<u>Lecture 20 Homework</u>

Definitions/Terminology, Graphing, one-to-one property and solving, compound interest, natural base, continuous compounding, applications

1.
Identify whether the following equation represents an exponential function.

$h(x) = -99x + 71$

- exponential function
- NOT an exponential function

2.
Determine whether the following equation represents an exponential growth or exponential decay.

$y = 280 \cdot (1.81)^x$

- exponential growth
- exponential decay

3.
Determine whether the following equation represents exponential growth or exponential decay.

$y = \left(\frac{13}{29}\right) \cdot \left(\frac{2}{27}\right)^x$

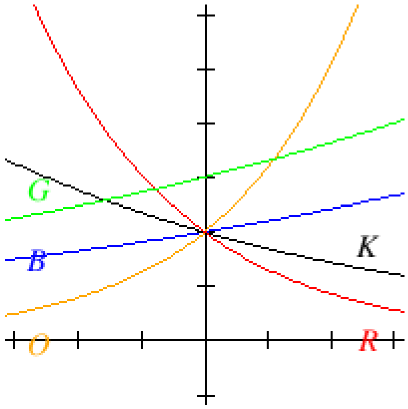
- exponential decay
- exponential growth

4.
Introduction to Exponential Functions

Which of the following are true regarding $f(x) = a(b)^x$? Assume a > 0 .

- The Range of the exponential functions is All Real Numbers .
- The Range of the exponential functions is $f(x) > 0$.
- The Domain of the exponential functions is All Real Numbers .
- The Horizontal Asymptote is the point $(0, a)$.
- The Horizontal Asymptote is the line $y = 0$.
- The Horizontal Asymptote is the line $x = 0$.
- The Domain of the exponential functions is $x > 0$.

5.



If all the graphs above have equations with form $y = ab^x$,

Which graph has the largest value for b?

- blue (B)
- red (R)
- green (G)
- orange (O)
- black (K)

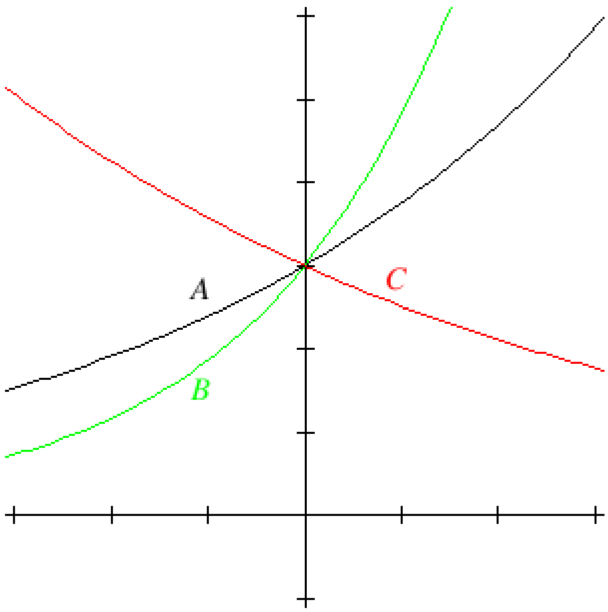
Which graph has the smallest value for b?

- blue (B)
- black (K)
- orange (O)
- green (G)
- red (R)

Which graph has the largest value for a?

- blue (B)
- green (G)
- orange (O)
- black (K)
- red (R)

6.
Match the graph with its function



- $3(0.84)^x$
- $3(1.6)^x$
- $3(1.25)^x$

7.
An exponential function $f(x) = ab^x$ passes through the points (0, 8) and (3, 512). What are the values of a and b ?

a =________
b =________

8.
An exponential function $f(x) = ab^x$ passes through the points (0, 7000) and (3, 5103). What are the values of a and b ?

a =________
b =________

9.
Find a formula for the exponential function passing through the points (-3, $\frac{3}{64}$) and (3, 192).

y =________

10.
The fox population in a certain region has an annual growth rate of 6 percent per year. It is estimated that the population in the year 2000 was 28100.

(a) Find a function that models the population t years after 2000 ($t = 0$ for 2000).
Your answer is $P(t) =$_________

(b) Use the function from part (a) to estimate the fox population in the year 2008.
Your answer is (the answer should be an integer) _________

11.
A population numbers 12,000 organisms initially and grows by 3.4% each year.

Suppose P represents population and t represents the number of years of growth. An exponential model for the population can be written in the form $P = ab^t$, where

$P =$_________

12.
A population numbers 19,000 organisms initially and decreases by 8% each year.

Suppose P represents population, and t the number of years of growth. An exponential model for the population can be written in the form $P = ab^t$, where

$P =$_________

13.
A vehicle purchased for $ 29800 depreciates at a constant rate of 4 %.

Determine the approximate value of the vehicle 12 years after purchase.

Round to the nearest whole number.

14.
A radioactive substance decays exponentially. A scientist begins with 110 milligrams of a radioactive substance. After 30 hours, 55 mg of the substance remains.

How many milligrams will remain after 39 hours?

_________mg

Give your answer accurate to at least one decimal place.

15.
A car was valued at $36,000 in the year 1995. The value depreciated to $10,000 by the year 2000.

A) What was the annual rate of change between 1995 and 2000?
r = _________ *Round the rate of decrease to 4 decimal places.*

B) What is the correct answer to part A written in percentage form?
r = _________ %.

C) Assume that the car value continues to drop by the same percentage. What will the value be in the year 2003 ?
value = $_________ *Round to the nearest 50 dollars.*

16.
The population of the world in 2010 was 7 billion and the annual growth rate was estimated at 1.2 percent per year. Assuming that the world population follows an exponential growth model, find the projected world population in 2057. Round your answer to one decimal place.
The population is estimated to be _________ billion in 2057.

17.
A house was valued at $105,000 in the year 2012. The value appreciated to $145,000 by the year 2018.

A) What was the annual growth rate between 2012 and 2018 as a decimal? Round to four decimal places.
r =_________

B) What is the annual growth rate as a percent? Round to two decimal places.
r = _________%.

C) Assuming that the house value continues to grow at the same rate, what will the value equal in the year 2021? Round to the nearest thousand dollars.
value = $_________

18.
A car was valued at $29,000 in the year 2010. The value depreciated to $13,000 by the year 2016.

A) What was the annual growth rate between 2010 and 2016 in decimal form? Round to four decimal places.
r = _________

B) What was the annual growth rate in percentage form? Round to two decimal places.
r = _________%.

C) Assuming that the car's value continues to drop at the same rate, what will the value be in the year 2019? Round to the nearest fifty dollars.
value = $_________

19.

In April 1986, a flawed reactor design played a part in the Chernobyl nuclear meltdown. Approximately 14252 becqurels (Bqs), units of radioactivity, were initially released into the environment. Only areas with less than 800 Bqs are considered safe for human habitation. The function $f(x) = 14252(0.5)^{\frac{x}{32}}$ describes the amount, $f(x)$, in becqurels, of a radioactive element remaining in the area x years after 1986.

Find $f(40)$ to determine the amount of becqurels in 2026. _________

Determine if the area is safe for human habitation in the year 2026.

- No, because by 2026, the radioactive element remaining in the area is greater than 800 Bqs.
- No, because by 2026, the radioactive element remaining in the area is less than 800 Bqs.
- Yes, because by 2026, the radioactive element remaining in the area is greater than 800 Bqs.
- Yes, because by 2026, the radioactive element remaining in the area is less than 800 Bqs.

20.
The half-life of caffeine in the human body is about 5.6 hours. A cup of coffee has about 110 mg of caffeine.

a. Write an equation for the amount of caffeine in a person's body after drinking a cup of coffee? Let $\boldsymbol{C}$ be the milligrams of caffeine in the body after $\boldsymbol{t}$ hours.
$C(t) =$_________

b. How much caffeine will remain after 10 hours?
_________ mg

c. Estimate the time until there are only 20 mg remaining
_________ hours

21.
Starting with the graph of $f(x) = 9^x$, write the equation of the graph that results when:

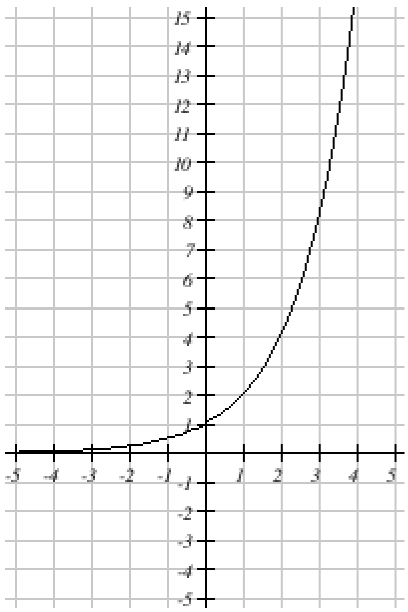
(a) $f(x)$ is shifted 4 units upward. $y =$_________

(b) $f(x)$ is shifted 1 units to the right. $y =$_________

(c) $f(x)$ is reflected about the x-axis. $y =$_________

22.

