MAC 1147 — Fall 2021 — EXAM 4A

Questions 1–23 are worth 4 points each.

1. Select the equation that is *not* a Pythagorean identity.

   A. $\sin^2\theta + \cos^2\theta = 1$

   B. $1 + \cot^2\theta = \csc^2\theta$

   C. $1 + \sec^2\theta = \cot^2\theta$

   D. $\tan^2\theta + 1 = \sec^2\theta$

   E. All of these are Pythagorean identities.

$(\tan\theta + 1)^2 = 0$

$\tan\theta = -1$

$\theta = \frac{3\pi}{4}$

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2. Suppose that $\frac{\pi}{2} < \theta < \pi$ and $\tan^2\theta + 2\tan\theta + 1 = 0$. What is the value of $\sin\theta + \cos\theta$?

   A. 0 B. $\sqrt{2}$ C. $-\sqrt{2}$ D. $\frac{1+\sqrt{3}}{2}$ E. $\frac{1-\sqrt{3}}{2}$

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3. Find all solutions in the interval $[0, 2\pi)$

$-2\sin^2\theta = -6$

$$-2\sin^2\theta + 7 = 1$$

$\sin^2\theta = 3$

A. $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

B. $\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

C. $\theta = 0, \pi$

D. $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

E. There are no solutions.

$\sin\theta = \pm\sqrt{3}$

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4. $\sin^4 x - \cos^4 x =$

$(\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$= (1 - \cos^2 x - \cos^2 x) =$

A. $1 - 2\cos^2 x$

B. $1 - 2\sin^2 x$

C. $1 - 2\tan^2 x$

D. $1 - 2\cot^2 x$

E. $1 - 2\csc^2 x$

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5. Select the expression that is equivalent to $\cos\left(\frac{2\pi}{9}\right)$

$= \sin\left(\frac{\pi}{2} - \frac{2\pi}{9}\right) = \sin\left(\frac{5\pi}{18}\right)$

A. $-\sin\left(\frac{5\pi}{18}\right)$ B. $\cos\left(\frac{5\pi}{18}\right)$ C. $\sin\left(\frac{5\pi}{18}\right)$ D. -cos $\left(\frac{5\pi}{18}\right)$ E. $\sin\left(-\frac{5\pi}{18}\right)$

$5\pi = \frac{2\pi}{b} \to b = \frac{2}{5}$

$\frac{2\pi}{3} = \frac{c}{b} = \frac{c}{2/5} \to c = \frac{4\pi}{15}$

6. Identify the function that has period 5π and phase shift $\frac{2\pi}{3}$.

A. $f(x) = \cos\left(\frac{2}{5}x - \frac{4\pi}{15}\right)$ B. $f(x) = \cos\left(\frac{1}{5}x - \frac{4\pi}{15}\right)$ C. $f(x) = \cos\left(\frac{2}{5}x - \frac{4}{15}\right)$

D. $f(x) = \cos\left(\frac{1}{5}x - \frac{4}{15}\right)$ E. None of these.

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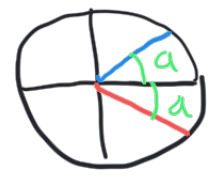
7. Suppose that $0 < \theta < \frac{\pi}{2}$ and $\cos\theta = \cos\left(\frac{8\pi}{5}\right)$. What is the value of θ?



$a = \frac{2\pi}{5}$

A. $\frac{\pi}{5}$ B. $\frac{2\pi}{5}$ C. $\frac{3\pi}{5}$ D. $\frac{4\pi}{5}$ E. $\frac{11\pi}{5}$

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8. $\sin\left(\arcsin\left(-\frac{4}{9}\right)\right) =$

A. $\frac{4}{9}$

B. $-\frac{4}{9}$

C. $\frac{9}{4}$

D. $-\frac{9}{4}$

E. The expression is undefined.

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9. Select the *true* statement.

A. The graph of $f(x) = \sin x$ is the same as the graph of $g(x) = \cos(x + \frac{\pi}{2})$.

B. $f(\theta) = -2\cos(3\theta - 2) + 4$ and $g(\theta) = 4\csc(3\theta - 1) + 2$ have the same period.

C. If $f(x) = a\sin x$ then a is the amplitude of the graph for any value of a.

D. $f(x) = x \cdot \cos(x)$ has an x-intercept at $x = \pi$.

E. The range of $f(x) = 5\sin(3x - 2) + 1$ is $[-5, 5]$.

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10. What is the range of $f(x) = \arccos(x)$?

A. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ B. $[-1, 1]$ C. $(-\infty, \infty)$ D. $[0, \pi]$ E. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

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11. Select the expression that is equivalent to $\frac{1+\cos x}{1-\cos x}$

$= \frac{(1+\cos x)^2}{1-\cos^2 x} = \frac{(1+\cos x)^2}{\sin^2 x}$

A. $(\csc x + \cot x)^2$

B. $(\sec x + \cot x)^2$

C. $(\csc x + \tan x)^2$

D. $(\sec x + \tan x)^2$

E. $(\csc x + \sec x)^2$

$= \left(\frac{1+\cos x}{\sin x}\right)^2 = (\csc x + \cot x)^2$

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12. How many solutions does the equation below have in the interval $[0, 2\pi)$?

$$4\sin^3 x - 2\sin x = 0$$

F. 6

A. 1 B. 2 C. 3 D. 4 E. 5

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13. Find an expression for all solutions to the equation.

$$5\sin(3x) - 5 = 0$$

$\sin(3x) = 1$

A. $x = \frac{\pi}{6} + 2\pi k$, k an integer

B. $x = \frac{\pi}{3} + \frac{2\pi}{3}k$, k an integer

C. $x = \frac{\pi}{2} + 2\pi k$, k an integer

D. $x = \frac{\pi}{6} + \frac{2\pi}{3}k$, k an integer

E. $x = \frac{\pi}{2} + \frac{2\pi}{3}k$, k an integer

$3x = \frac{\pi}{2} + 2\pi k$

$x = \frac{\pi}{6} + \frac{2\pi}{3}k$

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14. Select the function that has the same graph as $f(x) = 2\cot(x) + 4$

A. $g(x) = 2\tan\left(x - \frac{\pi}{2}\right) + 4$

B. $g(x) = 2\tan\left(x + \frac{\pi}{2}\right) + 4$

C. $g(x) = 2\tan\left(x - \frac{\pi}{2}\right) - 4$

D. $g(x) = -2\tan\left(x - \frac{\pi}{2}\right) + 4$

E. $g(x) = -2\tan\left(x + \frac{\pi}{2}\right) - 4$

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15. What are the x-intercepts of $f(x) = \tan(x)$?

$\tan 0 = 0$

$\text{Per } \tan x = \pi$

A. $x = 0 + \pi k$ where k is an integer

B. $x = \frac{\pi}{2} + \pi k$ where k is an integer

C. $x = 0 + 2\pi k$ where k is an integer

D. $x = \frac{\pi}{2} + 2\pi k$ where k is an integer

E. $f(x)$ does not have any x-intercepts

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16. Simplify the expression to a single trigonometric function:

$$\frac{1+\tan t}{1+\cot t} = \frac{1+\tan t}{1+\frac{1}{\tan t}} = \frac{\tan t(1+\tan t)}{\tan t + 1}$$

A. $\tan t$ B. $\sin t$ C. $\sec t$ D. $\cot t$ E. $\cos t$

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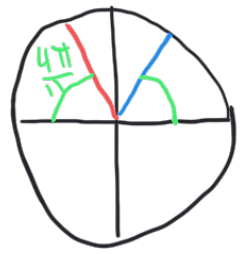
17. $\arcsin\left(\sin\left(\frac{7\pi}{11}\right)\right) =$



A. $-\frac{4\pi}{11}$ B. $\frac{4\pi}{11}$ C. $\frac{7\pi}{11}$ D. $\frac{\pi}{11}$ E. $-\frac{7\pi}{11}$

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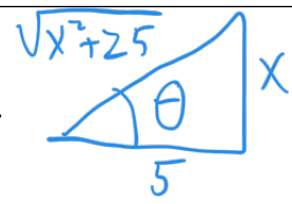
18. Rewrite $\cos\left(\arctan\left(\frac{x}{5}\right)\right)$ as an algebraic expression.



