Exam 3 Review

1. Find the radius of convergence and interval of convergence for the series.

a. $\sum_{n=1}^{\infty} \frac{(-1)^n 4^n}{\sqrt{n}} x^n$ b. $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)2^n} (x-1)^n$ c. $\sum_{n=1}^{\infty} \frac{n! x^n}{1 \cdot 3 \cdot 5 \cdots (2n-1)}$ d. $\sum_{n=1}^{\infty} \frac{n^2 x^n}{2 \cdot 4 \cdot 6 \cdots (2n)}$

2. Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -4$ and diverges when $x = 6$. What can be said about the convergence or divergence of the following series?

a. $\sum_{n=0}^{\infty} c_n$ b. $\sum_{n=0}^{\infty} c_n 8^n$ c. $\sum_{n=0}^{\infty} c_n (-3)^n$ d. $\sum_{n=0}^{\infty} c_n (-1)^n 9^n$

3. Let $S(x) = \sum_{n=0}^{\infty} c_n x^n$ be a power series with radius of convergence R and suppose that $S(x)$ converges when $x = 2$ and diverges when $x = -7$. Which of the following conclusions must be true?

a. The series $c_0 - c_1 + c_2 - c_3 + \cdots$ converges
b. $R \geq 2$
c. $R < 7$
d. The series $c_0 - 2c_1 + 4c_2 - 8c_3 + \cdots$ converges
e. The series $c_0 + 7c_1 + 49c_2 + 343c_3 + \cdots$ diverges
f. The series $-c_0 + 9c_1 - 81c_2 + 729c_3 + \cdots$ diverges
g. The series $c_0 + 4c_1 + 16c_2 + 64c_3 + \cdots$ diverges

4. Find a power series representation for the series and determine the radius and interval of convergence.

a. $f(x) = \frac{4}{2x+3}$ b. $f(x) = \frac{x-1}{x+2}$ c. $f(x) = \frac{x^2}{x^2+16}$ d. $f(x) = \ln(5-x)$

e. $f(x) = x^2 \arctan x$ f. $f(x) = \frac{x}{(1+4x)^2}$ g. $f(x) = \ln(1+x^4)$ h. $f(x) = e^{3+x^2}$

i. $g(x) = \frac{3}{x^2+x-2}$

5. Evaluate the indefinite integral as a power series.

a. $\int \frac{t}{1-t^8}\,dt$ b. $\int x^2 \ln(1+x)\,dx$ c. $\int \frac{\tan^{-1} x}{x} dx$ d. $\int x \ln(1+x^2)\,dx$

e. $\int x^2 \sin(x^2)\,dx$ f. $\int \frac{\cos x - 1}{x}\,dx$ g. $\int \arctan(x^2)\,dx$ h. $\int \frac{\cos(x^2)-1}{x}\,dx$

6. Find the Taylor series for $f(x)$ centered at the given value of a.

a. $f(x) = \sqrt[3]{x},\ a = 8$ b. $f(x) = \ln x,\ a = 1$ c. $f(x) = e^{4x},\ a = 4$

d. $f(x) = \sqrt{x},\ a = 16$ e. $g(x) = \frac{1}{x+4},\ a = 4$ f. $h(x) = \ln(1+2x),\ a = 4$

7. Find the first four nonzero terms of the series for $f(x)$ centered at the given value of a.

a. $f(x) = \sin x,\ a = \frac{\pi}{6}$ b. $f(x) = \cos x,\ a = \frac{\pi}{2}$

8. Use the series to evaluate the limit

a. $\lim_{x\to 0} \frac{x-\ln(1+x)}{x^2}$ b. $\lim_{x\to 0} \frac{1-\cos x}{1+x-e^x}$ c. $\lim_{x\to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5}$

9. Find the sum of the series

a. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{n!}$ b. $\sum_{n=0}^{\infty} (-1)^n \frac{\pi^{2n}}{6^{2n}(2n)!}$ c. $\sum_{n=0}^{\infty} (-1)^n \frac{3^n}{n5^n}$

d. $1 - \ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots$ e. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{8}{4!}$ f. $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{4^n}$

10. Approximate the value of the integrals

a. $\int_0^1 e^{-x^2} dx$ with an error no greater than 5×10^{-4}.

b. $\int_0^1 x \cos x dx$ with an error no greater than 10^{-3}.