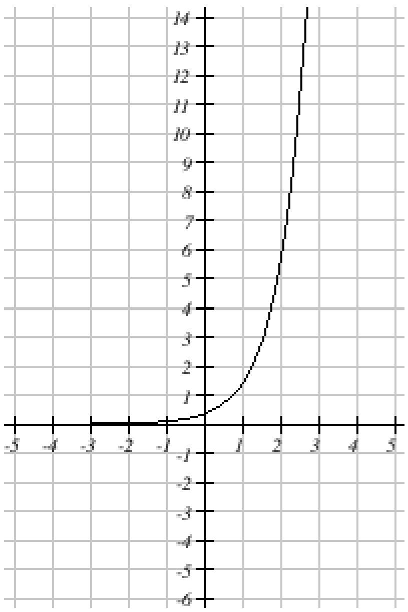
What function is graphed below?



- $f(x) = \left(\frac{1}{5}\right)^x$
- $f(x) = \left(\frac{1}{2}\right)^x$
- $f(x) = 2^x$
- $f(x) = 3^x$
- $f(x) = 5^x$
- $f(x) = \left(\frac{1}{3}\right)^x$
- $f(x) = \left(\frac{1}{4}\right)^x$
- $f(x) = 4^x$

23.

The function below has the form $f(x) = a \cdot b^x$.

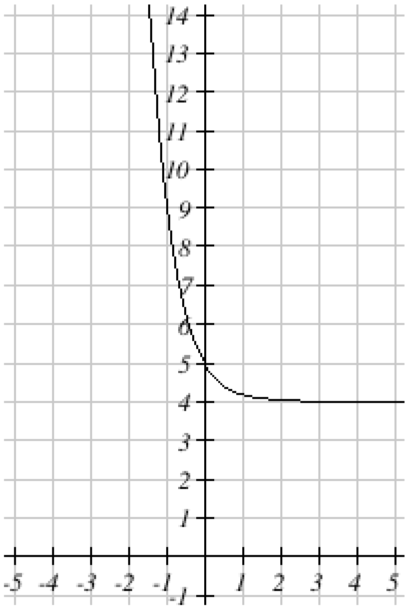


Which of the following functions is shown on the graph?

- $f(x) = 3 \cdot 4^x$
- $f(x) = 3 \cdot \left(\frac{1}{4}\right)^x$
- $f(x) = \frac{1}{3} \cdot 4^x$
- $f(x) = -\frac{1}{3} \cdot 4^x$
- $f(x) = -3 \cdot 4^x$
- $f(x) = -\frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$
- $f(x) = \frac{1}{3} \cdot \left(\frac{1}{4}\right)^x$
- $f(x) = -3 \cdot \left(\frac{1}{4}\right)^x$

24.

The function below has the form $f(x) = b^x + k$.

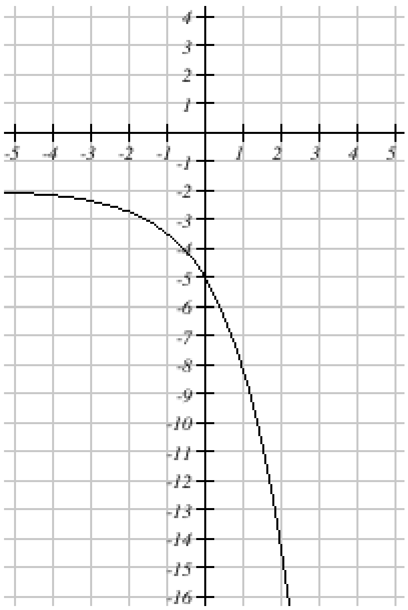


Which of the following functions is shown on the graph?

- $f(x) = \left(\frac{1}{5}\right)^x + 3$
- $f(x) = 5^x + 5$
- $f(x) = \left(\frac{1}{5}\right)^x + 5$
- $f(x) = 5^x + 3$
- $f(x) = \left(\frac{1}{5}\right)^x + 4$
- $f(x) = \left(\frac{1}{5}\right)^x - 4$
- $f(x) = 5^x + 4$
- $f(x) = 5^x - 4$

25.

The function below has the form $f(x) = a \cdot b^x + k$.

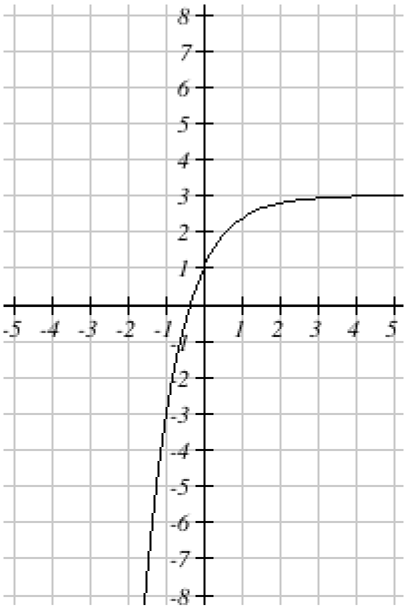


Which of the following functions is shown on the graph?

- $f(x) = 3 \cdot 2^x + 2$
- $f(x) = -\frac{1}{3} \cdot 2^x - 1$
- $f(x) = 3 \cdot 2^x - 2$
- $f(x) = -3 \cdot \left(\frac{1}{2}\right)^x - 3$
- $f(x) = 3 \cdot \left(\frac{1}{2}\right)^x + 2$
- $f(x) = -\frac{1}{3} \cdot \left(\frac{1}{2}\right)^x + 2$
- $f(x) = -3 \cdot 2^x - 2$
- $f(x) = 3 \cdot 2^x - 3$

26.

Find an equation for the graph sketched below



$f(x)$ =__________

27.

Describe the long run behavior of $f(n) = 2(4)^n + 3$

As $n \to -\infty$, $f(n) \to$

- ∞
- -∞
- 0
- 3

As $n \to \infty$, $f(n) \to$

- ∞
- -∞
- 0
- 3

28.

Describe the long run behavior of $f(t) = 2\left(\frac{1}{3}\right)^t - 1$

As $t \to -\infty$, $f(t) \to$

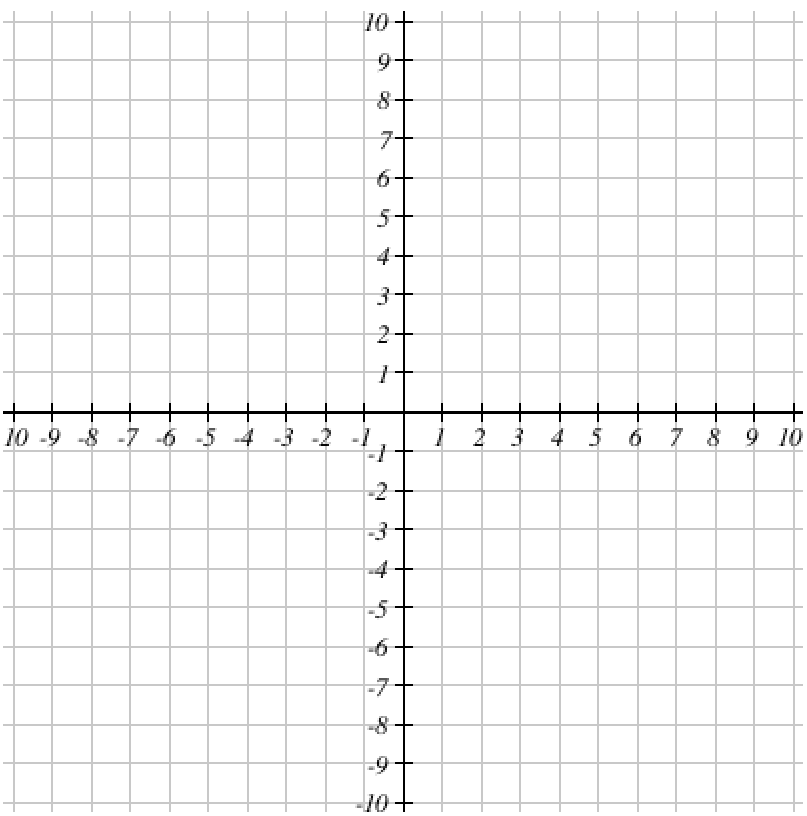
- ∞
- -∞
- 0
- -1

As $t \to \infty$, $f(t) \to$

- ∞
- -∞
- 0
- -1

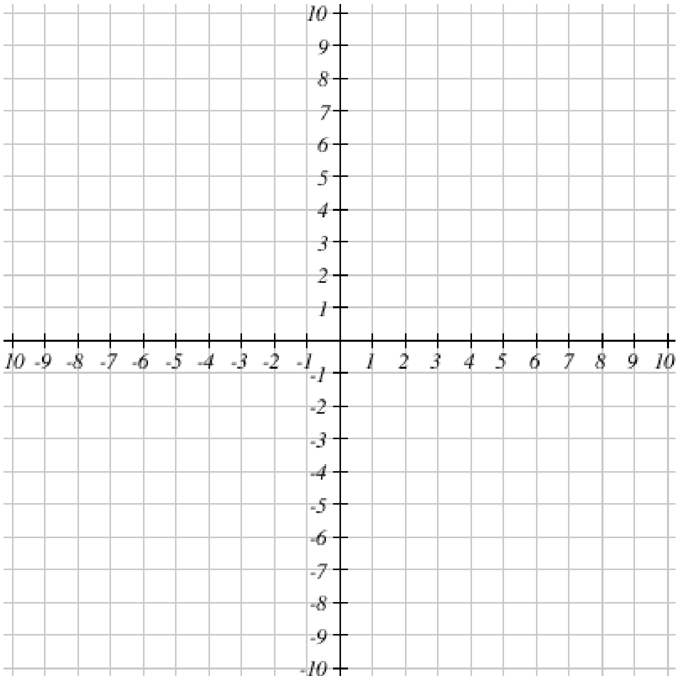
29.

