MAC2313, Calculus III
Exam 3 Review

This review is **not** designed to be comprehensive, but to be representative of the topics covered on the exam.

1. Convert the point $(1, -\sqrt{3}, -2\sqrt{3})$ from rectangular to
(1) cylindrical coordinates (2) spherical coordinates

2. Identify the surface in cylindrical coordinates.
(1) $r = 2\sin\theta$ (2) $z = r^2\cos(2\theta)$

3. Identify the surface in spherical coordinates.
(1) $\rho = 4\cos\phi$ (2) $\cos^2\phi - \sin^2\phi = 0$

4. (1) Describe the solid region E in cylindrical coordinates if E is bounded below by the plane $z = 0$, laterally by the circular cylinder $x^2 + (y-1)^2 = 1$, and above by the paraboloid $z = x^2 + y^2$.
(2) Sketch the solid $E = \{\, (r, \theta, z) \mid 0 \le \theta \le \pi/2,\ r \le z \le 2 \,\}$.

5. (1) Describe the solid region E in spherical coordinates if E is the portion of the solid bounded by the sphere $x^2+y^2+z^2 = 4$ and the cone $z^2 = 3(x^2+y^2)$ that lies in the first octant.
(2) Identify the solid

$$E = \{\, (\rho, \theta, \phi) \mid 0 \le \theta \le \pi,\ 0 \le \phi \le \pi/3,\ 1/\cos\phi \le \rho \le 2 \,\}.$$

6. Evaluate the following integrals:

(1) $\displaystyle\int_0^4 \int_0^5 \frac{1}{\sqrt{x+y}}\, dy\, dx$ (2) $\displaystyle\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$

(3) $\displaystyle\int_0^1 \int_{y^2}^1 y\sin(x^2)\, dx\, dy$ (4) $\displaystyle\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$

7. Convert the integral $\int_0^1 \int_x^{\sqrt{2x-x^2}} \frac{1}{\sqrt{x^2+y^2}}\, dy\, dx$ to polar coordinates.

8. Set up double integral(s) of the area of the region that
(1) lies inside both $r = 1 + \cos\theta$ and $r = 3\cos\theta$
(2) lies inside $r = 2\sin\theta$ and outside $r = 2\cos\theta$

9. Express the following integrals in polar coordinates:
(1) $\iint\limits_D (x^2+y^2)^{3/2}\, dA$, where D is the region in the first quadrant bounded by the lines $y = 0$ and $y = \sqrt{3}x$ and the circle $x^2 + y^2 = 9$.
(2) $\iint\limits_D \sqrt{x^2+y^2}\, dA$, where D is the closed disk with center $(0, 1)$ and radius 1.

10. Set up a triple integral for the volume of the solid in the first octant bounded by the coordinate planes and the plane $z = 6 - x - 2y$.

11. Rewrite the integral $\int_{-1}^1 \int_{x^2}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx$ as an iterated integral in the order $dx\ dy\ dz$.

12. Convert $\int_0^{2\pi} \int_0^{\sqrt{2}} \int_r^{\sqrt{4-r^2}} 3r\ dz\ dr\ d\theta$ to
(1) rectangular coordinates with the order of integration $dz\ dy\ dx$
(2) spherical coordinates
(3) evaluate one of the above integrals

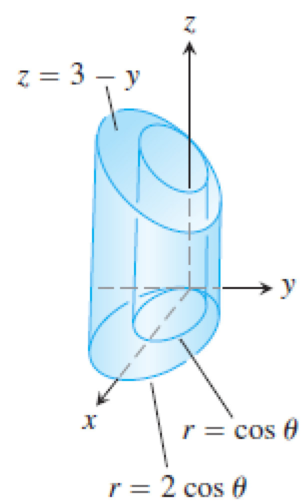
13. Convert $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 dz\ dy\ dx$ to spherical coordinates and then evaluate.

14. Express $\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^{x^2+y^2} f(x,y,z)\, dz\, dy\, dx$ in cylindrical coordinates.

15. Find the volume of the solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y$, $x = 0$, and $z = 0$ in the first octant.

16. Find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

17. Set up a triple integral for the volume of the solid whose base is the region between the circles $r = \cos\theta$ and $r = 2\cos\theta$ and whose top lies in the plane $z = 3 - y$.



18. Evaluate $\iint_R \cos\left(\frac{y-x}{y+x}\right) dA$, where R is the region bounded by the lines $x + y = 2$, $x + y = 4$, $x = 0$, and $y = 0$.

19. Evaluate $\iint_R \left(1 + \frac{x^2}{16} + \frac{y^2}{25}\right)^{3/2} dA$, where R is the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

20. Use the transformation $x = u^2$, $y = v^2$, and $z = w^2$ to set up an integral for the volume of the region bounded by $\sqrt{x}+\sqrt{y}+\sqrt{z} = 1$ and the coordinate planes.

21. True or False:

(1) For any region D in the plane, $\iint\limits_D dA \geq 0$.

(2) For any region D in the plane, $\iint\limits_D f(x,y)\, dA \geq 0$.

(3) If f is continuous on $[a,b]\times[c,d]$, then $\int_a^b \int_c^d f(x,y)dydx = \int_c^d \int_a^b f(x,y)dydx$.

(4) $\int_0^1 \int_0^x f(x,y)\, dy\, dx = \int_0^1 \int_0^y f(x,y)\, dx\, dy$.

(5) If the point P is on the surface $\phi = 0$, then P lies in the xy-plane.

(6) If the point P is on the surface $\theta = 0$, then P lies in the xz-plane.