11. Find the Taylor series expansion of

a. $f(x) = \sin x$ centered at $a = \frac{\pi}{4}$. Then use the Taylor Polynomial T_2 to approximate $\sin(47°)$.

b. $g(x) = 5x + e^{-3x}$ centered at $a = 0$. Then use the Taylor Polynomial T_2 to approximate $g(0.1)$.

12. Let $f(x) = \sin\left(\frac{x^2}{2}\right)$. Determine the value of $f^{(50)}(0)$.

13. Find the Taylor series for f centered at $a = -3$ and its radius of convergence if $f^{(n)}(-3) = -\frac{n!}{3^{n+1}}$.

14. Starting with the geometric series $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, (-1 < x < 1)$, use differentiation to find the sum of the series $\sum_{n=1}^{\infty} \frac{n}{4^n}$.

15. Eliminate the parameter t to find a Cartesian equation for the curve.

a. $x = t^2 - 3,\ y = t + 2,\ -3 \le t \le 3$

b. $x = \sin t,\ y = \cos t,\ 0 \le t \le 2\pi$

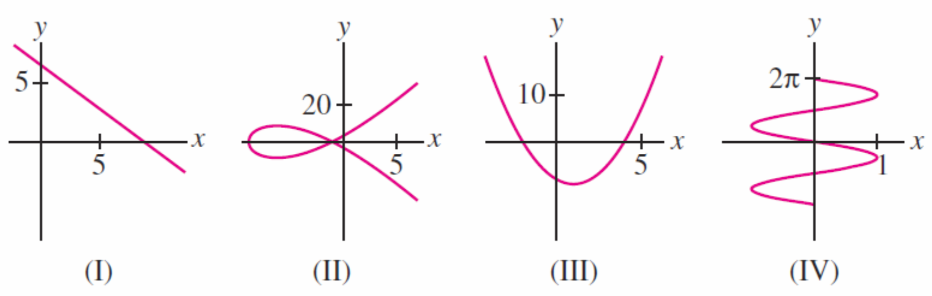
c. $x = \sqrt{t},\ y = 1 - t$

d. $x = t^2,\ y = t^3$

e. $x = t^2,\ y = \ln t$

f. $x = e^t,\ y = e^{2t}$

16. Match the parameteric equations with the graphs below.



a. $x = \sin t,\ y = -t$ b. $x = t^2 - 9,\ y = 8t - t^3$

c. $x = 1 - t,\ y = t^2 - 9$ d. $x = 4t + 2,\ y = 5 - 3t$

17. Find $\frac{d^2y}{dt^2}$ at the given value of t.

a. $x = t^3 + t,\ y = 7t^2 - 4,\ t = 2$ b. $x = t^{-1} + t,\ y = 4 - t^{-2},\ t = 1$

c. $x = \cos t,\ y = \sin t,\ t = \frac{\pi}{4}$

18. Find the points on the curve $x = 6\cos t,\ y = \sin(2t),\ 0 \le t \le 2\pi$, where the tangent line is

a. horizontal

b. vertical

19. Find the length of the curve.

a. $(3t + 1,\ 9 - 4t),\ 0 \le t \le 2$ b. $(e^t - t,\ 4e^{\frac{t}{2}}),\ 0 \le t \le 2$

c. $(3t,\ 4t^{\frac{3}{2}}),\ 0 \le t \le 1$ d. $(\sin 3t,\ \cos(3t)),\ 0 \le t \le \pi$

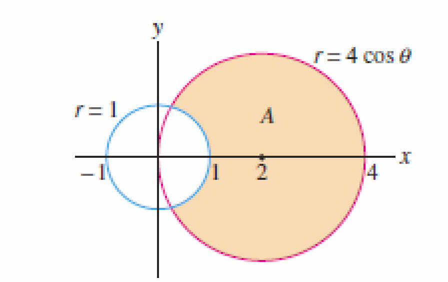
20. Which of the following are possible pairs of polar coordinates for the point with rectangular coordinates $(0, -2)$?

a. $(2, \frac{\pi}{2})$ b. $(2, \frac{7\pi}{2})$ c. $(-2, -\frac{3\pi}{2})$

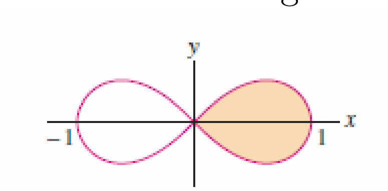
d. $(-2, \frac{7\pi}{2})$ e. $(-2, -\frac{\pi}{2})$ f. $(2, -\frac{7\pi}{2})$

21. Find the shaded areas below.

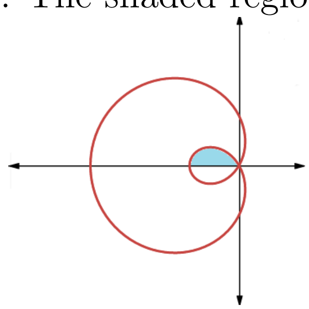
a. The shaded region inside $r = 4\cos\theta$ and outside $r = 1$.