Draw the graph of $f(x) = 3^{x-3}$



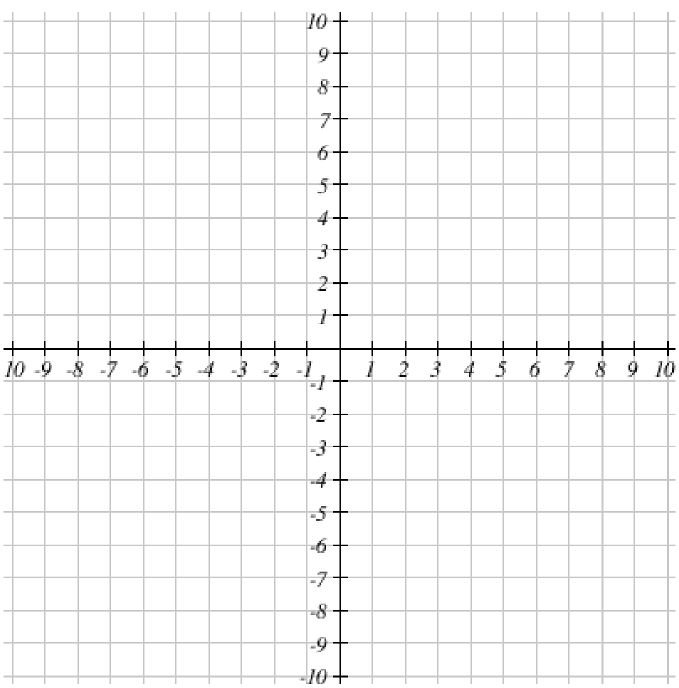
30.

Draw the graph of $f(x) = 4^{x+4}$



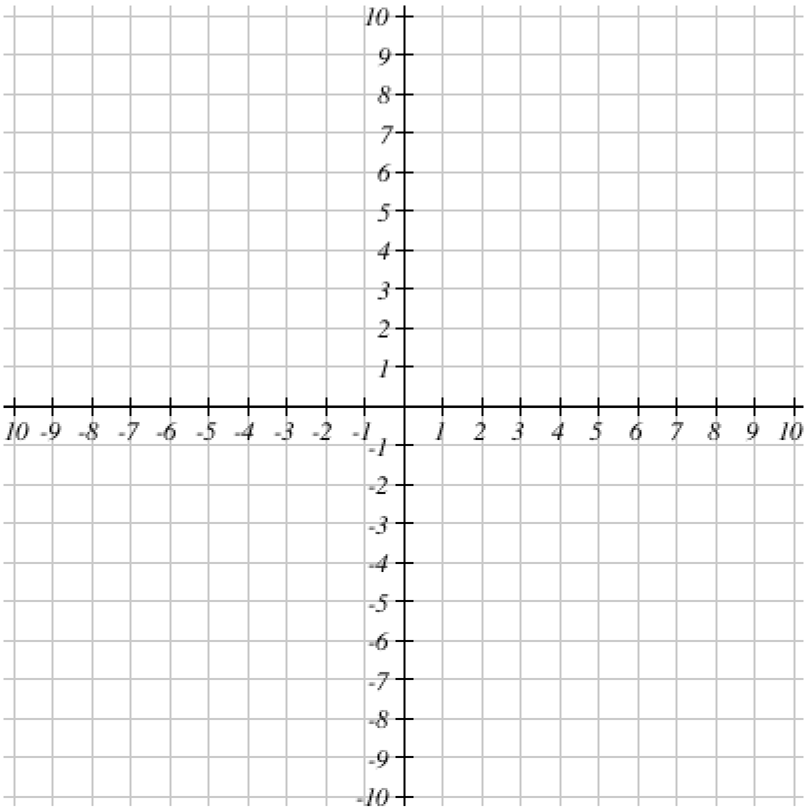
31.

Draw the graph of $f(x) = \left(\frac{1}{5}\right)^{x+2}$



32.

Draw the graph of $f(x) = -\left(\frac{1}{6}\right)^{x}$



33.
The temperature, $f(t)$ of a cup of coffee, in degrees Celsius, after t minutes can be determined by the equation $f(t) = 58(0.89)^{t} + 20$.

Estimate the temperature after 5 minutes:
_________ degrees

Determine the asymptote for the function:
The asymptote is _________

Interpret the asymptote in the context of the problem. Use complete sentences and correct units.

34.
Use the like-bases property and exponents to solve the equation
$$\left(\frac{1}{10}\right)^{n} = 10000$$
$n =$_________

35.
Use the like-bases property and exponents to solve the equation
$10^{n+10} = 10^{6n+6}$
$n =$_________

36.
Use the like-bases property and exponents to solve the equation
$6(10)^x - 23 = 59977$
$x =$_________

37.

Use the like bases property to solve the equation. Give your answer as an integer or reduced fraction.

$\left(\frac{1}{64}\right) = 2^{5x-8}$

$x =$_________

38.
Solve the equation $125^{z+5} = 25^{z+4}$.

$z =$_________

39.

$9 \cdot 5^{x+4} = 625 \cdot 3^{x+2}$

$x =$ _________

40.

$2 \cdot 2^{x-1} + 2^{x+8} - \frac{257}{16} = 0$

$x =$ _________

Lecture 21 Homework

Definitions/Terminology, Graphing, one-to-one property and solving, compound interest, natural base, continuous compounding, applications

1.

Approximate e^3 to 4 decimal places. _________

Approximate $e^{-1.48}$ to 4 decimal places. _________

2.
A) $e^5 \cdot e^8 = e^p$ where p =_________

B) $(e^5)^8 = e^r$ where r = _________

3.

The expression
$$\frac{e^7(2e)^6}{e^3}$$
equals ce^f where
the coefficient c is _________, the exponent f is _________.

4.
Solve the given equation. Enter the exact value of the solution, in simplified form.

$e^{4b+3} = 1$

b =_________

5.
Determine whether the following equation represents an exponential growth or exponential decay.

$y = 164.5 \cdot (e)^{-1.52 \cdot x}$

- exponential decay
- exponential growth

6.
Determine whether the following equation represents an exponential growth or exponential decay.

$y = 43 \cdot (e)^{1.53 \cdot x}$

- exponential growth
- exponential decay

7.
\$4000 are invested in a bank account at an interest rate of 9 percent per year.

Find the amount in the bank after 7 years if interest is compounded annually. _________

Find the amount in the bank after 7 years if interest is compounded quarterly. _________

Find the amount in the bank after 7 years if interest is compounded monthly. _________

Finally, find the amount in the bank after 7 years if interest is compounded continuously. _________

8.
A bank features a savings account that has an annual percentage rate of $r = 2.2$ % with interest compounded <u>quarterly</u>. Stephanie deposits \$2,500 into the account.

The account balance can be modeled by the exponential formula $A(t) = a\left(1 + \frac{r}{k}\right)^{kt}$, where A is account value after t years , a is the principal (starting amount), r is the annual percentage rate, k is the number of times each year that the interest is compounded.

(A) What values should be used for a , r, and k ?
$a =$_________ , $r =$_________ , $k =$_________

(B) How much money will Stephanie have in the account in 7 years?
Amount = \$_________
Round answer to the nearest penny.

(C) What is the annual percentage yield (APY) for the savings account? (The APY is the actual or effective annual percentage rate which includes all compounding in the year).
$APY =$ _________%
Round answer to 3 decimal places.

9.
The fox population in a certain region has a continuous growth rate of 8 percent per year. It is estimated that the population in the year 2000 was 15400.

(a) Find a function that models the population t years after 2000 ($t = 0$ for 2000).
Your answer is $P(t)$ =_________

(b) Use the function from part (a) to estimate the fox population in the year 2008.
Your answer is _________ (the answer must be an integer)

10.
Convert the equation $f(t) = 336e^{0.2t}$ to the form $f(t) = ab^t$

a =_________

b =_________

Give answers accurate to three decimal places

11.
Convert the equation $f(t) = 178e^{0.835t}$ to the form $f(t) = a(b)^t$. Round b to three decimal places.
$f(t)$ =_________

12.
The number of bacteria in a culture is given by the function
$n(t) = 960e^{0.25t}$
where t is measured in hours.
(a) What is the relative rate of growth of this bacterium population?
Your answer is _________ percent
(b) What is the initial population of the culture (at t=0)?
Your answer is _________
(c) How many bacteria will the culture contain at time t=5?
Your answer is _________

13.
For each nominal exponential growth/decay described below, find the effective annual growth rate and express it as a percentage rounded to one decimal place.

A quantity's size after t years is given by $A(t) = (1.07)^t$. Its effective growth rate is ________% per year.

A quantity shrinks at a continous rate of 40% per year. Its effective growth rate is ________% per year.

A quantity grows at a rate of 20% compounded monthly. Ifs effective growth rate is ________% per year.

A quantity has a half-life of 12 years. Its effective annual growth rate is ________% per year.

A quantity has a tripling time of 7 years. Its effective annual growth rate is ________% per year.

14.
A bacteria culture initially contains 2500 bacteria and doubles every half hour.

Find the size of the baterial population after 40 minutes. ________

Find the size of the baterial population after 8 hours. ________

15.
The half-life of Radium-226 is 1590 years. If a sample contains 400 mg, how many mg will remain after 2000 years? ________

16.
You want to have $700,000 when you retire in 30 years. If you can earn 8% interest compounded monthly, how much would you need to deposit now into the account to reach your retirement goal?

