MAC 2233
Fall 2019

Final Exam A

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)
2) UF ID Number
3) Section Number

C. Under "special codes", code in the test ID number 4, 1.

1 2 3 ● 5 6 7 8 9 0

● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E.
1) This test consists of 20 five-point multiple choice questions.
2) The time allowed is 120 minutes.
3) You may write on the test.
4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. **KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.**

G. When you are finished:

1) Before turning in your test, check for transcribing errors. Any mistakes you leave in are there to stay.
2) Bring your test, scratch paper, bubble sheet, and any tearoff sheets to your discussion leader or proctor to turn them in. Be prepared to show your UF ID card.
3) Answers will be posted in CANVAS after the test.

Multiple Choice are worth 5 points each.

1. If $f(x) = \frac{x^4}{4} - \frac{x^3}{3}$, the graph of f is both **decreasing** and **concave up** on which of the following intervals?

   A. $(0, 1)$

   B. $\left(-\infty, \frac{2}{3}\right)$

   C. $\left(-\infty, 0\right) \cup \left(\frac{2}{3}, \infty\right)$

   D. $\left(-\infty, 0\right) \cup \left(\frac{2}{3}, 1\right)$

   E. $(-\infty, 0)$

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2. Find the absolute maximum and minimum values of $f(x) = \frac{8x}{x^2+1}$ on $[0, 2]$.

   A. $4, 0$ B. $4, -4$ C. $\frac{16}{5}, -4$ D. $\frac{16}{5}, 0$ E. $4, \frac{16}{5}$

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3. The demand and cost functions for a certain product are $p(x) = 750 - 0.5x^2$ and $C(x) = 150x + 37,500$. Find the production level x that will yield maximum profit. What price should be charged for the product?

   A. $x = 20$; price is \$295 B. $x = 15$; price is \$638 C. $x = 10$; price is \$700

   D. $x = 15$; price is \$480 E. $x = 20$; price is \$550

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4. Evaluate $\int_1^{e^3} \frac{\sqrt{1+\ln(x)}}{x}\, dx$.

   A. 3 B. $\frac{14}{3}$ C. $\frac{16}{3}$ D. $\frac{3}{e^2}$ E. 7

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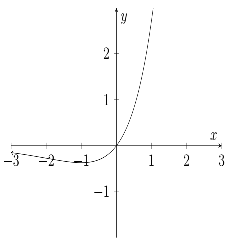
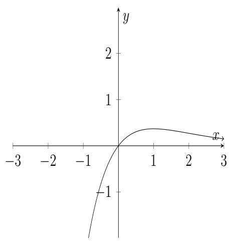
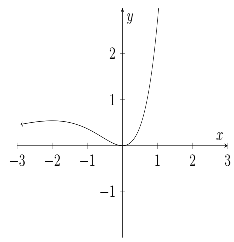
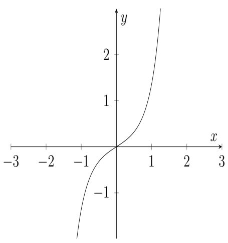
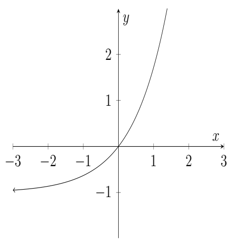
5. Which graph best represents $f(x) = xe^x$? Note: $f'(x) = e^x(x+1)$ and $f''(x) = e^x(x+2)$.

A. 

B. 

C. 

D. 

E. 

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6. Find the value of K so that the function $f(x) = \begin{cases} 6x + K & x \leq \frac{1}{2} \\ \dfrac{6x^2 - 3x}{2x^2 + x - 1} & x > \frac{1}{2} \end{cases}$

is continuous at $x = \frac{1}{2}$.

A. -1 B. 0 C. 2 D. -2 E. 1

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7. Evaluate $\lim\limits_{x \to -3} \dfrac{2 - \sqrt{1-x}}{x+3}$.

