MAC 1147 — Fall 2021 — EXAM 3A

Questions 1–23 are worth 4 points each.

$b = 1.08 \quad r = b - 1 = .08$

1. The size of a quantity after t days is given by $A(t) = 200 \cdot (1.08)^t$. What is the effective growth rate of the quantity?

A. 108% per day B. 92% per day C. -8% per day

D. -92% per day E. 8% per day

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2. Select the expression that is equivalent to

$$\log_b\left(\frac{8x^7}{yz^4}\right)$$

$= \log_b 8 + \log_b x^7 - \log_b y - \log_b z^4$

$=$

A. $7\log_b x - \log_b y - 4\log_b z + \log_b 8$

B. $7\log_b x + \log_b y - 4\log_b z + \log_b 8$

C. $7\log_b x - \log_b y + 4\log_b z + \log_b 8$

D. $7\log_b x - \log_b y + 4\log_b z - 7\log_b 8$

E. $7\log_b x - \log_b y - 4\log_b z + 7\log_b 8$

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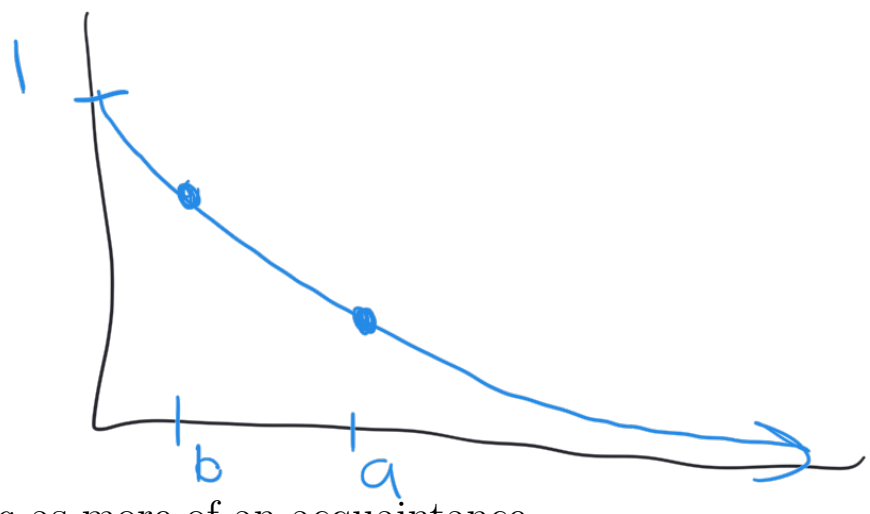
3. Suppose that a and b are real numbers such that $\left(\frac{2}{3}\right)^a < \left(\frac{2}{3}\right)^b < 1$. Select the statement that *must* be true about a and b.
Hint: Sketch the graph of $f(t) = \left(\frac{2}{3}\right)^t$



A. $a < 0$

B. $a > 1$

C. $a > b$

D. $a + b > 1$

E. a considers b a friend, but b thinks of a as more of an acquaintance.

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4. $\tan\left(\frac{7\pi}{4}\right) =$

$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right) \quad \frac{y}{x} = -1$

A. 1 B. $\sqrt{3}$ C. -1 D. $-\sqrt{3}$ E. 0

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5. Find the sum of all solutions to the equation:

$$(\log_3 x)^2 = 4\log_3 x$$

A. 82 B. 81 C. 65 D. 64 E. 1

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6. Identify any vertical asymptotes of the function:

$$f(x) = \log_4(3x - 15) - 2$$

A. $x = -2$

B. $x = 15$

C. $x = \frac{31}{3}$

D. $x = 5$

E. $f(x)$ has no vertical asymptotes.

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7. What is the length of the arc traced along the unit circle by an angle of $\frac{-2\pi}{3}$?

