

### Review 3 - Answers

3.  $\frac{16}{3}$  4. 60 5. (1)  $\frac{1}{3}$ , (2)  $2\int_0^1 dx \int_0^{1-x} (1-x-y) dy$ , (3)  $\int_0^1 dx \int_0^{1-x} dy \int_0^{2-2x-2y} 1 dz$

6. (a)  $\left(\int_0^1 2y dy\right)\left(\int_0^3 2xe^{x^2} dx\right)$ , (b)  $e^9 - 1$ ; 7. (a)  $\frac{8}{33}$ , (b)  $\int_0^1 \int_{y^2}^1 x\sqrt{y} dx dy$

8. (a)  $V = \iint_D x^2 dA$ , (b)  $\int_0^1 \int_y^{2-y} x^2 dx dy$ , (c)  $\frac{7}{6}$  9.  $\sin e - \sin 1$

10.  $\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} rf(r) dr d\theta$ ,  $\int_0^2 \int_{-\cos^{-1}(r/2)}^{\cos^{-1}(r/2)} rf(r) d\theta dr = \int_0^2 2\cos^{-1}(r/2) rf(r) dr$

11.  $D = \left\{(\theta, r) \mid -\frac{\pi}{6} \leq \theta \leq \frac{\pi}{6}, \frac{a}{\sqrt{2}} \leq r \leq a\sqrt{\cos(2\theta)}\right\}$ , Area =  $\frac{a^2}{12}(3\sqrt{3} - \pi)$

12.  $(r, \theta, z) = \left(\sqrt{2}, \frac{3\pi}{4}, 1\right)$  and  $(\rho, \varphi, \theta) = \left(\sqrt{3}, \cos^{-1}(1/\sqrt{3}), \frac{3\pi}{4}\right)$

13.  $S = \{(x, y, z) \mid 0 \leq x \leq \sqrt{2}, 0 \leq y \leq \sqrt{2-x^2}, x^2 + y^2 \leq z \leq 4-x^2-y^2\}$ ;  $V = \pi$

14. (a)  $\int_0^{2\pi} \cos\theta \sin^2\theta d\theta \int_0^2 r^4 dr \int_2^5 z dz = 0$

(b)  $\int_0^\pi \cos\theta \sin^2\theta d\theta \int_0^{\pi/4} \sin^4\varphi \cos\varphi d\varphi \int_0^3 \rho^6 d\rho = 0$

(c)  $\int_{-\pi/2}^{\pi/2} \cos\theta \sin^2\theta d\theta \int_0^2 r^4 dr \int_0^{4-r^2} z dz$

(d)  $\int_0^\pi \cos\theta (\sin^2\theta) d\theta \int_0^{\pi/3} \sin^4\varphi \cos\varphi d\varphi \int_1^{2\cos\theta} \rho^6 d\rho = 0$

15.  $\frac{\pi}{10}$  16.  $\frac{15}{32}\left(\arctan 2 - \frac{\pi}{4}\right)$  17.  $\frac{2\pi}{5}(\sqrt{2}-1)$  18.  $\frac{7\pi}{48}$

19. (a)  $\{(x, y, z) \mid 0 \leq x \leq 2, 0 \leq y \leq \sqrt{4-x^2}, \sqrt{x^2+y^2} \leq z \leq 2\}$  (b)  $\int_0^{\pi/2} d\theta \int_0^2 r dr \int_r^2 dz$ ,

(c)  $V = \frac{2\pi}{3}$  20. (a)  $T^{-1}(x, y) = (u, v)$ , where  $u = \frac{x^2}{y}, v = \frac{y^2}{x}$ , (c)  $J(u, v) = \frac{1}{3}$ , (d) Area =  $\frac{2}{3}$

21.  $J(u, v, w) = \frac{1}{u}$  22. (a) The region in the first quadrant bounded by the coordinate axes and the curve  $x^{2/3} + y^{2/3} = 1$  (a branch of an astroid), (b)  $J(u, v) = 3u \cos^2 v \sin^2 v$

23.  $\frac{3\pi}{2}$  24. (a)  $\frac{8}{15}$ , (b)  $\frac{3}{7}$  25. Half of a disk:  $x^2 + (y-1)^2 \leq 1, x \leq 0$