A. $\frac{x}{\sqrt{x^2+25}}$ B. $\frac{\sqrt{25-x^2}}{x}$ C. $\frac{5}{\sqrt{x^2+25}}$ D. $\frac{\sqrt{25-x^2}}{5}$ E. $\frac{\sqrt{x^2+25}}{x}$

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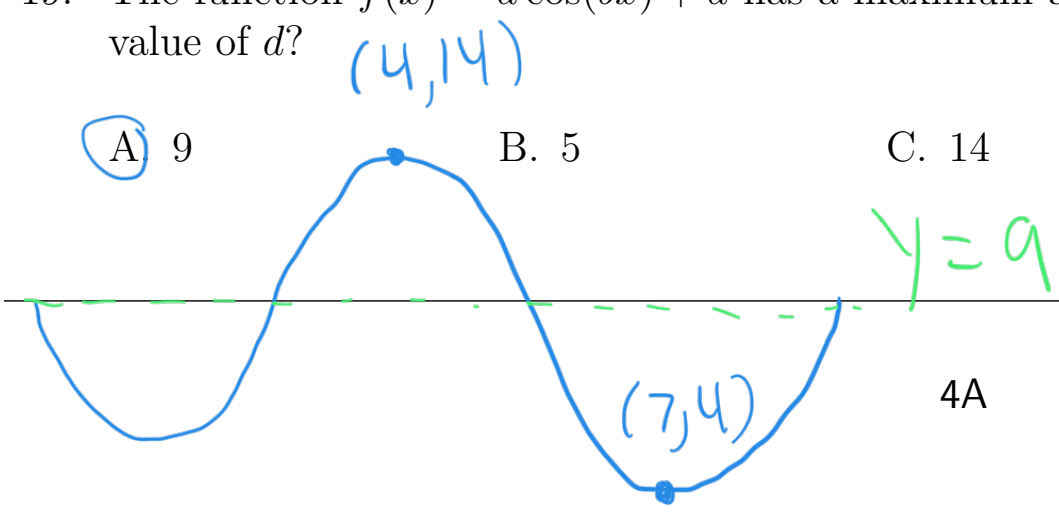
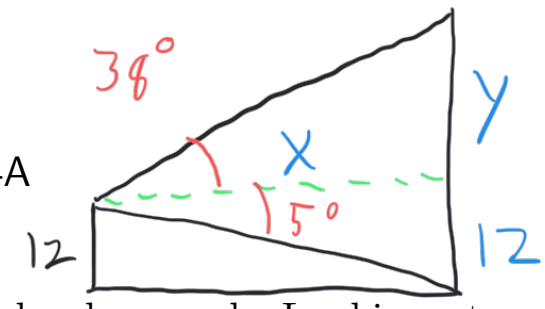
19. The function $f(x) = a\cos(bx) + d$ has a maximum at $(4, 14)$ and a minimum at $(7, 4)$. What is the value of d?



A. 9 B. 5 C. 14 D. 4 E. 0

$\tan 5^\circ = \frac{12}{x}$ $x = \frac{12}{\tan 5^\circ}$



20. You are standing on top of a 12-foot ladder that is placed on level ground. Looking at a nearby building, you note that the angle of elevation to the top of the building is 38°, while the angle of depression to the bottom of the building is 5°. What is the height of the building?

A. $\frac{12\tan 38^\circ}{\tan 5^\circ}$ feet

B. $\frac{12\tan 5^\circ}{\tan 38^\circ} + \frac{12}{\tan 5^\circ}$ feet

C. $12(\tan 5^\circ + \tan 38^\circ)$ feet

D. $\frac{12\tan 5^\circ}{\tan 38^\circ}$ feet

E. $\frac{12\tan 38^\circ}{\tan 5^\circ} + 12$ feet

$\tan 38^\circ = \frac{y}{x} \rightarrow y = \frac{12\tan 38^\circ}{\tan 5^\circ}$

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21. Select the function whose graph passes through the origin.