$________

17.
A radioactive substance decays exponentially. A scientist begins with 160 milligrams of a radioactive substance. After 25 hours, 80 mg of the substance remains. How many milligrams will remain after 45 hours?

________ mg

Give your answer accurate to at least one decimal place

18.
Let $P(t) = 25(1 - e^{-kt}) + 53$ represent the expected score for a student who studies t hours for a test. Suppose $k = 0.17$ and test scores must be integers.

What is the highest score the student can expect? _________

If the student does not study, what score can he expect? _________

Lecture 22 Homework

Definitions/Terminology, Properties (Basic, inverse, one-to-one), Graphs, Natural log, Common log

1.
Write the equation in exponential form.
Assume that all constants are positive and not equal to 1.

$\log_y(m) = b$

2.
Write the equation in exponential form.
Assume that all constants are positive and not equal to 1.

$\log(s) = c$

3.
Write the equation in logarithmic form.
Assume that all constants are positive and not equal to 1.

$9^r = n$

4.
Write the equation in logarithmic form.
Assume that all constants are positive and not equal to 1.

$10^c = v$

5.
Fill in each box below with an integer or a reduced fraction.

(a) $\log_2 8 = 3$ can be written in the form $2^A = B$ where
$A =$_________ and $B =$_________

(b) $\log_5 25 = 2$ can be written in the form $5^C = D$ where
$C =$_________ and $D =$_________

6.
Fill in each box below with an integer or a reduced fraction.

(a) $\log_4 2 = \frac{1}{2}$ can be written in the form $A^B = C$ where
$A =$________, $B =$________, and $C =$________

(b) $\log_2\left(\frac{1}{4}\right) = -2$ can be written in the form $D^E = F$ where
$D =$________, $E =$________, and $F =$________

7.
Express the equation in logarithmic form:
(a) $e^x = 4$ is equivalent to $\ln A = B$. Then
$A =$________
and
$B =$________

(b) $e^3 = x$ is equivalent to $\ln C = D$. Then
$C =$________
and
$D =$________

8.

Express the following equations in logarithmic form:

(a) $3^4 = 81$ is equivalent to the logarithmic equation: ________

(b) $10^{-3} = 0.001$ is equivalent to the logarithmic equation: ________

9.

Express equation equation in logarithmic form.

(a) $e^x = 8$ is equivalent to the logarithmic equation: ________

(b) $e^5 = x$ is equivalent to the logarithmic equation: ________

10.
Write the equation in exponential form. Assume that all constants are positive and not equal to 1.

$$\log_{343}(7) = \frac{1}{3}$$

11.
Simplify without a Calculator

$\log_8(512) =$ ________

12.
Simplify without a Calculator

$\log\left(\frac{1}{10}\right) =$ ________

13.
Find the logarithm.

$\log_5\left(\frac{1}{5}\right) =$________

14.
Find the logarithm.

$\log_5\left(\frac{1}{3125}\right) =$________

15.
Evaluate the following expressions. Your answers must be exact and in simplest form.

(a) $\ln e^{-5} =$________

(b) $e^{\ln 3} =$________

(c) $e^{\ln\sqrt{2}} =$________

(d) $\ln\left(\frac{1}{e^5}\right) =$________

16.

Evaluate the following expressions without using a calculator.

(a) $\ln\left(\frac{1}{e^7}\right)$ =________

(b) $\ln\left(\sqrt[5]{e^2}\right)$ =________

(c) $\ln(e^5)$ =________

(d) $e^{\ln(5)}$ =________

(e) $e^{\ln(\sqrt{3})}$ =________

17.
Solve: $\log_6(t) = 7$

$t =$________

18.
If $\log_2(5x+2) = 6$, then $x =$ ________.

19.
If $\ln(5x+5) = 4$, then $x =$ ________.

20.
Solve for x :

$\log_2(x^9) = 8$

$x =$________

21.
Solve for x :

$\left(\log_4(\log_3 x)\right) = -3$

$x =$________

22.
Simplify

$\log_z(z^9) =$ ________

23.
Solve for x:

$\log_x(2) = 1$

$x =$________

24.
Find the logarithm.

$\log_3\left(3^{\frac{1}{5}}\right) =$________

25.
Find the logarithm.

$\log(10{,}000) =$________

26.
Find the logarithm.

$\log\left(\frac{1}{100{,}000}\right) =$________

27.
Evaluate the following expressions.

(a) $\log_7 7^{12} =$________

(b) $\log_2 32 =$________

(c) $\log_4 1024 =$________

(d) $\log_4 4^4 =$________

28.
Simplify

$\log_x \left(\frac{1}{(x^6)^7}\right) =$ ________

29.
Solve for x in each equation below. It may be helpful to convert the equation into exponential form.

(A) $\log_a a^{-2} = x$

$x =$________

(B) $\log_a a^4 = x$

$x =$________

(C) $\log_a a = x$

$x =$________

(D) $\log_a a^n = x$

$x =$________

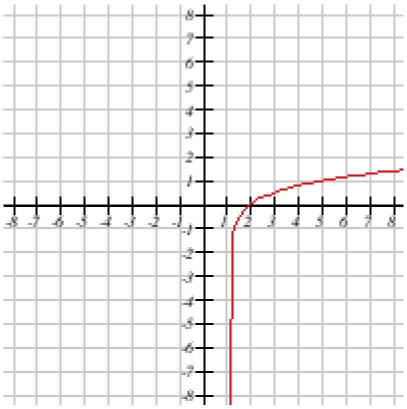
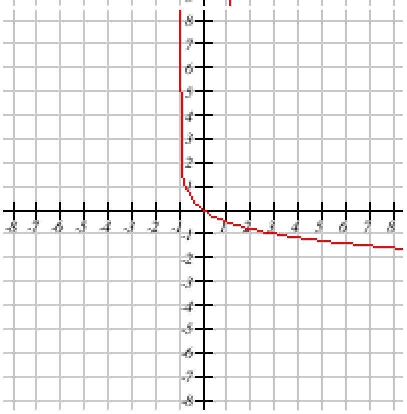
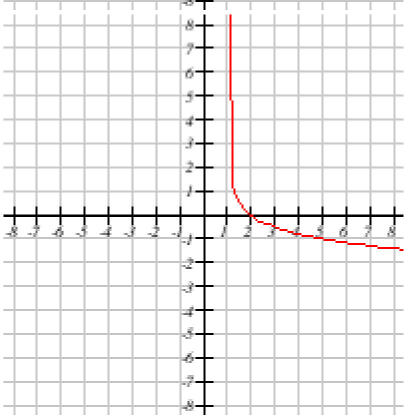
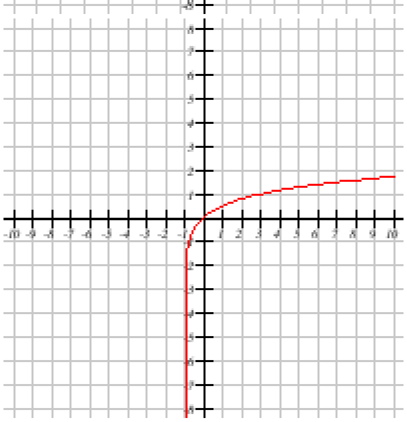
30.
Evaluate using your calculator, giving at least 3 decimal places:

log(680) =________

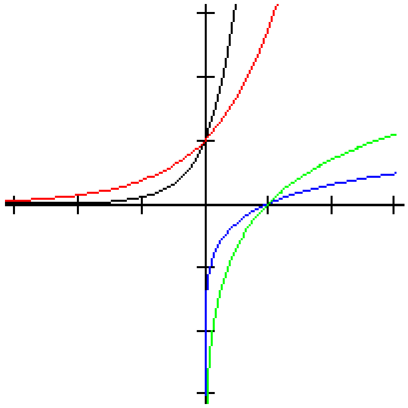
31.
Find the equation of the vertical asymptote $f(x) = \log(x - 4)$

32.

Choose the graph of $y = \log_{\frac{1}{4}}(x + 1)$

- 
- 
- 
- 

33.



Match each equation with a graph above:

- $\ln(x)$
- e^x
- $\log(x)$
- 10^x

1. black
2. red
3. blue
4. green

34.
Find the domain of $y = \log(5 - 5x)$.

The domain is: ________

35.

Find the domain of $\log(x^2 + x - 2)$ in interval notation.

36.
Find the domain of $\log(x^2 + 2x - 3)$ in interval notation.

37.
Find the domain and range of $y = \log_2(5 + 2x)$.

The domain is: ________

The range is : ________

38.
The Richter Scale reading of an earthquake is based on a logarithmic equation:

$$R = \log\left(\frac{A}{A_0}\right)$$

where
A - the measure of the amplitude of the earthquake wave
A_0 the amplitude of the smallest detectable wave (or standard wave).

An earthquake is measured with a wave amplitude $A = 0.0982$ while the smallest detectable was A_0 is measured at 0.0001 cm. What is the magnitude of this earthquake using the Richter scale, to the nearest tenth?

The earthquake registered _________ on the Richter scale.

39.
The pH scale for acidity is defined by $\text{pH} = -\log_{10}[\text{H}^+]$ where $[\text{H}^+]$ is the concentration of hydrogen ions measured in moles per liter (M).