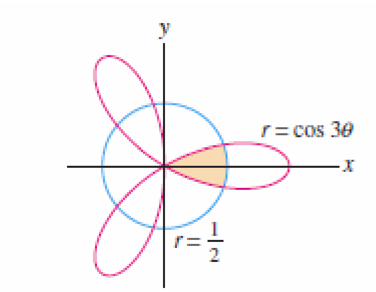
b. The shaded region of the lemniscate $r^2 = \cos(2\theta)$



c. The shaded region of the cardioid $r = 1 - 2\cos\theta$



d. The shaded area enclosed by $r = \cos 3\theta$ and $r = \frac{1}{2}$



e. The area of the region that lies inside both curves $r_1 = 3 + 2\cos\theta$ and $r_2 = 3 + 2\sin\theta$.

22. Match the equations with the graphs

a. $r = 3\sin 4\theta$

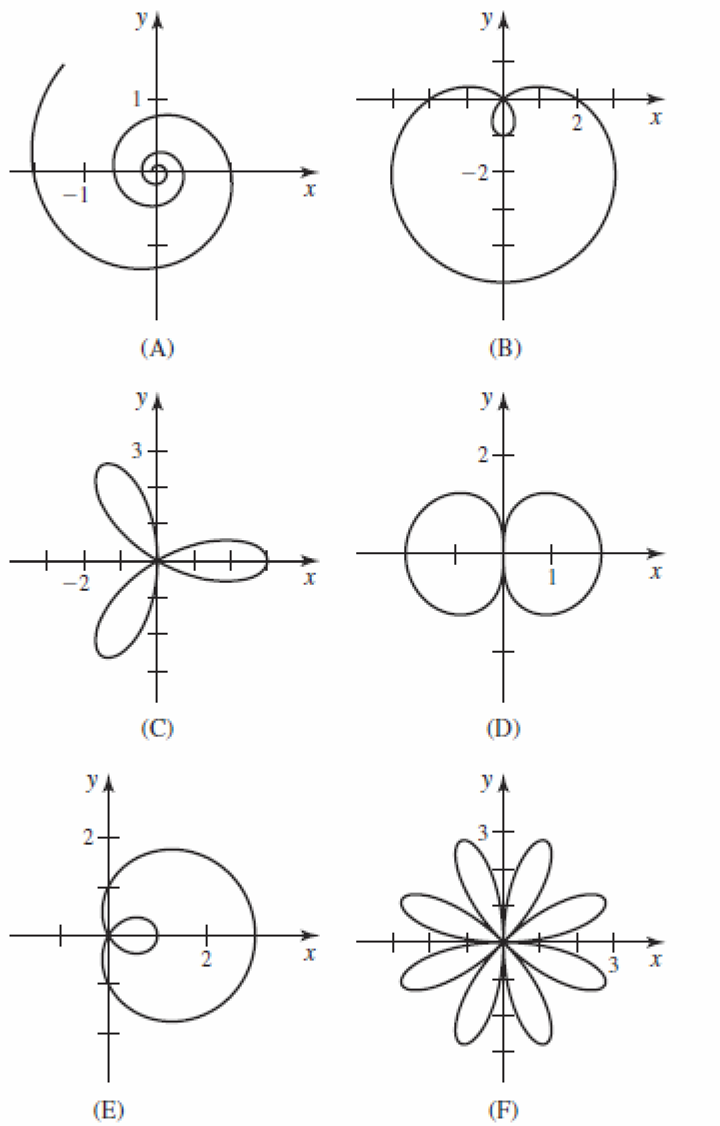
b. $r^2 = 4\cos\theta$

c. $r = 2 - 3\sin\theta$

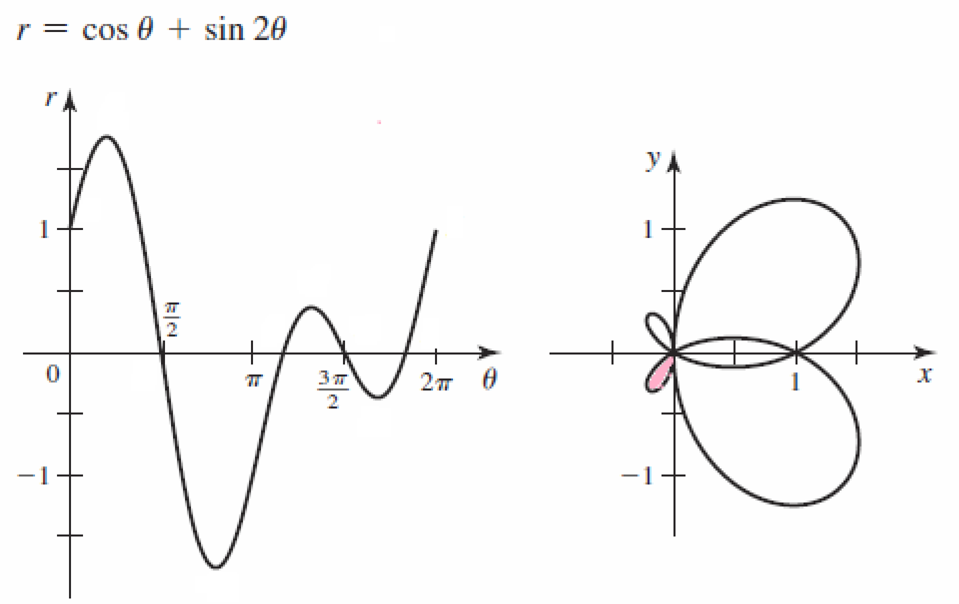
d. $r = 1 + 2\cos\theta$

e. $r = 3\cos 3\theta$

d. $r = e^{-\frac{\theta}{6}}$



23. Given flat and polar curves of $r = \cos\theta + \sin 2\theta$, let $\int_a^b \frac{1}{2}(\cos\theta + \sin 2\theta)^2 d\theta$ be the area of the shaded region. Find a and b.



Exam 3 - Review

- Taylor series of a function f centered at $x = a$ is given by $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ where $c_n = \dfrac{f^n(a)}{n!}$. When $a = 0$, we call it a Maclaurin series.
- c_n = Taylor coefficient(or just coefficient) of $(x-a)^n$.
- Taylor Polynomial: $T_N(x) = c_0 + c_1(x-a) + c_2(x-a)^2 + ... + c_N(x-a)^N$.
- The answers provided here are in the simplified form accepted on the exam with similar questions.

1. Recall the Maclaurin series of the following functions. **Make sure you know them before you go to the exam!!!** (even though this list will be provided on the exam.)