A. $\frac{1}{4}$

B. 4

C. 1

D. 0

E. The limit does not exist.

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8. The horizontal tangent lines of $f(x) = \dfrac{\sqrt{2x-1}}{x}$ occur at which values of x?

A. $\dfrac{1}{2}$ only

B. $\dfrac{2}{3}$ only

C. 1 only

D. $\dfrac{1}{2}$ and 1 only

E. $\dfrac{1}{2}$ and $\dfrac{2}{3}$ only

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9. Write the equation of the tangent line to $f(x) = \dfrac{x^4-4}{x^2}$ at $x = 2$.

A. $y = 3$

B. $y = 8x - 13$

C. $y = 5x - 13$

D. $y = 8x - 7$

E. $y = 5x - 7$

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10. Find the slope of the tangent line to $y^2 \ln x = 3y - 2$ at the point $(e, 1)$.

A. $2 + \dfrac{1}{e}$

B. $\dfrac{1}{e}$

C. $\dfrac{e}{2}$

D. $e - 1$

E. $\dfrac{2}{e} - 3$

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11. If $f(x) = \begin{cases} -1 & x \leq 0 \\ 3 - x & 0 < x < 2 \\ \dfrac{1}{3-x} & x > 2 \end{cases}$, then which of the following statements is/are true?

I. $\lim_{x \to 3^+} f(x) = -\infty$

II. $f(x)$ is continuous at $x = 0$.

III. $\lim_{x \to 0^+} f(x) = f(0)$.

IV. $f(x)$ can be made continuous at $x = 2$ by defining $f(2) = 1$.

A. II and III

B. I and II

C. I and IV

D. I, III, and IV

E. II and IV

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12. If $f(x) = \dfrac{x^2 - 3x - 10}{x^2 + 2x}$, let $p = \lim_{x \to \infty} f(x)$ and let $q = \lim_{x \to 0^+} f(x)$. Then

A. $p = -\infty$ and $q = +\infty$

B. $p = +\infty$ and $q = -5$

C. $p = 1$ and $q = +\infty$

D. $p = 1$ and $q = -\infty$

E. $p = +\infty$ and $q = 0$

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13. The acceleration of an object moving in a straight line is given by $a(t) = 6t - 2$. If the initial velocity of the object is 3 feet per second and the position of the object at $t = 1$ second is 5 feet, find the position of the object at $t = 2$ seconds.

A. 10 feet B. 12 feet C. 9 feet D. 11 feet E. 8 feet

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14. If $f(x) = \ln x$ and $g(x) = \dfrac{e^x}{e^x - 2}$, find $(g \circ f)(x)$ and its domain.
Consider the domain of the individual functions as necessary.

A. $(g \circ f)(x) = \dfrac{x}{x-2}$ domain: $(-\infty, 2) \cup (2, \infty)$

B. $(g \circ f)(x) = x - \ln(e^x - 2)$ domain: $(0, \ln 2) \cup (\ln 2, \infty)$

C. $(g \circ f)(x) = \dfrac{e^x \ln x}{e^x - 2}$ domain: $(0, \ln 2) \cup (\ln 2, \infty)$

D. $(g \circ f)(x) = x - \ln(e^x - 2)$ domain: $(\ln 2, \infty)$

E. $(g \circ f)(x) = \dfrac{x}{x-2}$ domain: $(0, 2) \cup (2, \infty)$

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15. A spherical balloon is inflated with a gas at a rate of 20 ft^3/min. How fast is the radius of the balloon changing with the radius is 3 feet? (Recall: $V = \frac{4}{3}\pi r^3$.)

A. $\dfrac{5}{9\pi}$ ft/min B. $\dfrac{9\pi}{5}$ ft/min C. $\dfrac{5}{6\pi}$ ft/min

D. $\dfrac{20}{9\pi}$ ft/min E. $\dfrac{5\pi}{9}$ ft/min

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16. Find the slope of the tangent line to $f(x) = x^{4x}$ at $x = e$.