A solution has a pH of 12.6.

Calculate the concentration of hydrogen ions in moles per liter (M).

The concentration of hydrogen ions is _________ moles per liter.

40.
The pH reading of a sample of each substances is given. Calculate the hydrogen ion concentration of the substance.
(a) Vinegar: pH = 3.0.
Your answer is _________.
(b) Milk: pH = 6.5.
Your answer is _________.

Lecture 23 Homework

Properties (Multiplication, division, exponent), expanding/combining expressions, common mistakes, change of base formula, measuring with logs

1.
Find the exact value for the following expression without a calculator

$\frac{\log_2(625)-\log_2(625)}{\log_2(125)-\log_2(125)} =$ ________

2.
Which of the following statements is TRUE? Select ALL that apply.

- $\log(x+y) = \log(x) + \log(y)$
- $\log_7(xy) = \log_7(x) + \log_7(y)$
- $\log_2\left(\frac{x}{y}\right) = \log_2(x) - \log_2(y)$
- $\ln\left(\frac{x}{y}\right) = \frac{\ln(x)}{\ln(y)}$
- $\log_4(xy) = \log_4(x) \cdot \log_4(y)$
- $\log_b\left(\frac{1}{5}\right) = -\log_b(5)$

3.
Which of the following are equivalent to $\log_b(3)$? Choose all that apply.

- $-\log_b\left(\frac{1}{3}\right)$
- $\log_b\left(\frac{1}{10}\right) + \log_b(30)$
- $\frac{1}{2}\log_b(9)$
- $\frac{1}{3}\log_b(27)$
- $\log_b(21) - \log_b(7)$

4.
Simplify

$\log_y\left(\frac{y^7}{y^7}\right) =$ ________

5.
Simplify the given expression.

$e^{\ln(N)} =$ ________

6.

Write the following as the sum and/or difference of logarithms. Assume all variables are positive.

$\log\left(\frac{5c}{11}\right) =$________

7.

Write expression $\log\left(\frac{x^{20}y^{11}}{z^2}\right)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$\log\left(\frac{x^{20}y^{11}}{z^2}\right) =$________

8.

Expand the given logarithm and simplify. Assume when necessary that all quantities represent positive real numbers. Be sure to factor out any common factors in your final answer.

$\ln(x^{139}y^{-41}) =$________

9.

Expand the given logarithm and simplify. Assume when necessary that all quantities represent positive real numbers. Be sure to factor out any common factors in your final answer.

$\log_9\left(\frac{729}{x^2-9}\right) =$________

10.

Write the expression $\ln\left(\frac{w^{20}x^8}{\sqrt[5]{z+2}}\right)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$\ln\left(\frac{w^{20}x^8}{\sqrt[5]{z+2}}\right) =$ ________

11.

Write the expression $\log_3(9x^{15}y^8)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$\log_3(9x^{15}y^8) =$________

12.

Write the expression $\log\sqrt[3]{\frac{y^3w^{11}}{x^{10}}}$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$\log\sqrt[3]{\frac{y^3w^{11}}{x^{10}}} =$ ________

13.

Let $\log(A) = 12$, $\log(B) = 7$, and $\log(C) = 6$. Evaluate the following logarithms using logarithmic properties.

- $\log\left(\frac{A^2}{\sqrt{B}}\right) =$ ________
- $\log\left(\frac{B}{C^5}\right) =$ ________
- $\log\left(\frac{A}{B^3C}\right) =$ ________

14.
Evaluate using your calculator and round to 4 decimal places.

$\log_{19} 44 =$________

15.
Simplify the expressions given. Use exact values. (Hint: Using the change-base formula might make these problems quite easy.)

$\log_8(32) =$________

$\log_8(\sqrt{32}) =$________

$\log_8\left(\frac{1}{\sqrt{32}}\right) =$ ________

16.
Given that f is defined by $f(t) = -5(2^t) + 6$, which of the following is a formula for f^{-1} ?

- $f^{-1}(t) = \frac{1}{-5(2^t)+6}$
- $f^{-1}(t) = -5\left(\frac{\ln t}{\ln 2}\right) + 6$
- $f^{-1}(t) = \frac{\ln\left(\frac{t-6}{-5}\right)}{\ln 2}$
- $f^{-1}(t) = \ln(-5(2^t) + 6)$
- $f^{-1}(t) = \frac{\frac{t}{-5}-6}{\ln 2}$

17.
Write the following as a single logarithm. Assume all variables are positive.

$\log_3(5) + 5\log_3(b) =$________

18.
Write the following sum as a single logarithm. Assume all variables are positive.

$\log_3(b) + \log_3(b+5) =$________

19.
Simplify the following into a single logarithm:
$3\log(7) - 1\log(x)$

- $\log(7^3 x^1)$
- $\log\left(\frac{7^3}{x^1}\right)$
- $\log\left(\frac{3 \cdot 7}{1x}\right)$
- $\log(3 \cdot 7 \cdot 1x)$
- $\log(3 \cdot 7 \cdot x^1)$

20.
Write the expression $20\log_4(w) + 5\log_4(x)$ as a single logarithm.

$20\log_4(w) + 5\log_4(x) =$________

21.
Write the expression $4\log_3(w) + 9\log_3(x) - \frac{1}{2}\log_3(y+17)$ as a sum or difference of logarithms with no exponents. Simplify your answer completely.

$4\log_3(w) + 9\log_3(x) - \frac{1}{2}\log_3(y+17) =$________

Lecture 24 Homework

Solving exponential equations, solving logarithmic equations, finding inverses of exponential/log functions

1.
Use a calculator to find the natural logarithm.

$\ln(4.53)$

2.
Simplify mentally using the properties of logarithms

$\ln(e^8)$

3.
Simplify mentally using the properties of logarithms.

$e^{\ln(2)}$

4.
Use the like-bases property and exponents to solve the equation
$\left(\frac{1}{5}\right)^{n+3} = 5^{3n-4}$
$n =$_________

5.
If $\ln x + \ln(x-2) = \ln(3x)$, then $x =$_________.

6.
5000 dollars is invested in a bank account at an interest rate of 9 percent per year, compounded continuously. Meanwhile, 36000 dollars is invested in a bank account at an interest rate of 3 percent compounded annually.

To the nearest year, When will the two accounts have the same balance?

The two accounts will have the same balance after _________ years.

7.
If 7000 dollars is invested in a bank account at an interest rate of 7 per cent per year, compounded continuously. How many years will it take for your balance to reach 10000 dollars?

NOTE: Give your answer to the nearest tenth of a year.

8.
Find the time required for an investment of 5000 dollars to grow to 8400 dollars at an interest rate of 7.5 percent per year, compounded quarterly.
Your answer is $t =$ ________ years.

9.
Solve for x :

$11(1.09^x) = 18(1.12^x)$

$x =$________

10.
Solve for x :

$9(1.17^x) = 20(1.09^x)$

$x =$________

11.

Find the solution of the exponential equation $4e^x - 3 = 13$

The exact solution, in terms of the natural logarithm is: $x =$________

The approximate solution, accurate to 4 decimal places is: $x =$________

12.

A computer purchased for $1,250 loses 19% of its value every year. The computer's value can be modeled by the function $v(t) = a \cdot b^t$, where v is the dollar value and t the number of years since purchase.

(A) Give the function that models the decrease in value of the computer: $v(t) =$________

(B) In how many years will the computer be worth half its original value? *Round answer to 1 decimal place.* ________ years

13.

Find the solution of the exponential equation $18e^{x+5} = 5$

The exact solution (using natural logarithms) is: $x =$_________

The approximate solution, rounded to 4 decimal places is: $x =$_________

14.
Solve correct to 2 decimal places.

$$40257 = \frac{2600((1.09)^n - 1)}{\frac{9}{100}}$$

$n =$_________

15.
Solve for x : $4^x = 23$

The exact solution is $x =$_________

The solution rounded to 4 decimal places is $x =$_________

16.

Find the solution of the exponential equation $e^{4x-1} = 13$

The exact solution is: $x =$_________

The approximate solution, correct to four decimal places is $x =$_________

17.

Solve the equation $4^{\frac{x}{3}} = 3$

The exact solution is $x =$_________

The solution, rounded to 4 decimal places is $x =$_________

18.
Solve for x . Round answers to four decimal places.

$\ln(x) = 6$

19.
Solve for x . Round answers to four decimal places.

$3\ln(x+4) = 9$

20.
Solve for x . Round answers to four decimal places.

$5e^{x} + 3 = 2e^{x} + 338$

21.
A culture of bacteria grows according to the continuous growth model

$B = f(t) = 200e^{0.058t}$

where B is the number of bacteria and t is in hours.

Find
$f(0) =$_________

To the nearest whole number, find the number of bacteria after 6 hours. _________

To the nearest tenth of an hour, determine how long it will take for the population to grow to 600 bacteria. _________

22.
Solve the equation, enter your answer as a decimal approximation.
$5e^{14t} = 12 + 11e^{14t}$

- One solution: _________
- No solution

23.
Solve:

$20(4^{3x}) = 14$

$x =$_________ (Answer DNE if no solution exists)

24.
Suppose $\ln n = 10$. Find $\log_{14} n$.

$\log_{14} n =$________

25.
Solve: $e^{6x} - 1e^{3x} = 12$.

$x =$ ________ (If no solution exists, answer DNE)

26.
Solve: $9^x - 3^{x+3} = 324$.

$x =$ ________ (If no solution exists, answer DNE)

27.