   (a) $\dfrac{1}{1-x} =$
   (b) $e^x =$
   (c) $\ln(1+x) =$
   (d) $\ln(1-x) =$
   (e) $\arctan(x) =$
   (f) $\sin(x) =$
   (g) $\cos(x) =$
2. Find the ROC and IOC of the followings:
   (a) $\sum_{n=1}^{\infty} \dfrac{(-1)^n (x-5)^n}{4^n n}$
   (b) $\sum_{n=0}^{\infty} \dfrac{x^{2n}}{(2n)!}$
3. Find Taylor series expansion of the following functions centered at given point **WITHOUT SETTING UP THE TABLE of derivatives**. Then find the radius of convergence and the interval of convergence for each.
   (a) $f(x) = \dfrac{x^2}{(2-x^3)^2} \quad ; x = 0$
   (b) $f(x) = e^{x+2} \quad ; x = 0$
   (c) $f(x) = (x-1)e^{3x} \quad ; x = 1$
4. Find the Taylor series expansion of the following functions centered at given point.
   (a) $f(x) = \dfrac{1}{x^2} \quad ; x = 2$

(b) $f(x) = \sqrt{3x+4}$; $x = 7$, Use second Taylor Polynomial, $T_2(x)$ to approximate $\sqrt{25.3}$.

5. Evaluate the following sums:

   (a) $\dfrac{\pi^5}{2^5 5!} - \dfrac{\pi^7}{2^7 7!} + \dfrac{\pi^9}{2^9 9!} - \dfrac{\pi^{11}}{2^{11} 11!} + \ldots$

   (b) $-\dfrac{\pi^6}{2^6 5!} + \dfrac{\pi^8}{2^8 7!} - \dfrac{\pi^{10}}{2^{10} 9!} + \ldots$

   (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n} - \sum_{n=1}^{\infty} \frac{n3^n}{7^n}$ (hint for the second series: differentiate a geometric series)

6. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-4)^n x^{2n}}{(2n+1)!}$. Note that the radius of convergence of this series is ∞. Evaluate $f(\pi/12)$.

