# Exam 2 Review

*Disclaimer:* Exam 2 covers Chapters 8.1-8.8. This review may not cover all the material that will be on the exam. Exam 2 can cover any material from the lectures, homework, quizzes, etc.
Solutions to these problems will not be released. This is to focus your studying on understanding the process of solving them, rather than the final answer. Feel free to use the discussion boards to discuss these problems.

1. Find the domain of each function.

   (a) $f(x, y) = \ln(x + 3y)$

   (b) $g(x, y) = \dfrac{xy}{x^2 - y^2}$

   (c) $h(x, y) = \sqrt{16 - x^2 - y^2}$

2. Sketch the level curves of the functions corresponding to the given $z$ values.

   (a) $f(x, y) = e^x - y$ with $z = -2, -1, 0, 1, 2.$

   (b) $g(x, y) = xy$ with $z = -2, -1, 1, 2.$

   (c) $h(x, y) = \ln(x - y)$ with $z = -2, -1, 0, 1, 2$

3. Let $f(x, y) = x^2 - xy + 5y^2$. Compute

$$\lim_{h\to 0} \frac{f(x+h, y) - f(x, y)}{h} \text{ and } \lim_{h\to 0} \frac{f(x, y+h) - f(x, y)}{h}.$$

   What do these limits represent?

4. Let $f(x, y) = x^3 \ln(y) + 4y^2 e^x$. Find the indicated function or value.

   (a) $f_y(x, y)$

   (b) $f_x(x, y)$

   (c) $f_{yy}(x, y)$

   (d) $f_{xx}(x, y)$

   (e) $f_{xy}(x, y)$

   (f) $f_{yy}(-1, 1)$

   (g) $f_{yx}(-1, 1)$

   (h) $f_{xx}(-1, 1)$

5. Find the critical points of the function, and then use the second derivative test to classify the nature of each point. Determine any relative extrema of the function.

   (a) $f(x, y) = x^3 + y^2 - 6xy$

   (b) $g(x, y) = e^{x^2+y^2}$

   (c) $h(x, y) = xy + \ln x + 2y^2$

6. A firm produces two types of earphones per year: $x$ thousand of type $A$ and $y$ thousand of type $B$. If the revenue $R$ and cost $C$ for the year (in millions of dollars) are

$$\begin{aligned} R(x, y) =& 2x + 3y \\ C(x, y) =& x^2 - 2xy + 2y^2 + 6x - 9y + 5 \end{aligned}$$

   determine how many of each type of earphone should be produced per year to maximize profit. What is the maximum profit?

7. The table below shows the price of a stock $p$ at the end of each month $t$. Assuming the price of the stock increases linearly, determine an equation for the price of the stock at time $t$. Predict what the price of the stock will be at the end of month 6.

| $t$ (months) | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $p$ (price) | 3 | 5 | 5 | 7 | 8 |

   Note: The least squares line $y = ax + b$ that best fits the points $(x_1, x_2), (x_2, y_2), \ldots, (x_n, y_n)$ is given by

$$a = \frac{n(\sum x_i y_i) - (\sum x_i)(\sum y_i)}{n \sum x_i^2 - (\sum x_i)^2} \qquad b = \frac{\sum y_i - a \sum x_i}{n}.$$

8. Use the method of Langrange multipliers to minimize or maximize the function subject to the constraint.

   (a) $f(x, y) = 3x^2 + 5y^2$ subject to $2x + 3y = 6$.

   (b) $f(x, y) = xy$ subject to $3x + y = 720$.

   (c) $f(x, y) = 2x^2 + y^2 + 2$ subject to $x^2 + 4y^2 = 4$.

   (d) $f(x, y) = e^{3x-5y}$ subject to $x^2 + y^2 = 1$.

9. Let $f(x, y) = xe^{xy}$ and suppose $(x, y)$ changes from $(1, 0)$ to $(0.9, 0.01)$. Compute $dz$ and $\Delta z$. Compare the values of $\Delta z$ and $dz$. How close is the approximation?

10. The price-earnings ratio (PE ratio) of a stock is given by

$$R(x, y) = \frac{x}{y}$$

   where $x$ denotes the price per share of the stock and $y$ denotes the earnings per share. Estimate the change in the PE ratio $\Delta R$ of a stock if its price increases from \$62/share to \$65/share while its earnings decrease from \$3/share to \$2.60/share. (Round your answer to two decimal places.)

11. Find the average value of $f(x, y) = y + 2x$ over $R$, where $R$ is the rectangle defined by $1 \le x \le 2$ and $0 \le y \le 1$.

12. Evaluate the double integral $\displaystyle\iint_R 2xy\, dA$ and $R$ is the region bounded by the graphs of $y = -x$, $y = x^2$, and $x = 1$, where $x \ge 0$.

13. Find the volume of the solid bounded above by the surface $z = f(x, y) = 2x + y$ and below by the plane region $R$ where $R$ is the triangle bounded by $y = 2x$, $y = 0$, and $x = 3$.

14. Reverse the order of integration for each integral. Evaluate the integral with the order reversed. Do not attempt to evaluate the integral in the original form.

    (a) $\displaystyle\int_0^2 \int_{x^2}^4 \frac{4x}{1+y^2}\, dy\, dx$

    (b) $\displaystyle\int_0^1 \int_y^1 \sqrt{1-x^2}\, dx\, dy$