Solve $5^{7x+4} = 3^{x+2}$ for x. Give both the exact answer and the decimal approximation to the nearest hundredth. If logarithms are needed to solve the equation, use "ln."

Exact answer: ________

Decimal approximation: ________

28.

Solve $6^{8x+5} = 5^{x-3}$ for x. Give both the exact answer and the decimal approximation to the nearest hundredth. If logarithms are needed to solve the equation, use "ln."

Exact answer: ________

Decimal approximation: ________

29.
Solve exactly, then give the approximate decimal solution rounded to four decimal places.

$7^{x-9} = 6$

$x =$ ________

$x \approx$________

30.
If $\ln x + \ln(x-7) = \ln(2x)$, then $x =$ ________.

31.
If $\log_2(6x+5) = 4$, then $x =$ ________.

32.
Solve for n in the equation below. It may be helpful to convert the equation into exponential form. Write answer as an integer or reduced fraction.

$-\log_6(n) + 25 = 23$

$n =$________

33.
Solve the equation for x : $\log_{12} x + \log_{12}(x-1) = 1$.

Answer DNE if there is no solution

34.
Solve for x :

$\left(\log_4(\log_4 x)\right) = -4$

$x =$________

Lecture 25 Homework

Exponential growth/decay examples, Doubling time and half-life

1.
A population of bacteria is growing according to the equation $P(t) = 1650e^{0.1t}$. Estimate when the population will exceed 2161.

t =_________

Give your answer accurate to at least one decimal place.

2.
The amount of money in an investment is modeled by the function $A(t) = 800(1.0471)^t$. The variable A represents the investment balance in dollars, and t the number of years.

What is the doubling time for the investment? *Round answer to 1 decimal place.*

Answer = _________ years

3.
Find the time required for an investment of 5000 dollars to grow to 7900 dollars at an interest rate of 7.5 percent per year, compounded quarterly.

Your answer is $t =$ _________ years.
Round to 2 decimal places.

4.
A bacteria culture initially contains 2500 bacteria and doubles every half hour.

Find the size of the baterial population after 100 minutes. _________

Find the size of the baterial population after 8 hours. _________

5.
A computer purchased for \$1,600 loses 15% of its value every year.

The computer's value can be modeled by the function $v(t) = a \cdot b^t$, where v is the dollar value and t the number of years since purchase.

(A) In the exponential model $a =$_________ and $b =$ _________.

(B) In how many years will the computer be worth half its original value? *Round answer to 1 decimal place.*

The answer is _________ years

6.
You go to the doctor and he gives you 12 milligrams of radioactive dye. After 24 minutes, 7.25 milligrams of dye remain in your system. To leave the doctor's office, you must pass through a radiation detector without sounding the alarm.

If the detector will sound the alarm if more than 2 milligrams of the dye are in your system, how long will your visit to the doctor take, assuming you were given the dye as soon as you arrived?

Give your answer to the nearest minute.

You will spend _________ minutes at the doctor's office.

7.
The half-life of Radium-226 is 1590 years. If a sample contains 200 mg, how many mg will remain after 1000 years?

_________ mg

Give your answer accurate to at least 2 decimal places.

8.
A wooden artifact from an ancient tomb contains 50 percent of the carbon-14 that is present in living trees.

How long ago, to the nearest year, was the artifact made? (The half-life of carbon-14 is 5730 years.)

_________years

9.

An object with initial temperature $130°F$ is submerged in large tank of water whose temperature is $60°F$. Find a formula for $F(t)$, the temperature of the object after t minutes, if the cooling constant is $k = -0.2$

$F(t)$ =_________

10.

The temperature, $f(t)$ of a cup of coffee, in degrees Celsius, after t minutes can be determined by the equation $f(t) = 60(0.88)^t + 21$.

Estimate the temperature after 35 minutes:
_________ degrees

Determine the asymptote for the function:
The asymptote is_________

Interpret the asymptote in the context of the problem. Use complete sentences and correct units.

11.

You currently have $8,200 (Present Value) in an account that has an interest rate of 6% per year compounded semi-annually (2 times per year). You want to withdraw all your money when it reaches $12,300 (Future Value). In how many years will you be able to withdraw all your money?

The number of years is _________.
Round your answer to 1 decimal place.

12.

The function $f(t) = \frac{575,000}{1+4000e^{-t}}$ describes the number of people, $f(t)$, who have become ill with ebola t weeks after the initial outbreak in a particular community.

How many people became ill with ebola when the epidemic began? _________

Round to the nearest whole number of people.

How many people were infected 6 weeks after the initial breakout? Round to the nearest whole number of people. _________

What is the limiting size of the infected population? _________

Round to the nearest whole number of people.

13.

The function $P(x) = \frac{120}{1+372e^{-0.133x}}$ models the percentage, $P(x)$, of Americans who are x years old and have some degree of heart disease. What is the percentage, to the nearest tenth, of 29-year olds with some degree of heart disease?

_________%

14.

A conservation organization releases 100 animals of an endangered species into a game preserve. The organization believes that the preserve has a carrying capacity of 1000 animals and that the growth of the herd will follow the logistic curve $P = \frac{1000}{1+9e^{-0.165t}}$ where t is measured in months.

What is the population after 5 months? _________

Lecture 26 Homework

Definitions (standard position, central angle, positive/negative), coterminal angles, radian measure and conversion, complementary, supplementary, arc length, sector area, linear and angular speed

1.

In which quadrant does an angle of $\frac{25}{6}\pi$ terminate? Assume the vertex of the angle is at the origin and one leg of the angle is on the positive x -axis. Find the quadrant of the other leg of the angle.

2.

Let $\theta = 1$ rad be the measure of an angle. State the exact value and the approximate value of the angle measure in degrees. For the approximate value, round your answer to two decimal places.

Exact Value: _________degrees
Approximate Value: _________degrees

Conversion Formula
Let $\theta = x$ rad be the measure of an angle. State the conversion formula to degree measure.
x rad = _________degrees.
Let $\theta = 4$ rad be the measure of an angle. State the exact value and the approximate value of the angle measure in degrees. For the approximate value, round your answer to two decimal places.
Exact Value: _________degrees
Approximate Value: _________degrees

3.

Convert the angle $\frac{11\pi}{6}$ from radians to degrees.

_________degrees

4.

Convert the angle $\frac{-7\pi}{6}$ from radians to degrees:

$\frac{-7\pi}{6} =$ _________°

5.

Convert the angle 120° to radians. Give the exact value.

6.
Convert 220 degrees to radians

_________radians

Convert $\frac{4\pi}{9}$ to degrees

_________degrees

Give a exact answers.

7.
Find the angle between 0° and 360° and is coterminal with a standard position angle measuring 1376°.

8.
Find an angle between 0° and 360° that is coterminal with the given angle.

$439°$ is coterminal to _________°
$-125°$ is coterminal to _________°
$-945°$ is coterminal to _________°
$11199°$ is coterminal to _________°

9.
Write an expression describing *all* the angles that are coterminal with 266°. (Please use the variable k in your answer. Give your answer in degrees.)

_________degrees

10.
Given an angle with measure $\frac{-2\pi}{3}$ find the following.

Find a coterminal angle between -4π and -2π . _________

Find a coterminal angle between 0 and 2π . _________

11.

Find an angle between 0° and 360° that is coterminal with a standard position angle measuring -240°.

____

Find an angle between -720°and -360° that is coterminal with a standard position angle measuring -240°.

____

12.

The angle between 0 and 2π in radians that is coterminal with the angle $-\frac{11\pi}{3}$ radians is ________.

13.
Find the complement of each of the following angles.

48° is complement to ________°

1° is complement to ________°

60° is complement to ________°

53° is complement to ________°

6° is complement to ________°

14.
$m\angle X = 35°$. $\angle X$ and $\angle Y$ are supplementary angles.

$m\angle Y =$ ________°

15.
Find the length of an arc that subtends a central angle of 5° in a circle of radius 11 in.
arc-length = ________in

Answer must be exact.

16.

In a circle of radius 2 miles, the length of the arc that subtends a central angle of 3 radians is ________ miles.

17.
On a circle of radius 8 feet, give the degree measure of the angle that would subtend an arc of length 5 feet. Round your answer to the nearest hundredth, or two decimal places. degrees

18.
A bicycle with 18-in.-diameter wheels has its gears set so that the chain has a 7-in. radius on the front sprocket and 3-in. radius on the rear sprocket. The cyclist pedals at 190 rpm.

Find the linear speed of the bicycle in in/min (correct to at least **two decimal places**) _________ in/min

How fast is the bike moving in mph (to two decimal places)? _________ mph

19.
A truck's 44-in.-diameter wheels are turning at 530 rpm.

Find the linear speed of the truck in mph:

_________miles/hour

Write answer as an exact expression using pi for π .

20.
Your car's speedometer is geared to accurately give your speed using a certain tire size: 14.5" diameter wheels (the metal part) and 4.5" tires (the rubber part).

(a) If your car's instruments are properly calibrated, how many times should your tire rotate per second if you are travelling at 45 mi/hr?
rotations =_________
Report answer accurate to 3 decimal places.

(b) You buy new 5.4" tires and drive at a constant speed of 55 mph (according to your car's instrument). However, a cop stops you and claims that you were speeding. How fast did the radar gun clock you moving?
actual speed = _________mph
Report answer accurate to the nearest whole number.