7. Let $F(x) = \displaystyle\int_0^1 e^{-t^2} dt$. Find the Maclaurian series for $F(x)$. Evaluate $F(1)$ to with an error of at most 0.01.

8. Find the Maclaurin series and the radius of convergence.

   (a) $F(x) = \displaystyle\int_0^x \frac{t - \arctan(t)}{t^3} dt$ (b) $\displaystyle\int_0^1 x e^{x^3}\, dx$

9. (a) Suppose the power series $\displaystyle\sum_{n=0}^{\infty} c_n (x+2)^n$ converges at $x = -4$ and diverges at $x = 3$. What can you say about the convergence of the following series?

   (i) $\displaystyle\sum_{n=0}^{\infty} (-1)^n c_n 7^n$ (ii) $\displaystyle\sum_{n=0}^{\infty} c_n$ (iii) $\displaystyle\sum_{n=0}^{\infty} c_n 2^n$ (iv) $\displaystyle\sum_{n=0}^{\infty} c_n 3^n$ (v) $\displaystyle\sum_{n=0}^{\infty} c_n 9^n$

   (b) Let $S(x) = \displaystyle\sum_{n=0}^{\infty} c_n (x-6)^n$ be a power series with radius of convergence $R > 0$. Which of the following statements are possible?

   (i) The series $c_0 - 5c_1 + 25c_2 - 125c_3 + \cdots$ diverges but $c_0 + 2c_1 + 4c_2 + 8c_3 + \cdots$ converges.

   (ii) The series $c_0 - c_1 + c_2 - c_3 + \cdots$ diverges but $c_0 + 2c_1 + 4c_2 + 8c_3 + \cdots$ converges.

   (iii) The series converges absolutely at $x = 1$, conditionally converges at $x = 9$.

10. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n (x-2)^{4n+3}}{2^{2n+1}(2n+3)!}$ and $g(x) = \arctan(x^2)$. Determine the followings:

    (a) $f^{(43)}(2)$, $g^{(43)}(0)$

(b) $f^{(18)}(2),\ g^{(18)}(0)$

11. Find the equation of the tangent line to the path $c(t) = (t^2 + 1, t^3 - 4t)$ at $t = 3$.

12. Convert from polar to rectangular coordinates.

(a) $\left(3, \frac{\pi}{6}\right)$

(b) $\left(5, \frac{-\pi}{2}\right)$

13. Convert from rectangular to polar coordinates.

(a) $(3, \sqrt{3})$

(b) $(-2, 2)$

14. Let $r = 2\cos(\theta) - 1$ on $[0, 2\pi]$.

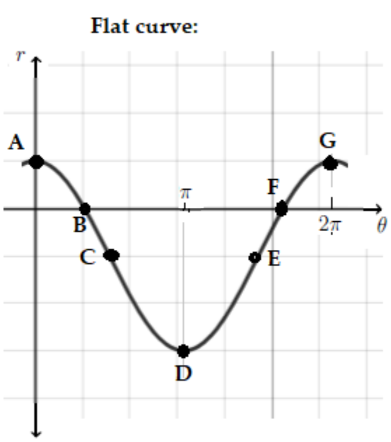
(a) Solve for $r = 0$, $[0, 2\pi]$

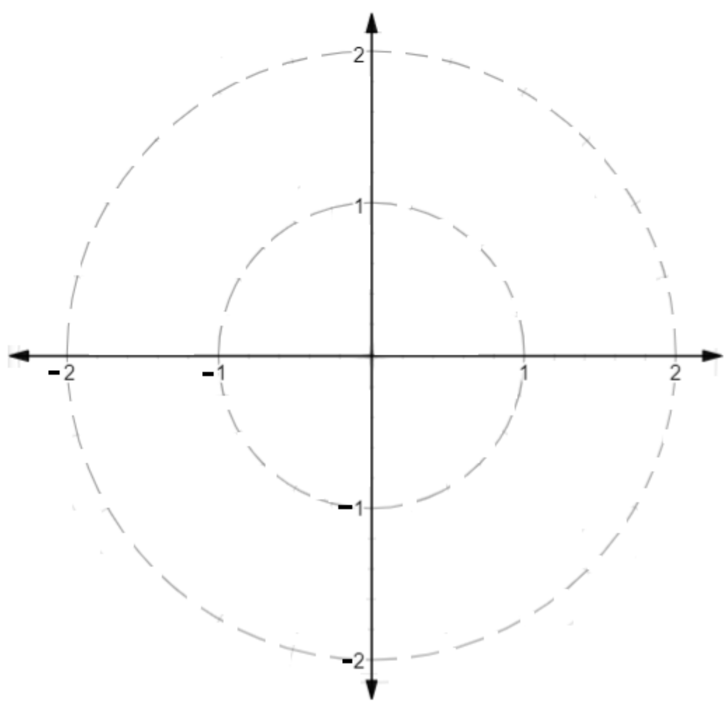
$\theta =$ ____________

(b) Identify the points in the flat curve below in polar coordinates (r, θ):

A:(___ , ___), B:(___ , ___), C:(___ , ___), D:(___ , ___), E:(___ , ___), F:(___ , ___), G:(___ , ___)

Label these points on the polar coordinates on the right.
Use the flat curve on the left to help sketch the polar curve r on the right.





(c) Set up an integral for the area inside r in the second quadrant.

$$A = \int_a^b \qquad\qquad d\theta, \quad a = \_\_, \; b = \_\_.$$

(d) Find the area above.

$A =$ ______________________

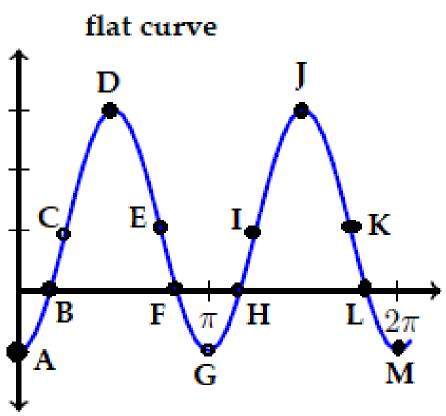
15. Find the area of the region **inside the curve** $r_1 = 1$ **and outside the curve** $r_2 = \cos(3\theta)$.

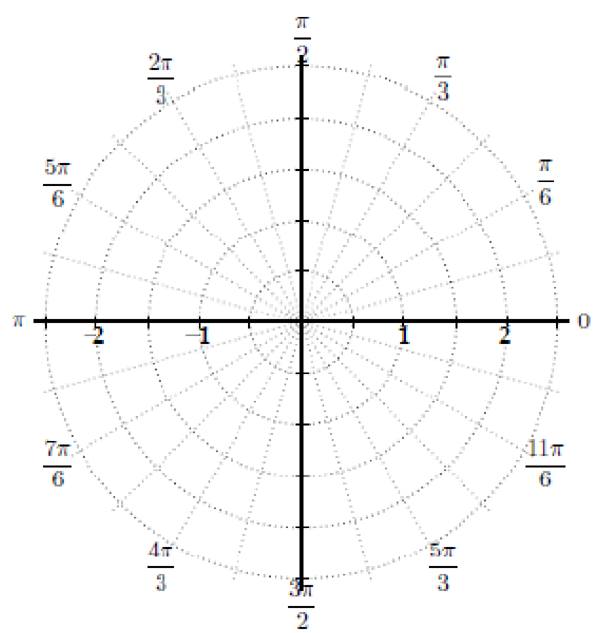
16. Find the arc length of the parametric curve.
    (a) $C(t) = (e^{-t}\cos t, e^{-t}\sin t)$ for $0 \leq t \leq 5$.
    (b) $C : x(t) = e^t - t,\ y(t) = 4e^{t/2},\ 0 \leq t \leq 2$
    (c) $C : x(t) = e^{2t} + e^{-2t},\ y(t) = 4t - 64,\ 0 \leq t \leq 1$

17. Find the area between the 2 curves:
    (a) $r_1^2 = 9\cos(2\theta),\ r_2 = 5\cos(\theta)$.
    (b) $r_1^2 = 9\cos(2\theta),\ r_2 = \dfrac{3}{\sqrt{2}}$.

18. Let C be the curve $x(t) = \cos^2 t + \cos t,\ y(t) = \sin t\cos t + \sin t$. Find the t values where the curve has HTL and VTL.

19. Let $r = 1 - 2\cos(2\theta)$ and its flat curve is given below on the left.Answer the same questions as in problem 14.