A. $4e^{4e-1}$ B. 8 C. $5e^{4e}$ D. 4 E. $8e^{4e}$

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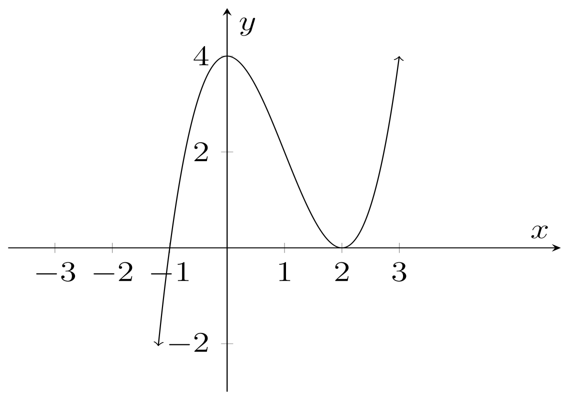
17. The graph of the **Second Derivative**, $f''(x)$, of a continous function is sketched below:



Which of the following statements is/are true?

I. According to the Second Derivative Test, if $f'(1) = 0$, then $f(x)$ has a relative minimum at $x = 1$.

II. $f(x)$ is concave down on the interval $(-\infty, -1)$ only.

III. $f(x)$ has inflection points at both $x = -1$ and $x = 2$.

A. II only B. I and II only C. II and III only

D. I and III only E. I, II and III

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18. Evaluate $\displaystyle\int \frac{x}{\sqrt[3]{3x^2+1}}\,dx$.

A. $\dfrac{1}{9(3x^2+1)^{4/3}} + C$ B. $9(3x^2+1)^{2/3} + C$ C. $\dfrac{(3x^2+1)^{2/3}}{4} + C$

D. $4\ln(3x^2+1) + C$ E. $\dfrac{\ln(3x^2+1)}{6} + C$

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19. The slope of the curve $y = f(x)$ at any point is $\dfrac{(x-2)^2}{x}$. If the curve passes through the point $(1, 1/2)$, find $f(x)$.

A. $\dfrac{1}{2}x^2 + 4\ln|x|$

B. $\dfrac{1}{2}x^2 - 4x + 4\ln|x| + 4$

C. $\dfrac{1}{2}x^2 - 4x + 4\ln|x| - \dfrac{1}{2}$

D. $\dfrac{1}{2}x^2 - 4x + \ln|4x| + 4$

E. $2x^2 - 4x + 4\ln|x| + \dfrac{5}{2}$

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20. Evaluate $\lim_{h\to 0} \dfrac{\dfrac{2}{(x+h)^5} - \dfrac{2}{x^5}}{h}$. Hint: Consider the definition of the derivative.

A. $\dfrac{2}{x^5}$ B. $-\dfrac{1}{10x^5}$ C. $\dfrac{1}{2x^6}$ D. $-\dfrac{10}{x^6}$ E. $-\dfrac{2}{5x^6}$

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21. Approximate the value of $\int_1^5 \sqrt{2x+1}\,dx$ using four rectangles of equal width and the midpoints of each subinterval to find the height.

A. $\sqrt{5}+\sqrt{7}+3+\sqrt{11}$

B. $\frac{1}{2}(\sqrt{5}+\sqrt{7}+3+\sqrt{11})$

C. $\sqrt{3}+\sqrt{5}+\sqrt{7}+3$

D. $\frac{1}{2}(\sqrt{6}+\sqrt{8}+\sqrt{10}+\sqrt{12})$

E. $2+\sqrt{6}+\sqrt{8}+\sqrt{10}$

22. If $f(x) = \begin{cases} 2-|x| & x<0 \\ \sqrt{4-x^2} & x \geq 0 \end{cases}$, evaluate $\int_{-2}^{2} f(x)\,dx$ using geometric areas. Be sure to sketch the graph of $f(x)$ and the corresponding region.

a. $2+\pi$ b. $1+\pi$ c. $4+2\pi$ d. $1+2\pi$ e. $2+4\pi$

Merry Christmas and Happy Holidays!