A. $\frac{2\pi}{3}$ B. $\frac{1\pi}{3}$ C. $\frac{-2\pi}{3}$ D. $\frac{-1\pi}{3}$ E. $\frac{4\pi}{3}$

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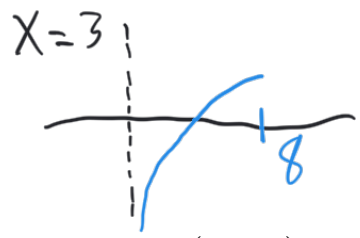
8. Suppose that $f(x) = \log_5(x - 3)$. Identify the largest interval where $f(x) < 1$.
Hint: Sketch the graph of $f(x)$



A. $(3, 28)$ B. $(0, 8)$ C. $(5, 8)$ D. $(3, 8)$ E. $(5, 28)$

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9. Select the angle that is coterminal with $\theta = \frac{2\pi}{7}$

A. $\frac{-12\pi}{7}$ B. $\frac{-17\pi}{14}$ C. $\frac{9\pi}{7}$ D. $\frac{3\pi}{14}$ E. $\frac{5\pi}{7}$

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10. Select the statement that is *not* true for all positive a, b.

A. $\ln(a+b) = \ln a \cdot \ln b$ B. $\ln(a^b) = b \ln a$ C. $\ln(ab) = \ln a + \ln b$

D. $\ln(\frac{a}{b}) = \ln a - \ln b$ E. $\log_b a = \frac{\ln a}{\ln b}$ where $b \neq 1$

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11. What is the growth factor of the function?

$$g(t) = 65e^{-0.19t} = 65\left(e^{-.19}\right)^t$$

A. 0.81 B. $e^{-0.19}$ C. -0.19 D. 65 E. 1.19

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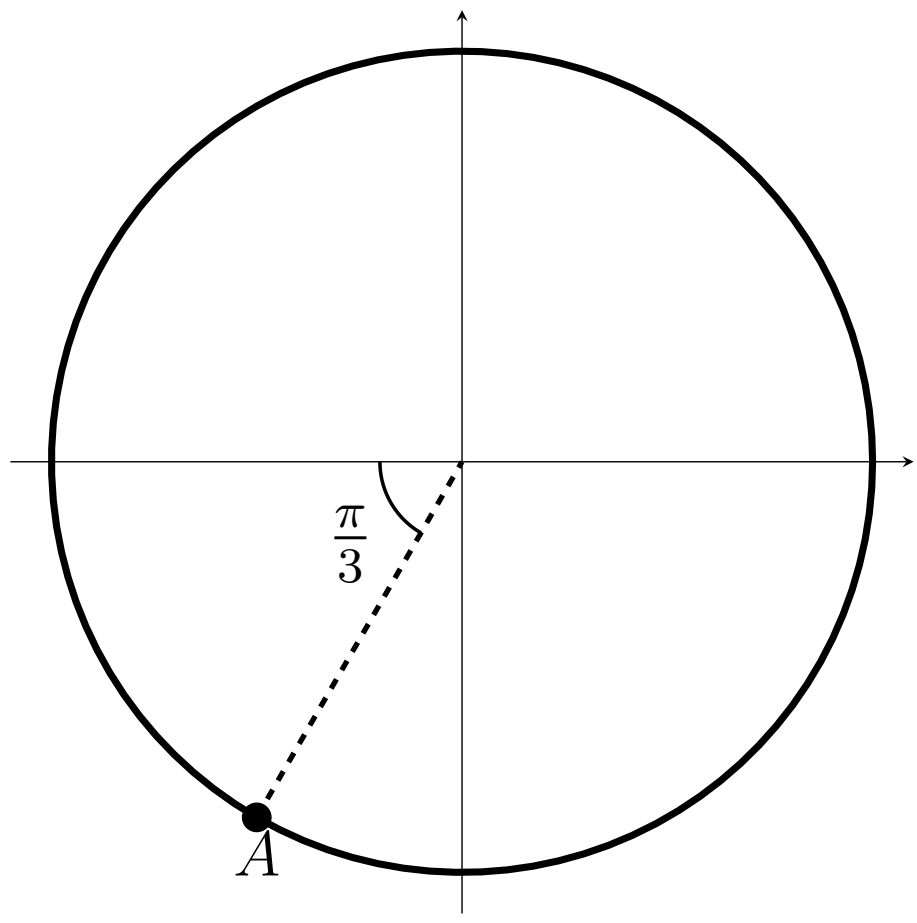
12. What are the coordinates of point A?