A. $f(x) = \sec x$

B. $f(x) = \cos x$

C. $f(x) = \sin x$

D. $f(x) = \csc x$

E. $f(x) = \cot x$

$+\cdot+$ or $-\cdot-$

22. Suppose that θ is an angle with $\sin\theta \cdot \cos\theta > 0$. In which quadrants could θ lie?

A. I only

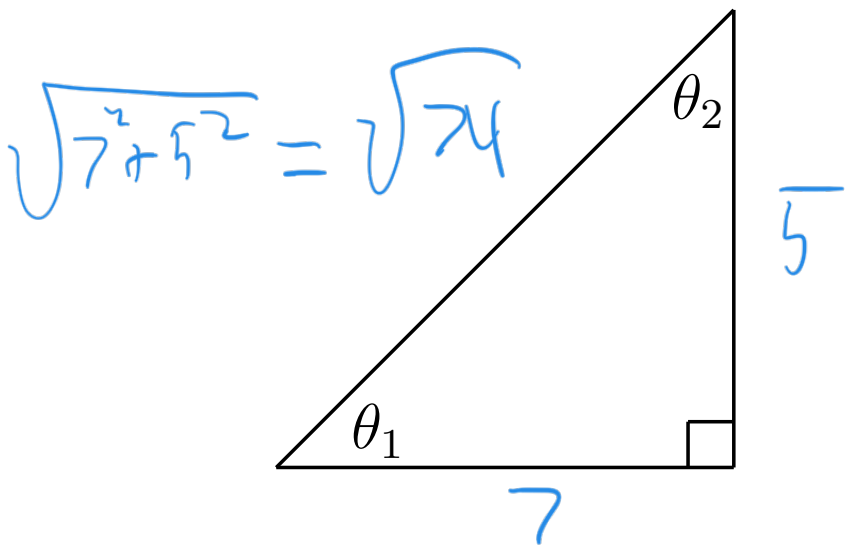
B. II only

C. I or III

D. II or IV

E. I or IV

23.



$\sqrt{7^2+5^2} = \sqrt{74}$

5

7

In the triangle above, $\theta_2 = \arctan\left(\frac{7}{5}\right)$. What is the value of $\sin(\theta_1)$?

A. $\frac{7}{\sqrt{74}}$

B. $\frac{5}{12}$

C. $\frac{7}{12}$

D. $\frac{5}{\sqrt{74}}$

E. $\sin(\theta_1)$ is undefined

MAC 1147 — Fall 2021 — EXAM 4A

T.A. _____ Disc. Per. ____ Name Bruce Wayne

Honor Pledge: "On my honor, I have neither given nor received unauthorized aid for this exam."

