MAC 2311 Exam 2 Review

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- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
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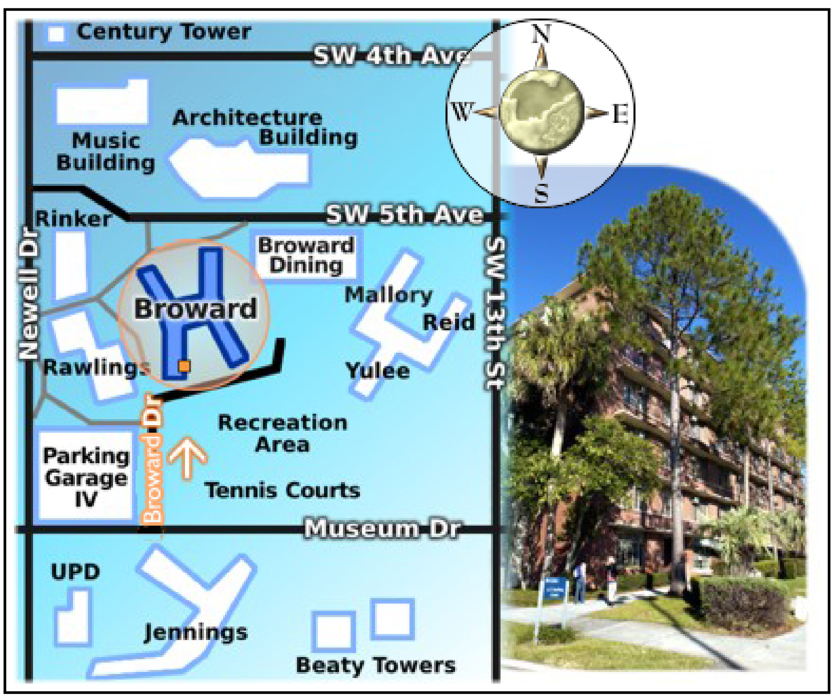

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1. Calculate the following derivatives:

   (a) $\dfrac{d}{dx}\, e^3$

   (b) $\dfrac{d}{dx}\, (\pi x^2)^{23}$

   (c) $\dfrac{d}{dx}\, \dfrac{1-x}{\sqrt{x}}$

   (d) $\dfrac{d}{dx}\, 7^x 49^x$

2. Evaluate the limit $\lim\limits_{h\to 0} \dfrac{(27+h)^{2/3}-9}{h}$.

3. Find the equation for the tangent line to $f(t) = \dfrac{t}{e^t - 1}$ at $t = 1$.

4. Which of the following derivatives requires the product or quotient rules? Which do not? Compute the derivatives.

   (a) $\dfrac{d}{dx}\, \pi^3 x^2$

   (b) $\dfrac{d}{dx}\, \dfrac{x^{1/3}}{e}$

   (c) $\dfrac{d}{dx}\, \dfrac{e^{2x}-7}{e^{x+1}}$

   (d) $\dfrac{d}{dx}\, \dfrac{(e^x+1)^2}{e^{-x}}$

   (e) $\dfrac{d}{dx}\, \dfrac{(\sqrt{x}-x)^2}{x^{5/2}}$

   (f) $\dfrac{d}{dx}\, \dfrac{(e^x+x)^2}{e^{-x}}$

5. Suppose $f(x)$ is a differentiable function such that $f(\pi) = \dfrac{3\pi}{4}$ and $f'(\pi) = 2018$. Consider the composite function $g(x) = \cot(f(x))$.

   (a) Calculate $g'(\pi)$

   (b) Suppose additionally that f is an even function. Calculate $g'(-\pi)$

   (c) Suppose additionally that f is an odd function. Calculate $g'(-\pi)$

6. Find the equation of the normal line to $f(x) = x\cot(x)$ at $x = -\dfrac{\pi}{6}$.

7. Find the slope for each of the following functions at the given point:

   (a) $f(x) = \cos(x)$ at $x = \pi/4$.
   (b) $g(x) = \ln(x-1)$ at $x = e$.
   (c) $h(x) = \dfrac{1}{x^2}$ at $x = 2$.

8. Let $f(x) = e^{\frac{1}{3}x^3 - x}$.
   (a) At what points (x, y), if any, does $f(x)$ have a horizontal tangent line?
   (b) Find the equation for the tangent line to $f(x)$ at $x = 0$.

9. At how many points does the curve $2y^3 + y^2 - y^5 = x^4 - 2x^3 + x^2$ have horizontal tangent lines?

10. Follow the steps below to find the derivative of $f(x) = \tan^{-1}(e^{x^2})$
    (a) Begin by writing y for $f(x)$, i.e., $y = \tan^{-1}(e^{x^2})$
    (b) Observe that the result from part (a) is equivalent to $\tan(y) = e^{x^2}$.
    (c) Differentiate the expression from part (b) implicitly, and find $f'(x)$.
    (d) Does $f(x)$ have any horizontal tangent lines? If so where?