A. $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ B. $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ C. $\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

D. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ E. $\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

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13. What is the domain of the function?

$$f(x) = \log_7(5x - 3)$$

A. $(0, \infty)$ B. $(\frac{3}{5}, \infty)$ C. $(7, \infty)$ D. $(\frac{5}{7}, \infty)$ E. $(3, \infty)$

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14. Suppose that θ is an angle in the unit circle with terminal point (a, b). Which angle has terminal point $(-a, -b)$?

A. $\theta + \frac{3\pi}{2}$ B. $\theta + \pi$ C. $\theta + \frac{\pi}{2}$ D. $\theta + 2\pi$ E. $\theta + \frac{\pi}{4}$

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15. What is the inverse of the function?

$$f(x) = \frac{3^x + 7}{11}$$

A. $f^{-1}(x) = 11\log_3(x) - 7$ B. $f^{-1}(x) = 11\log_3(x - 7)$ C. $f^{-1}(x) = \log_3(11x) - 7$

D. $f^{-1}(x) = \log_3(11x - 7)$ E. $f^{-1}(x) = \log_3(x) - 77$

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16. Select the *true* statement.

A. A quantity with a nominal growth rate of 14% per year will grow more in one year if its growth is compounded 4 times per year than if its growth is compounded 2 times per year.

B. If a is positive and $b > 1$ then $f(x) = ab^x$ does not have a horizontal asymptote.

C. The functions $f(t) = 70e^{.24t}$ and $g(t) = 70e^{.29t}$ have the same doubling time.

D. $A(t) = Pe^{.08t}$ models a quantity with an effective growth rate of 8% per year.

E. If a quantity's growth is compounded more than once per year, then its effective growth rate will be less than its nominal growth rate.

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17. Find the sum of all solutions to the equation.

$$\log_6 x = 2 - \log_6(x + 9)$$

A. 6 B. -9 C. 3 D. 0 E. 15

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18. A quantity shrinks with a continuous rate of 11% per year. What is its half-life?

A. $-\frac{\ln\left(\frac{1}{2}\right)}{\ln(0.89)}$ years

B. $-\frac{\ln\left(\frac{1}{2}\right)}{.11}$ years

C. $\frac{\ln\left(\frac{1}{2}\right)}{\ln(0.89)}$ years

D. $\frac{\ln\left(\frac{1}{2}\right)}{.11}$ years

E. $\frac{\ln\left(\frac{1}{2}\right)}{\ln(1.11)}$ years

$\frac{1}{2} = e^{-.11t}$

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19. Suppose that θ is an angle between $0°$ and $360°$ and $\sin(\theta) = \frac{-3}{4}$. Which of these is a possible value of $\tan(\theta)$?

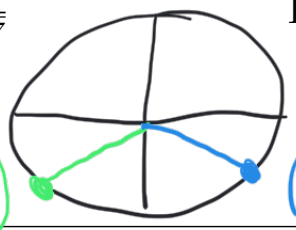
A. $\frac{4}{\sqrt{7}}$

B. $-\frac{7}{9}$

C. $\frac{3}{\sqrt{7}}$

D. $-\frac{\sqrt{7}}{3}$

E. $\frac{9}{7}$



$\left(-\frac{\sqrt{7}}{4}, -\frac{3}{4}\right)$ $\left(\frac{\sqrt{7}}{4}, -\frac{3}{4}\right)$ $\frac{y}{x} = \frac{3}{\sqrt{7}}$ $\frac{y}{x} = \frac{-3}{\sqrt{7}}$

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20. Solve for t.