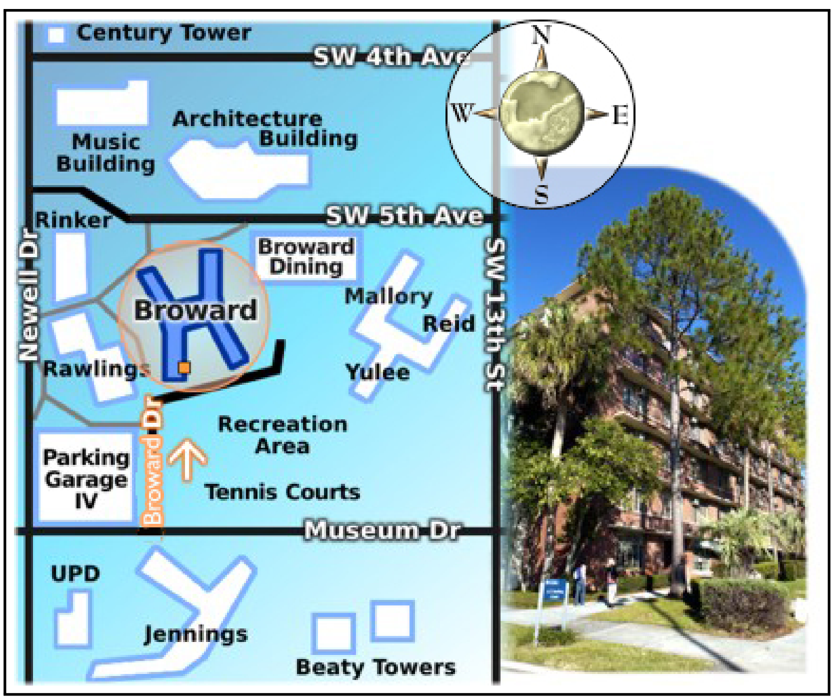


MAC 2312 Exam 3 Review

This review, produced by the Broward Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:



You can learn more about the services offered by the teaching center by visiting https://teachingcenter.ufl.edu/

1. The length of the parametric curve $x = e^t - t$, $y = 4e^{t/2}$, $t \in [-8, 3]$ is given by the integral:

   A. $\int_{-8}^{3} (e^t + 1)dt$ B. $\int_{-8}^{3} 2(e^t + 1)dt$ C. $\int_{-8}^{3} 4(e^t + 1)dt$

   D. $\int_{-8}^{3} 2(e^t - 1)dt$ E. $\int_{-8}^{3} \sqrt{2}(e^t + 1)dt$

2. The polar equation $r = 2\cos\theta$ can be expressed as:

   A. $(x-1)^2 + (y+1)^2 = 1$ B. $(x+1)^2 + (y-1)^2 = 1$

   C. $(x-1)^2 + y^2 = 1$ D. $(x-1)^2 + (y-1)^2 = 4$

   E. $(x+1)^2 + (y+1)^2 = 2$

3. Graph the following polar equation: $r = 5 - 4\cos\theta$

4. Calculate the Taylor series representation of e^{3x} centered at 3.

   A. $e^9 \sum_{n=0}^{\infty} \frac{3^n(x-3)^n}{n!}$ B. $\sum_{n=0}^{\infty} \frac{3^n(x-3)^n}{n!}$ C. $e^9 \sum_{n=0}^{\infty} \frac{(x-3)^n}{n!}$

   D. $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$ E. $\sum_{n=0}^{\infty} e^9 \frac{3^n(x-3)^n}{n}$

5. Write the Cartesian point $(2, \sqrt{12})$ in polar coordinates.

   A. $(4, \frac{\pi}{3})$ B. $(4, \frac{\pi}{6})$ C. $(\frac{\pi}{6}, 2)$ D. $(\frac{\pi}{3}, 2)$ E. $(2, \frac{\pi}{3})$

6. Consider $x = t^3 - t^2 + t$ and $y = 3e^t$. Calculate the slope and the sign (positive or negative) of the second derivative at the point $(0, 3)$.