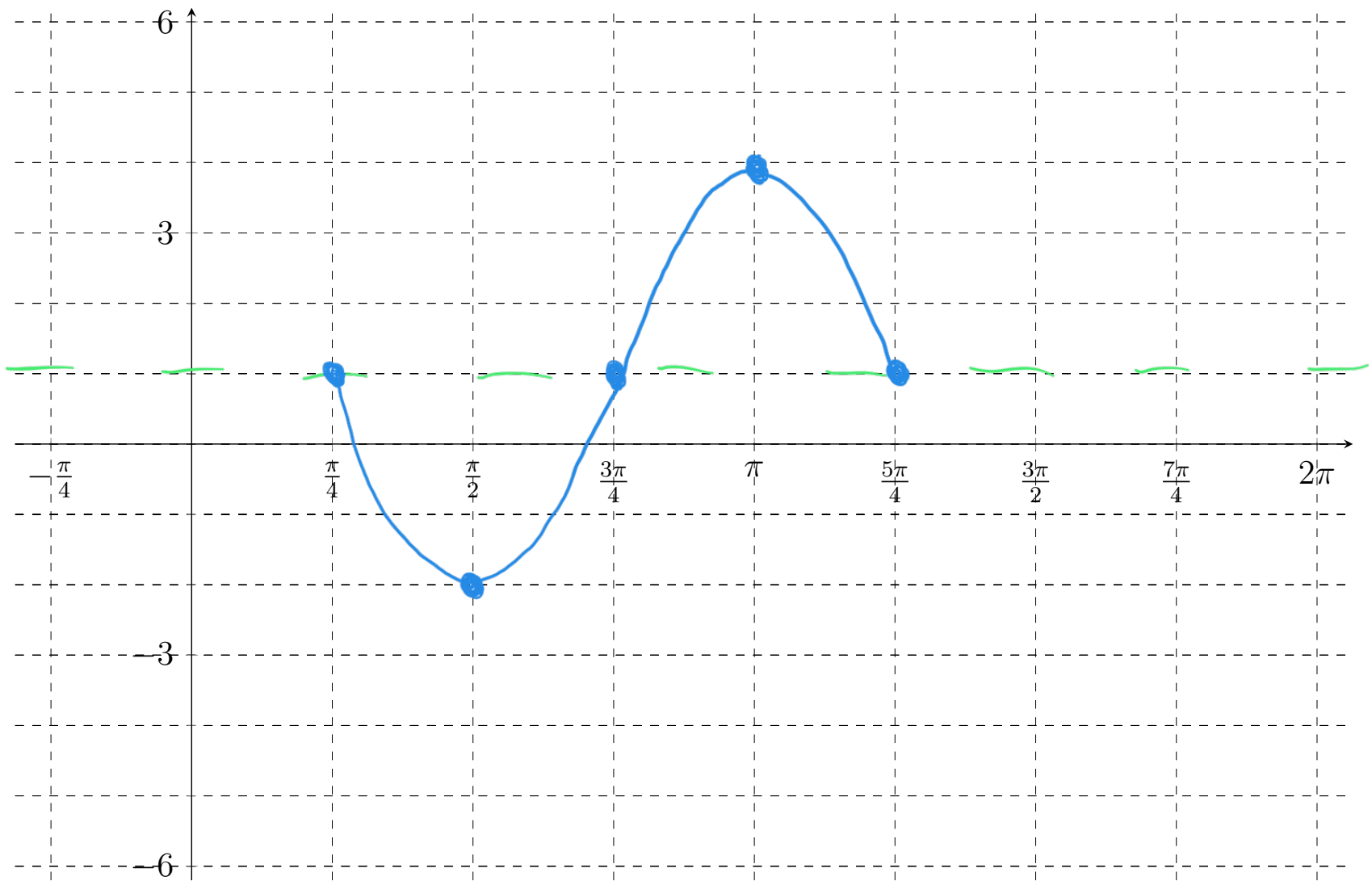
UF ID # _____ Signature Bertman

YOU MUST SHOW ALL WORK TO RECEIVE FULL CREDIT.

Free response questions 24-25 are worth 4 points each.

24. Sketch the graph of the given function. Your picture must be clear, legible and include at least one full period of the graph. You can fill in the blanks below as a guide, but your score will come from accurately portraying them on the graph.