$3\log_5 t = 36$

$\log_6(3\log_5(t)) = 2$

$\log_5 t = 12$

$t = 5^{12}$

A. $t = 5^{36}$

B. $t = 6^{\frac{25}{3}}$

C. $t = 5^{12}$

D. $t = 6^{25}$

E. $t = 6^{36}$

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21. A wheel with a diameter of 16 inches is spinning. A point on its edge has a linear velocity of 480π inches per minute. What is the angular velocity of the wheel in revolutions per minute?

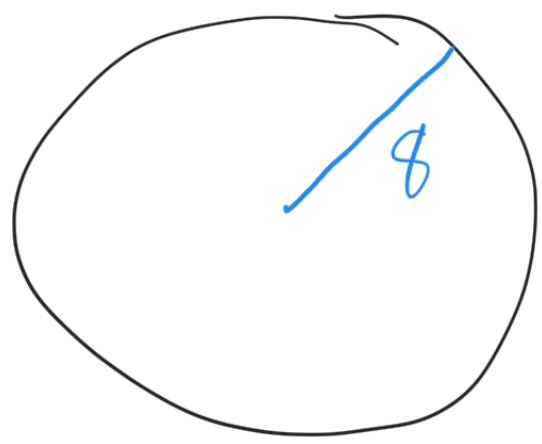
A. 30 revolutions per minute

B. 60π revolutions per minute

C. 30π revolutions per minute

D. 60 revolutions per minute

E. 15 revolutions per minute



$v = r\omega$

$480\pi = 8\omega$

$\omega = 60\pi \frac{\text{rad}}{\text{min}}$

$= 30 \frac{\text{rot}}{\text{min}}$

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22. A quantity has initial value a, and its size after t minutes is given by $A(t) = a \cdot (1.13)^t$. For which value of a will the quantity double in size fastest?

$2a = a(1.13)^t$

$2 = (1.13)^t$

A. 80

B. 110

C. 42

D. 7

E. The population will take the same amount of time to double for any a.

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23. $\log_3 \frac{1}{81} =$

A. 1 B. $\frac{1}{4}$ C. $-\frac{1}{4}$ D. 4 E. -4

$-4 \quad 1$

$3 = \sqrt{81}$

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T.A. _____ Disc. Per. ____ Name _____

Honor Pledge: "On my honor, I have neither given nor received unauthorized aid for this exam."

UF ID # _____ Signature _____

YOU MUST SHOW ALL WORK TO RECEIVE FULL CREDIT.

Free response questions 24-25 are worth 4 points each.

24. Solve for x. Give exact values, not decimal approximations. You must show your work to receive credit.

$$49^x = 24 - 5 \cdot 7^x$$

$(7^x)^2 + 5 \cdot 7^x - 24 = 0$

$k = 7^x$

$k^2 + 5k - 24 = 0$

$(k+8)(k-3) = 0$

$k = 3, -8$

$7^x = -8$ X

$7^x = 3$

$x = \log_7 3$

$x =$ $\log_7 3$

25. An invasive lizard species is causing problems in a local ecosystem. In response, a program to capture and relocate lizards is started. At the start of the project, 256 lizards are observed in the ecosystem. Two years later, the population is measured to be 144.

a.(2 pts) Construct an exponential function $P(t)$ that gives the size of the lizard population t years after the project started.

$(0, 256) \rightarrow a = 256$

$(2, 144)$

$P(t) = 256(b)^t$

$144 = 256\, b^2$

$b^2 = \frac{144}{256} = \frac{9}{16}$

$b = \pm \frac{3}{4}$

$P(t) = \underline{256\left(\frac{3}{4}\right)^t}$

b.(1 pt) According to this model, what will the lizard population be 5 years into the project? Leave your answer in exact form. You do not need to evaluate.

Population=$\underline{P(5) = 256\left(\frac{3}{4}\right)^5}$

c.(1 pt) According to this model, how long after the project starts will the population be 30 lizards? Give your answer in exact form, not decimal.

$30 = 256\left(\frac{3}{4}\right)^t$

$\frac{30}{256} = \left(\frac{3}{4}\right)^t$

$\frac{15}{128} = \left(\frac{3}{4}\right)^t$

$t = \underline{\log_{3/4}\left(\frac{15}{128}\right)}$ years