(c) Then you replace your tires with 3.8" tires. When your speedometer reads 30 mph, how fast are you really moving?
actual speed = _________mph
Report answer accurate to 1 decimal places.

21.
A saw uses a circular blade 10 inches in diameter that spins at 3480 rpm. How quickly are the teeth of the saw blade moving? Express your answer in several forms:

In **exact** feet per second: _________ ft/sec

In **approximate** feet per second, rounded to 1 decimal place: _________ft/sec

In **exact** miles per hour: _________mph

In **approximate** miles per hour, rounded to 1 decimal place: _________mph

22.
Find the area of a sector with a central angle of 0.5 rad in a circle of radius 10.9 m.
area = _________sq-m

Report answer accurate to 4 decimal places.

23.
The area of a sector of a circle with a central angle of $\frac{4}{5}\pi$ rad is 39 mm^2.

Find the radius of the circle.
$r =$ _________mm
Give an exact value.

24.
A sector of a circle has a central angle of 150° . Find the area of the sector if the radius of the circle is 17 cm.

_________cm^2

Lecture 27 Homework

Unit circle, Special angles and their unit circle coordinates, definition of trig functions, Domain/range/period of sin/cos

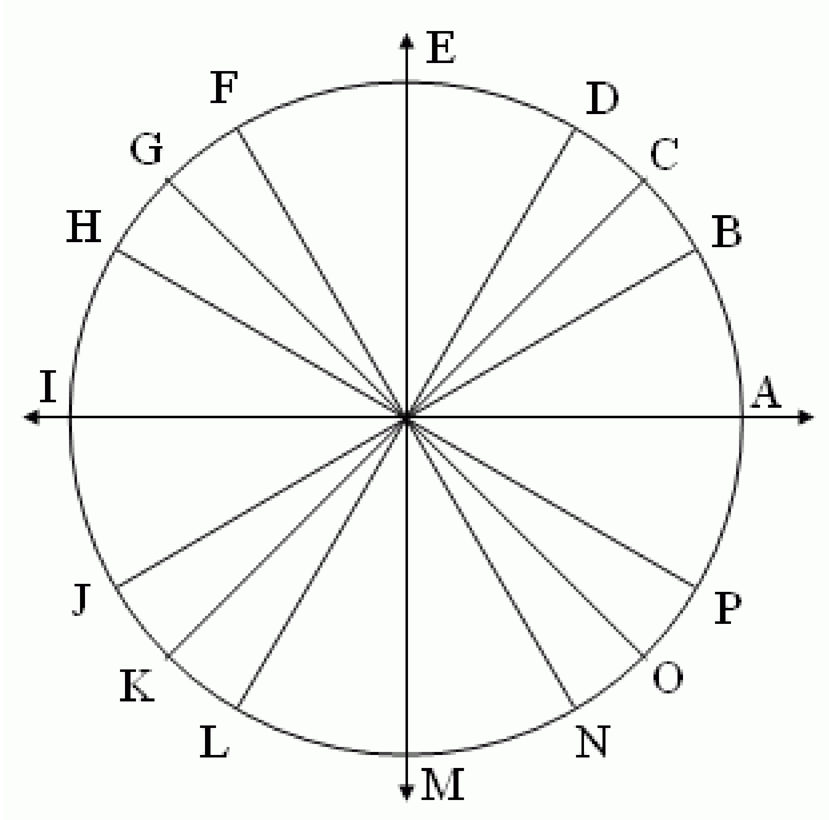
1.
From the information given, find the quadrant in which the terminal point determined by t lies. Input I, II, III, or IV.
(a) $\sin(t) < 0$ and $\cos(t) < 0$, quadrant _________;
(b) $\sin(t) > 0$ and $\cos(t) < 0$, quadrant _________;
(c) $\sin(t) > 0$ and $\cos(t) > 0$, quadrant _________;
(d) $\sin(t) < 0$ and $\cos(t) > 0$, quadrant _________;

2.
A point on the unit circle lies in quadrant III and has y -coordinate $\frac{9}{11}$. What is its x -coordinate? The x -coordinate is _________

3.
A point on the unit circle lies in quadrant II and has y -coordinate $\frac{7}{11}$. What is its x -coordinate? The x -coordinate is _________

4.
A point on the unit circle lies in quadrant I and has y -coordinate $\frac{7}{13}$. What is its x -coordinate? The x -coordinate is _________

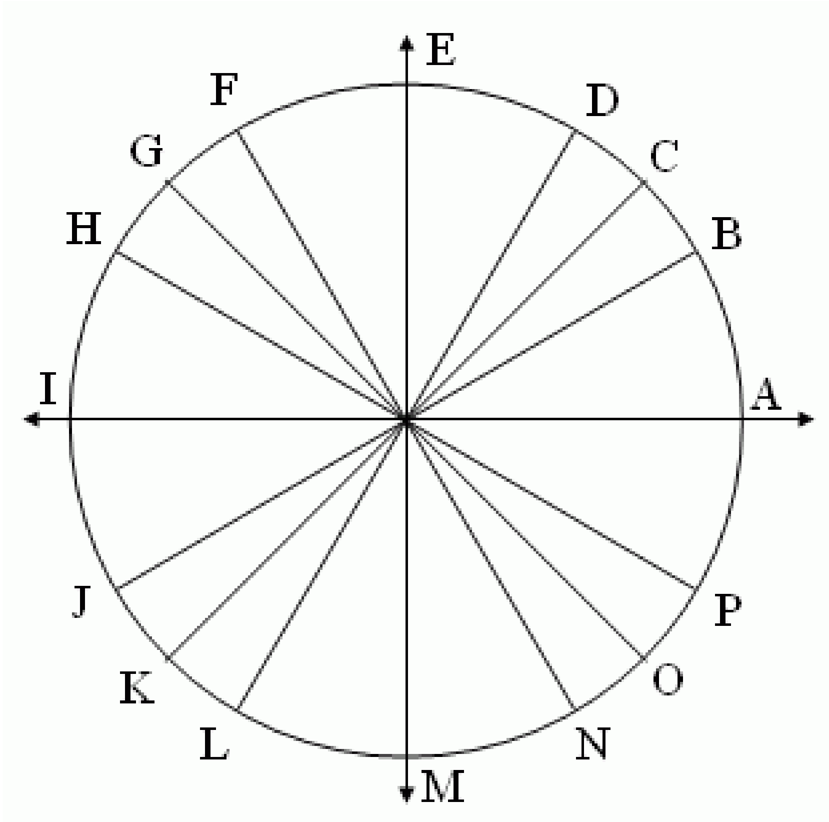
5.



Identify the special angles above. Give your answers in radians, using **pi** for π . No decimal values allowed.

A: __________
B: __________
C: __________
D: __________
E: __________
F: __________
G: __________
H: __________
I: __________
J: __________
K: __________
L: __________
M: __________
N: __________
O: __________
P: __________

6.

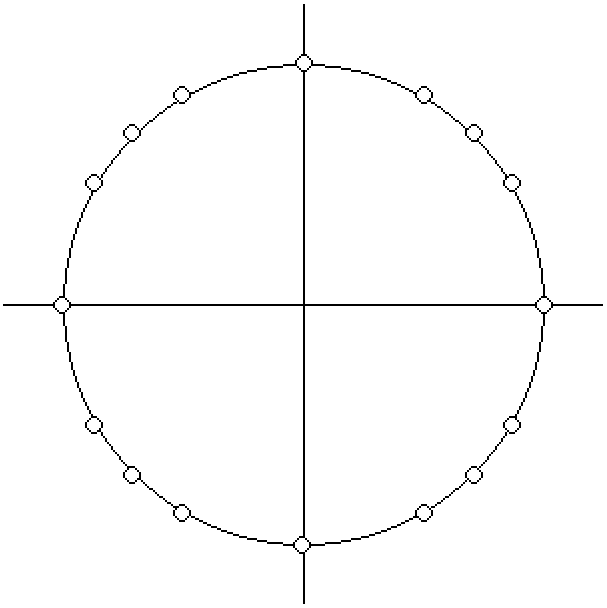


Identify the special angles above. Give your answers in degrees.

A: __________
B: __________
C: __________
D: __________
E: __________
F: __________
G: __________
H: __________
I: __________
J: __________
K: __________
L: __________
M: __________
N: __________
O: __________
P: __________

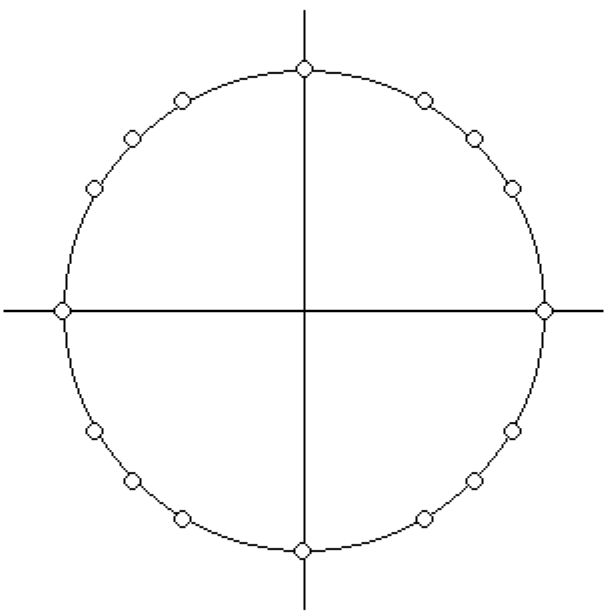
7.