11. Let $f(x) = \dfrac{(x^2+4)\cos(\pi x)e^{3x}}{\sin(3\pi x)\sqrt{x+3}}$. Find $f'(x)$. *Hint*: *use logarithmic differentiation*

12. Calculate the following derivatives.

    (a) $\dfrac{d}{dx}\ln(\ln(x))$
    (b) $\dfrac{d}{dx}\ln(\ln(\ln(x)))$
    (c) $\dfrac{d}{dx}\ln(\ln(\ln(\ln(x))))$

13. Evaluate the following derivatives.

(a) $\dfrac{d}{dx}\sin^{-1}(2\sqrt{x})$

(b) $\dfrac{d}{dx}\sec^{-1}(x^2)$

(c) $\dfrac{d}{dx}2x\tan^{-1}(x)$

(d) $\dfrac{d}{dx}\sqrt{\cos^{-1}(x)}$

14. Evaluate the limit $\displaystyle\lim_{h\to 0}\frac{\cos^{-1}\left(\frac{\sqrt{2}}{2}+h\right)-\frac{\pi}{4}}{h}$.

15. CẚyŘ ǘKŜ ŘŜꞪẚØł·ǘẚØŜă ƨŦ ǘKŜ ŦƨffƨǿẚyƎ ŦdzyǑǘẚƨyãŸ

(a) $f(x) = \mathrm{qin}^{-1}(2x-1)$

(b) $\mathrm{e}(x) = (1+x^2)\mathrm{ran}^{-1}(x)$

(c) $\mathrm{f}(x) = \mathrm{ran}^{-1}(\sin(x))$

16. CẚyŘ ǘKŜ ŘŜꞪẚØł·ǘẚØŜă ƨŦ ǘKŜ ŦƨffƨǿẚyƎ ŦdzyǑǘẚƨyãŸ

(a) f(x) = csc(5x)

(b) g(x) = $-4\sin^2(2x)$

(c) f(x) = 2xcot(x)

(d) g(x) = $\cos^3(2x^2-1)$

MAC 2311 Review - Exam 2

1) Differentiate the following functions.

a) $$f(x) = x\sqrt{x^2 - 3}$$

b) $$g(x) = \frac{x^3 + 2}{x^2 + 1}$$

c) $$h(x) = (3x^2 - 1)\left(x^2 - \frac{1}{x}\right)$$

d) $$r(x) = \frac{x^3 + 2x^2 + x - 1}{\sqrt{x}}$$

2) Differentiate the following functions:

a) $h(x) = \frac{x^2 - x + 1}{(x-1)^{2/3}}$

b) $k(x) = \begin{cases} -\frac{x^2-1}{x+2}, & x > -1 \\ x + 2, & x \leq -1 \end{cases}$

c) $n(x) = \sin^2(\cos(4x))$

3) Find...

$$\lim_{h \to 0} \frac{\sqrt[5]{x+h} - \sqrt[5]{x}}{h}$$

4) Given $f(x) = 3x^2\sqrt[3]{4 - x^2}$

a) Find $\frac{df}{dx}$

b) Where are the horizontal and vertical tangents?

5) For the following equation:

$$5x^2y - y^3 = 1 + x^2$$

a) Find $\frac{dy}{dx}$.

b) Find the equation of the tangent line to the curve at the point (1,2).

6) Find the derivatives of the following functions:

a) $f(x) = \frac{x}{x+3}$

b) $g(x) = \sin(3x)$

7) For what values of x does the function $g(x) = x + 2\sin(x)$ have horizontal tangent lines?

8) Suppose $f(x) = ax^2 + bx + c$ and that the tangent lines at $x = 1$ and $x = -1$ have slopes -8 and -1 respectively, and that the point $(2,15)$ is a point on the graph. What are the values of a, b, and c?

9) Find the values of a so that the tangent line to $y = x^2 - 2\sqrt{x} + 1$ is perpendicular to the line $ay + 2x = 2$ at $x = 4$.

10) Find the x values where the curve represented by the following equation has horizontal tangent lines.

$$x^2 + xy + y^2 = 6$$

11) Take the derivatives of the following functions [using logarithmic differentiation]:

a) $f(x) = 5^{\tan^2(x)}$

b) $g(x) = x^{\sin(x)}$

c) $h(x) = \frac{e^{3x+1}(x^2+3)^3}{\sqrt{2x-1}}$