MAC2313, Calculus III
Final Exam Review (Lectures 27–34)

The formula sheet (page L34-10) will be provided on the test. The review below is not designed to be comprehensive, but to be representative of the topics covered on the exam.

1. Find div$\vec{F}$ and curl$\vec{F}$. Determine whether the vector filed $\vec{F}$ is conservative and find its potential function f if it is conservative.

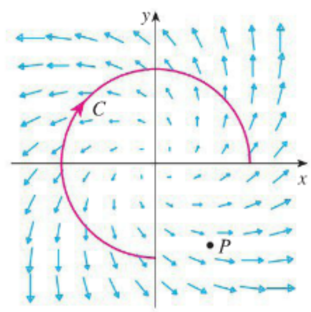
(1) $\vec{F} = \langle y, x+z, y \rangle$ (2) $\vec{F} = \langle 4xe^z, \cos(y), 2x^2e^z \rangle$

(3) $\vec{F} = \langle xe^{2x}, ye^{2z}, ze^{2y} \rangle$ (4) $\vec{F} = \left\langle \dfrac{y}{1+x^2}, \arctan(x), 2z \right\rangle$

2. A vector field $\vec{F}$, a curve C and a point P are shown.

(1) Is $\int_C \vec{F} \cdot d\vec{r}$ positive, negative, or zero?

(2) Is div $\vec{F}$ at P positive, negative, or zero?



3. Evaluate the line integral:

(1) $\int_C y\,dx + (x+y^2)\,dy$ if C is the ellipse $4x^2+9y^2=36$ with counterclockwise orientation.

(2) $\int_C (x^2+y^2+z^2)\,ds$ if C is the the curve $\vec{r}(t) = \langle t, \cos(2t), \sin(2t) \rangle$, $0 \leq t \leq 2\pi$.

(3) $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle (1+xy)e^{xy}, x^2e^{xy} \rangle$ and C is a curve moving from $(1,0)$ to $(0,2)$.

(4) $\int_C \left(y + e^{\sqrt{x}}\right) dx + \left(2x + \cos y^2\right) dy$, where C is the positively oriented boundary curve of the region enclosed by $y = x^2$ and $x = y^2$.

(5) $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = \langle x + yz, 2yz, x - y\rangle$ and C is the intersection of $x^2 + y^2 = 4$ and $x + y + z = 1$ with counterclockwise orientation when viewed from above.

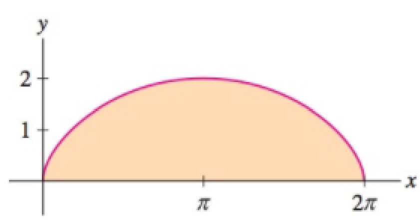
4. Evaluate the surface integral:

(1) $\iint_S (x^2z + y^2z)dS$, where S is the part of the plane $z = 4 + x + y$ that lies inside the cylinder $x^2 + y^2 = 4$.

(2) $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = \langle xz, -2y, 3x\rangle$ and S is the sphere $x^2 + y^2 + z^2 = 4$ with outward orientation.

(3) $\iint_S \vec{F} \cdot d\vec{S}$, where $\vec{F} = \langle x^2, xy, z\rangle$ and S is the part of paraboloid $z = x^2 + y^2$ below the plane $z = 1$ with upward orientation.

5. Find the area of the region between the x-axis and the cycloid $x = t - \sin t,\ y = 1 - \cos t,\ 0 \le t \le 2\pi$.



6. Consider the parametric surface S: $\vec{r}(u, v) = \langle v^2, -uv, u^2\rangle,\ 0 \le u \le 3,\ -3 \le v \le 3$.

(1) Find an equation of the tangent plane to the surface S at the point $(4, -2, 1)$.

(2) Set up an integral for the surface area of S.

7. Is there a vector field $\vec{G}$ on $\mathbb{R}^3$ such that curl $\vec{G} = \langle x, y, z\rangle$?

8. Find the work done by the force field $\vec{F} = \langle z, x, y \rangle$ in moving a particle from the point $(3, 0, 0)$ to the point $(0, \pi/2, 3)$ along

(1) a straight line (2) the helix $x = 3\cos t,\ y = t,\ z = 3\sin t$

9. Let $\vec{F} = \langle x^2 - y^2, 2xy \rangle$ be the velocity field of a two-dimensional fluid flow. If D is the region in the first quadrant bounded by $y = \sqrt{1 - x^2},\ x = 0$, and $y = 0$ with its boundary ∂D oriented counterclockwise, find:

(1) the circulation of $\vec{F}$ around the curve ∂D

(2) the flux of $\vec{F}$ through the curve ∂D

10. Compute the flux of the vector field $\vec{F}$ across the given surface.

(1) $\vec{F} = \langle \sin(y), \sin(z), yz \rangle$; S is the rectangular surface $0 \le y \le 2,\ 0 \le z \le 3$ in the yz-plane with a normal vector pointing in the negative x-direction

(2) $\vec{F} = \langle -x, -y, z^3 \rangle$; S is the part of the cone $z = \sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 3$ with downward orientation

(3) $\vec{F} = \langle 2x^3 + y^3, y^3 + z^3, 3y^2 z \rangle$; S is the surface of the solid bounded by paraboloid $z = 1 - x^2 - y^2$ and the xy-plane.

11. True or False:

(1) If $\vec{F} = \langle P, Q \rangle$ and $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ in an open region D, then $\vec{F}$ is conservative.

(2) $\displaystyle\int_{-C} f(x, y)\, ds = -\int_C f(x, y)\, ds.$

(3) $\displaystyle\int_{-C} \vec{F} \cdot d\vec{r} = -\int_C \vec{F} \cdot d\vec{r}.$

(4) If S is a sphere and $\vec{F}$ is a constant vector field, then $\displaystyle\iint_S \vec{F} \cdot d\vec{S} = 0.$

(5) The area of the region bounded by the positively oriented, piecewise smooth, simple closed curve C is $\displaystyle\oint_C y\, dx.$

(6) The flux of curl $\vec{F}$ through every oriented surface is zero.