Plot the point(s) where $\tan \theta = \sqrt{3}$



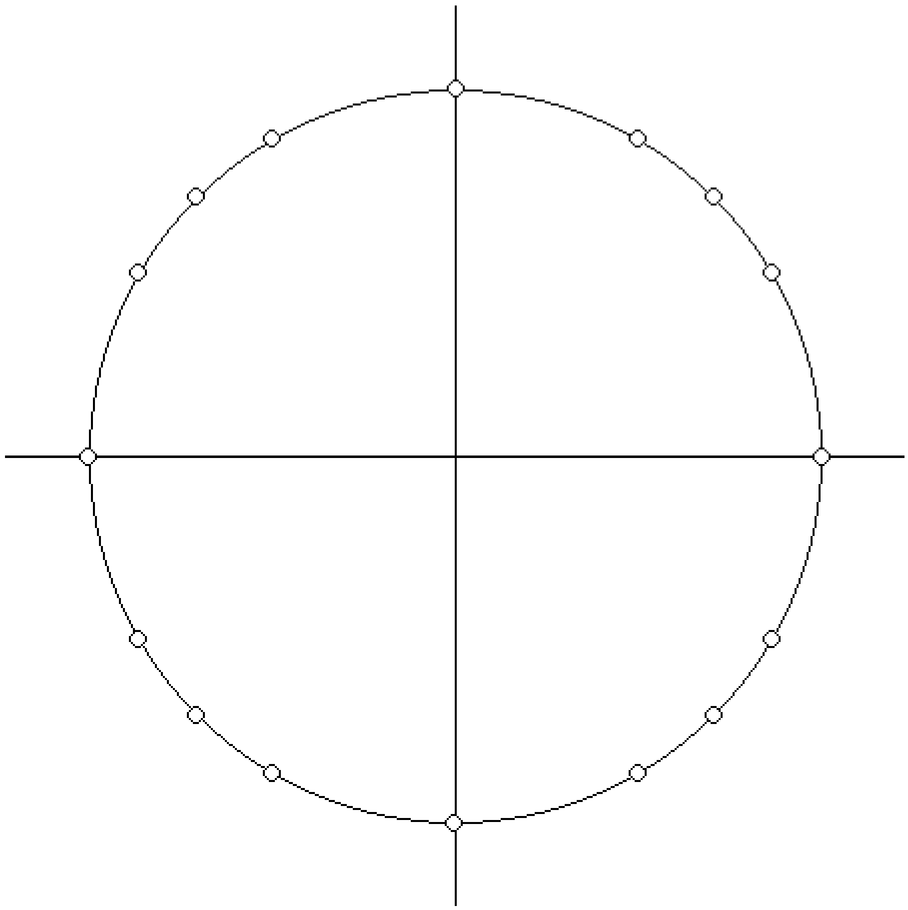
8.

Plot the point(s) where $\cot \theta = 1$



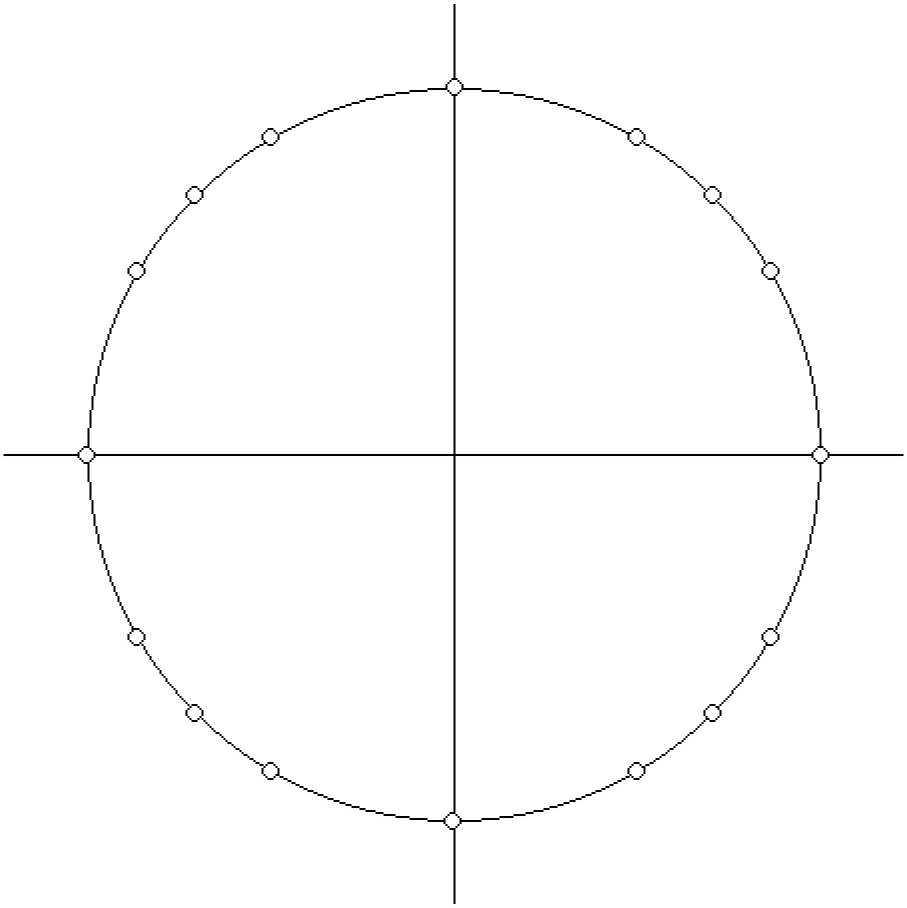
9.

Plot the point where $\theta = -\frac{2\pi}{3}$



10.

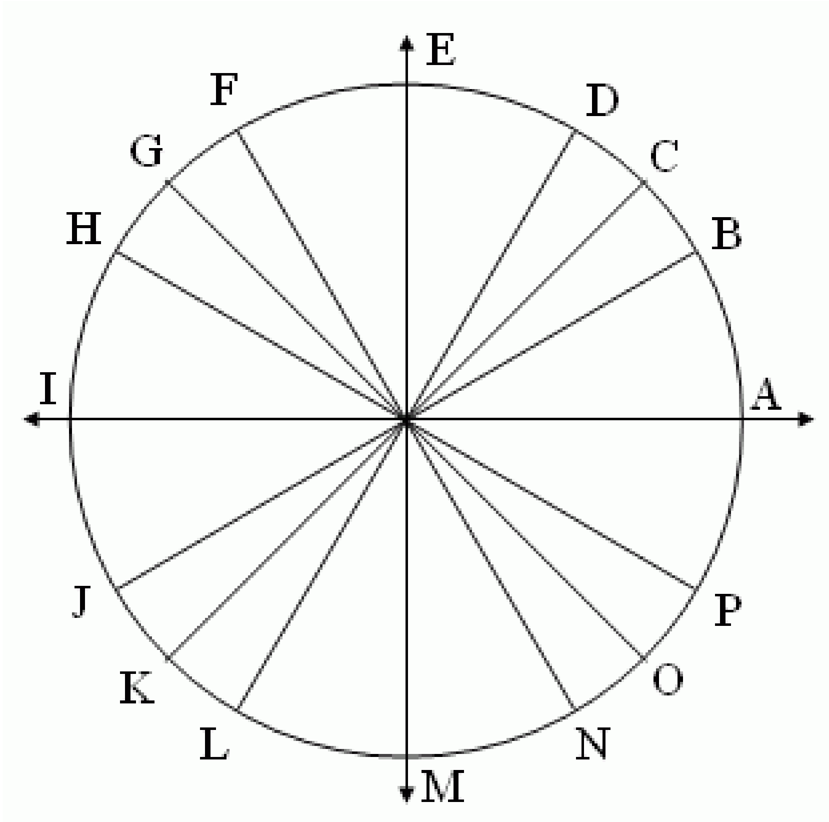
Plot the point where θ = -330°



11.

What angle is at position $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$? _________º

12.



Use the above figure to find exact values of the sine and cosine of the special angles listed below.
Enter $\sqrt{w}$ as sqrt(w).

sin(K) =________ cos(K) =________
sin(C) =________ cos(C) =________
sin(B) =________ cos(B) =________

13.

State the exact value of $\cos\left(\frac{\pi}{6}\right)$ =________

14.

State the exact value of $\tan\left(\frac{\pi}{4}\right)$ =________

15.

State the exact value of $\sin\left(\frac{\pi}{4}\right)$ =________

16.

State the exact value of $\tan\left(\frac{\pi}{4}\right)$ =________

17.

State the exact value of $\tan\left(\frac{\pi}{4}\right) =$_________

18.

csc 150º equals _________

19.

tan 60º equals _________

20.

Find an angle θ with $0^\circ < \theta < 360^\circ$ that has the same:

Sine as 60° : θ = _________degrees

Cosine as 60° : θ = _________degrees

21.

Find an angle θ with $0^\circ < \theta < 360^\circ$ that has the same:

Sine function value as 220°
θ = _________degrees

Cosine function value as 220°
θ = _________degrees

22.

To show that the point $\left(\frac{3}{5}, \frac{4}{5}\right)$ is on the unit circle, we need to prove that $\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 =$ _________.