to 3^+} \arctan\left(\frac{4}{x-3}\right)$

b) $\lim_{x\to -\infty} \frac{\sqrt{9x^2-5}}{2-x}$

c) $\lim_{x\to\infty} (e^{-x}\sin x)$

d) $\lim_{x\to\infty} \frac{\sin x - x}{2x}$

11. Find each vertical and horizontal asymptote of the following functions:

a) $f(x) = \frac{4x}{\sqrt{x^2+1}-2x}$

b) $f(x) = \frac{2e^x}{4e^x-3}$

12. Find the domain and all vertical & horizontal asymptotes and holes for the following:

a) $f(x) = \frac{x\sin x}{x^2-4x}$

b) $f(x) = \ln\left(\frac{x}{x+1}\right)$

13. Sketch the following graphs:

a) $y = (x+2)|x-2|$ b) If $f(x) = \sqrt{x}$, graph $g(x) = 2 - f(3-x)$.

14. Solve for x in $[0, 2\pi)$: $\cos(2x) + 5\cos(x) = 2$

15. Let $f(x) = \dfrac{(2x+1)^{1/3} - (x^2+1)(2x+1)^{-2/3}}{2x+1}$. Simplify the function and solve the equation $f(x) = 0$.

16. If $f(x) = \dfrac{|1-x|}{x^2 - x}$, find a) $\lim_{x\to 0} f(x)$ and b) $\lim_{x\to 1^+} f(x)$. Find and describe each discontinuity of $f(x)$ (jump, infinite or removable). Sketch the graph of $f(x)$.

17. Find the inverse of $f(x) = \sqrt[3]{2 + \ln x}$, and the domain and range of f and f^{-1}.

18. Use triangles to evaluate: a) $\tan\left(\sin^{-1}\dfrac{3}{4}\right)$ b) $\sin\left(2\cos^{-1}\dfrac{x}{2}\right)$

19. Find the domain: a) $f(x) = \ln\left(x^2 - \dfrac{8}{x}\right)$ b) $f(x) = \dfrac{x}{3 - 2\ln x}$

20. Find the solution set of the following: $\log_3(2x^2 - 5) - \log_3 x = 1$.

21. Consider the function $f(x) = \begin{cases} e^x + 1 & x < 0 \\ 2 - x & 0 < x < 2 \\ \ln(x-2) & x > 2 \end{cases}$

a) Sketch the graph of $f(x)$.

b) Find the following limits if possible.

$\lim_{x \to -\infty} f(x) =$ ____________ $\lim_{x \to 0^+} f(x) =$ ____________

$\lim_{x \to 0} f(x) =$ ____________ $\lim_{x \to 1} f(x) =$ ____________

$\lim_{x \to 2^-} f(x) =$ ____________ $\lim_{x \to 2} f(x) =$ ____________

c) List all discontinuities of $f(x)$ and state whether they are removable or nonremovable.

22. A farmer plans to spend $6000 to enclose a rectangular field with two types of fencing. Two opposite sides will require heavy-duty fencing that costs $3 per foot, while the other sides can be constructed with standard fencing that costs $2 per foot. Express the area of the field, A, as a function of x, the length of a side that requires heavy-duty fencing. If the area of the field is 375,000 square feet, what are its dimensions?

23. If an object is projected upward from the roof of a 200 foot building at 64 ft/sec, its height h in feet above the ground after t seconds is given by $h(t) = 200 + 64t - 16t^2$. Find the following:

a) The average velocity of the object from time $t = 0$ until it reaches its maximum height (hint: consider the graph of the function)

b) The average velocity of the object on the time interval $[1, 1 + h]$

c) The instantaneous velocity of the object at time $t = 1$ second