A. Slope $= 3$, second derivative is negative

B. Slope $= 1$, second derivative is negative

C. Slope $= \frac{1}{3}$, second derivative is negative

D. Slope $= \frac{1}{3}$, second derivative is positive

E. Slope $= 3$, second derivative is positive

7. Which of the following describes the graph of the parametric curve given by the equations below?

$$\begin{cases} x = 2\sin(t) \\ y = \cos(t) \end{cases} \quad 0 \leq t \leq 2\pi$$

A. An ellipse drawn clockwise

B. A circle drawn counterclockwise

C. A circle drawn clockwise

D. An ellipse drawn counterclockwise

E. None of the others

8. Find a Taylor series for $\ln(1 + 4x)$ centered at 1.

A. $\ln 5 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4^n (x-1)^n}{5^n n}$

B. $\ln 5 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4^n (x-1)^n}{5^n}$

C. $\ln 5 + \sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4^n (x-1)^n}{5^n n!}$

D. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} 4^n x^n}{n}$

E. $\sum_{n=0}^{\infty} \frac{(-1)^{n+1} 4^n (x-1)^n}{5^n n!}$

9. Find the point (x, y) where the tangent line to the curve $c(t) = (3t^2 - 2, 3t^2 + 2t)$ is <u>horizontal</u>.

A. $\left(-\frac{14}{9}, \frac{52}{9}\right)$ B. $(10, 16)$ C. $\left(-\frac{5}{3}, -\frac{1}{3}\right)$

D. $\left(-\frac{23}{12}, -\frac{1}{4}\right)$ E. DNE

10. Graph the following:

(a) $r = \cos(3\theta)$

(b) $r = \sin(2\theta)$

11. Find the Maclaurin series of $f(x) = \dfrac{x}{(3-x)^2}$ and determine its radius of convergence R.

A. $f(x) = \sum_{n=1}^{\infty} \dfrac{nx^n}{3^{n+1}}$ and $R = \dfrac{1}{3}$ B. $f(x) = \sum_{n=1}^{\infty} \dfrac{-nx^n}{3^{n+1}}$ and $R = \dfrac{1}{3}$

C. $f(x) = \sum_{n=1}^{\infty} \dfrac{nx^n}{3^{n+1}}$ and $R = 3$ D.$f(x) = \sum_{n=1}^{\infty} \dfrac{-nx^n}{3^{n+1}}$ and $R = 3$

E. $f(x) = \sum_{n=1}^{\infty} \dfrac{-nx^{n-1}}{3^{n+1}}$ and $R = \dfrac{1}{3}$

12. Suppose that $\sum c_n(x-2)^n$ converges for $x = 4$ and diverges for $x = -2$. Which of the following must be correct?

A. $\sum c_n(-2)^n$ converges B. $\sum c_n 4^n$ diverges

C. $\sum c_n 3^n$ converges D. $\sum c_n(-1)^n$ converges

E. $\sum c_n \left(\frac{5}{2}\right)^n$ diverges

13. If $f(x) = \sin^2(x)$, find $f^{(102)}(0)$.

A. $-\dfrac{102!}{205!}$ B. $\dfrac{102!}{205!}$ C. $-\dfrac{102!}{51!}$ D. $\dfrac{102!}{205!}$ E. $\dfrac{205!}{102!}$

14. Set up an integral for the area of the region that lies inside $r = \sqrt{3}\sin\theta$ and outside $r = \cos\theta$.

A. Area $= \displaystyle\int_{\pi/3}^{2\pi} \frac{1}{2}(\sqrt{3}\sin\theta)^2 d\theta - \int_{\pi/3}^{\pi} \frac{1}{2}(\cos\theta)^2 d\theta$

B. Area $= \displaystyle\int_{\pi/6}^{2\pi} \frac{1}{2}(\sqrt{3}\sin\theta)^2 d\theta - \int_{\pi/6}^{\pi/2} \frac{1}{2}(\cos\theta)^2 d\theta$

C. Area $= \displaystyle\int_{\pi/6}^{2\pi} \frac{1}{2}(\sqrt{3}\sin\theta)^2 d\theta - \int_{\pi/6}^{\pi} \frac{1}{2}(\cos\theta)^2 d\theta$

D. Area $= \displaystyle\int_{\pi/6}^{\pi} \frac{1}{2}(\sqrt{3}\sin\theta)^2 d\theta - \int_{\pi/6}^{\pi/2} \frac{1}{2}(\cos\theta)^2 d\theta$

E. Area $= \displaystyle\int_{\pi/3}^{\pi} \frac{1}{2}(\sqrt{3}\sin\theta)^2 d\theta - \int_{\pi/3}^{\pi/2} \frac{1}{2}(\cos\theta)^2 d\theta$

15. Graph the following polar equation: $r = 2 - 2\sin\theta$