$$f(x) = -3\sin\left(2x - \tfrac{\pi}{2}\right) + 1.$$



Amplitude: 3

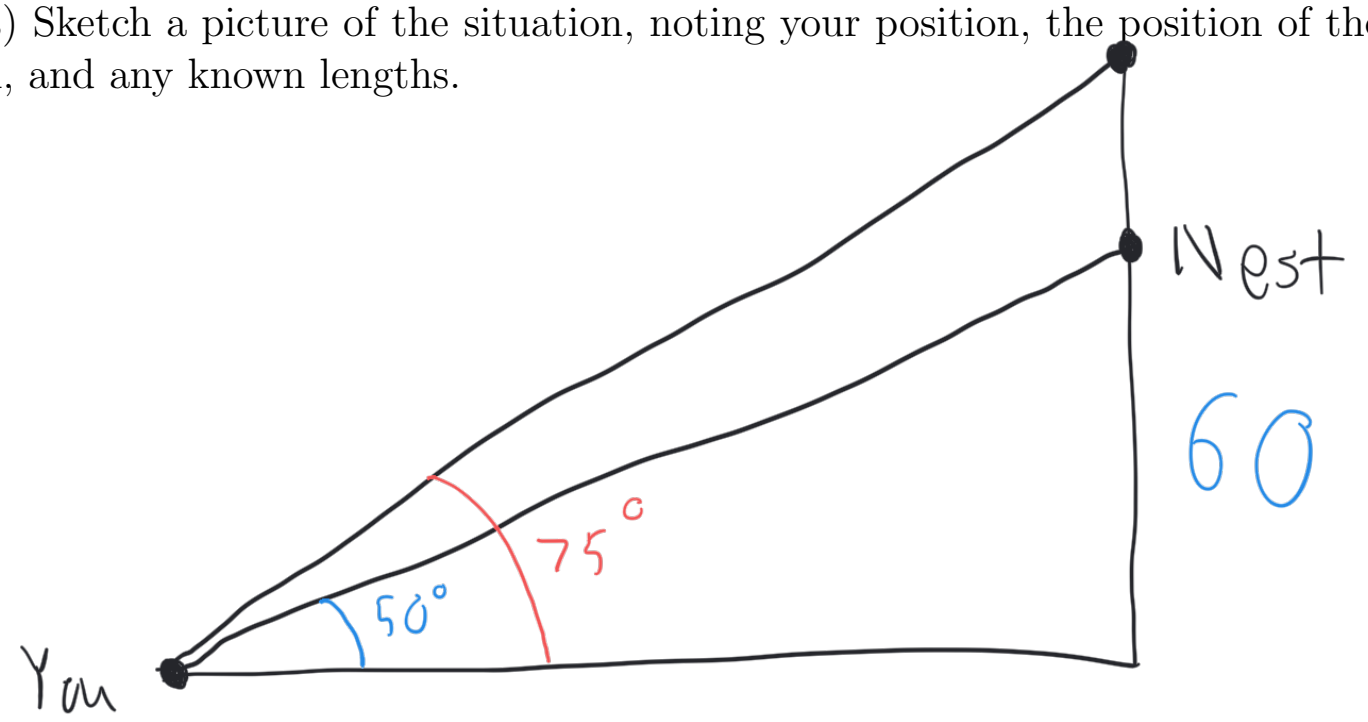
Period: $\frac{2\pi}{2} = \pi$

Phase Shift: $\frac{\frac{\pi}{2}}{2} = \frac{\pi}{4}$

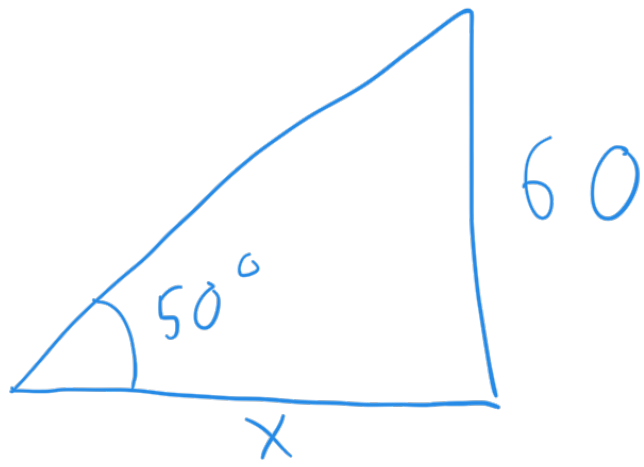
Midline: $y = 1$

25. You are walking across a level field of trees. In a nearby tree, you spot a Mathingbird sitting in its nest partway up the tree trunk. The Mathingbird is known for always building nests exactly 60 feet from the ground. You note that your angle of elevation to the Mathingbird's nest is 50°, while your angle of elevation to the top of the tree is 75°.

a. (2 pts) Sketch a picture of the situation, noting your position, the position of the nest, angles of elevation, and any known lengths.



b. (1 pt) How far are you from the base of the tree? Show your work and give an exact value for your answer.

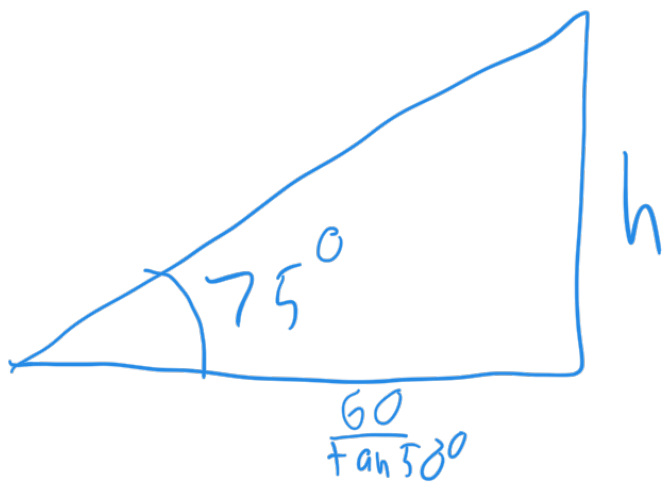


$\tan 50^\circ = \frac{60}{x}$

$x = \boxed{\frac{60}{\tan 50^\circ} \text{ ft}}$

Distance = _____ feet

c. (1 pt) What is the height of the tree? Show your work and give an exact value for your answer.



$\tan 75^\circ = \frac{h}{60/\tan 50^\circ}$

$h = \boxed{\frac{60 \tan 75^\circ}{\tan 50^\circ} \text{ ft}}$

Height = _____ feet