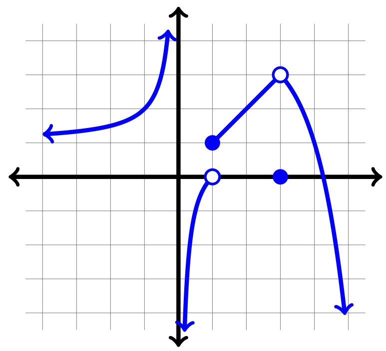
MAC 2233: Unit 2 Review
Exam covers Lectures 10 – 19

1. Use the following graph of a function $f(x)$ to evaluate the limits and function value if possible. If the limit does not exist, write "dne".



a) $\lim_{x\to 0^-} f(x)$ b) $\lim_{x\to 0^+} f(x)$ c) $\lim_{x\to 0} f(x)$

d) $\lim_{x\to 1+} f(x)$ e) $\lim_{x\to 1^-} f(x)$ f) $\lim_{x\to 1} f(x)$

g) $\lim_{x\to 3} f(x)$ h) $f(3)$ i) $\lim_{x\to -1} f(x)$

2. Use the properties of limits to evaluate $\lim_{x\to a} \dfrac{(fg)(x)}{\sqrt[3]{g(x)} - 1}$ if $\lim_{x\to a} f(x) = -\dfrac{1}{3}$ and $\lim_{x\to a} g(x) = 9$.

3. Evaluate (a) $\lim_{x\to -1} \dfrac{x + \sqrt{x+2}}{x+1}$ and (b) $\lim_{x\to 2} \dfrac{\frac{2}{x} - 1}{x-2}$.

4. If $f(x) = \begin{cases} \dfrac{x^2 - 16}{x^2 + 3x - 4} & x \neq -4 \\ 0 & x = -4 \end{cases}$

find $p = \lim_{x\to -4} f(x)$ and $q = \lim_{x\to 1^-} f(x)$.

5. Sketch the graph of $f(x) = \dfrac{|6 - 2x|}{x - 3}$. Hint: rewrite as a piecewise function without absolute value bars. Use the graph to find: (a) $\lim_{x\to 3^-} f(x)$, (b) $\lim_{x\to 3^+} f(x)$, and (c) $\lim_{x\to 3} f(x)$.
Now find those limits algebraically without using the graph.

6. If $f(x) = \dfrac{x^3 + 3x^2 + 2x}{x - x^3}$, find a) $\lim_{x\to 0^+} f(x)$ b) $\lim_{x\to -1^+} f(x)$, c) $\lim_{x\to 1^-} f(x)$ and d) $\lim_{x\to -\infty} f(x)$. Find each vertical and horizontal asymptote of $f(x)$.

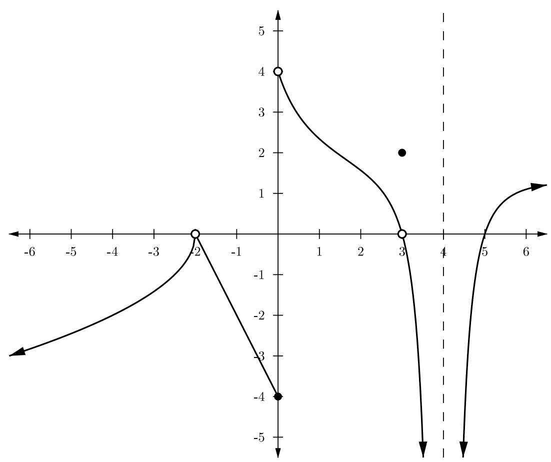
7. If $f(x) = \dfrac{2}{e^{-x} - 3}$, find if possible:

1) $\lim\limits_{x \to -\infty} f(x)$ 2) $\lim\limits_{x \to +\infty} f(x)$ 3) Each asymptote of the graph of $f(x)$.

8. The Intermediate Value Theorem guarantees that the function $f(x) = x^3 - \dfrac{1}{x} - 5x + 3$ has a zero on which of the following intervals?

a) $[-1, 1]$ b) $[1, 3]$ c) $[3, 5]$ d) $[-3, -2]$

9. Consider a function $f(x)$ which has the following graph.



(a) On which interval(s) is $f(x)$ continuous?
(b) $f(x)$ has a jump discontinuity at $x =$________.
(c) $f(x)$ has an infinite discontinuity at $x =$________.
(d) $f(x)$ has a removable discontinuity at $x =$________.
(e) How would you define or redefine $f(x)$ at the point(s) in part (d) in order to make $f(x)$ continuous?
(f) Find each value at which $f(x)$ is continuous but not differentiable.
(g) Find $f'(-1)$. (h) Which is larger, $f'(-5)$ or $f'(-3)$?

10. Use the definition of derivative to evaluate $f'(x)$ if $f(x) = \sqrt{2x - 1}$. Check your answer using a derivative rule.

11. (a) Use the **definition of derivative** to find $f'(x)$ if $f(x) = \dfrac{x}{2x - 1}$. Check your answer using the Quotient Rule.

(b) Find each interval over which $f(x)$ is differentiable.

(c) Write the equation of the tangent line to $f(x) = \dfrac{x}{2x - 1}$ at $x = -1$.

12. Indicate whether each of the following statements is true or false.

    (a) If f is continuous at $x = a$, then f is differentiable at $x = a$.

    (b) If f is not continuous at $x = a$, then f is not differentiable at $x = a$.

    (c) If f has a vertical tangent line at $x = a$, then the graph of $f'(x)$ has a vertical asymptote at $x = a$.

13. Let $f(x) = \begin{cases} 2 - x|x| & x < 0 \\ 3x + 2 & x \geq 0 \end{cases}$.

    (a) Use the limit definition of continuity to show that $f(x)$ is continuous at $x = 0$.

    (b) Find $f'(0)$ if possible using the **limit definition** of derivative at a point $f'(0) = \lim\limits_{x \to 0} \dfrac{f(x) - f(0)}{x - 0}$.

14. If an object is projected upward from the roof of a 200 foot building at 64 ft/sec, its height h in feet above the ground after t seconds is given by $h(t) = 200 + 64t - 16t^2$. Find the following:

    (a) The average velocity of the object from time $t = 0$ until it reaches its maximum height (hint: consider the graph of the function)

    (b) The instantaneous velocity of the object at time $t = 1$ second using the limit definition.

15. Find each value at which $f(x) = \dfrac{x^3}{3} - \dfrac{x^2}{2} - 2x$ is parallel to the line $2y - 8x + 9 = 0$.

16. Find the value of a so that the tangent line to $y = x^2 - 2\sqrt{x} + 1$ is perpendicular to the line $ay + 2x = 2$ when $x = 4$.

17. If $f(x) = (x^3 - 2x)(2\sqrt{x} + 1)$, find $f'(x)$ two ways: rewriting $f(x)$ and differentiating, and using the Product Rule.

18. Find each value of x at which $f(x) = (1 - x)^5(5x + 2)^4$ has a horizontal tangent line.

19. Let $f(x) = \dfrac{(\sqrt{x} - 1)^2}{x}$. Find $f'(x)$ and write as a single fraction. Write the equation of the tangent line to $f(x)$ at $x = 4$.

20. Write the equation of the tangent line to $f(x) = \left(x - \dfrac{6}{x}\right)^3$ at $x = 3$.

21. Find each value of x at which the function $f(x) = \dfrac{\sqrt[3]{6x+1}}{x}$ has

(a) horizontal and (b) vertical tangent lines.

Write the equation of each of those lines.

22. Suppose that $f(4) = -1$, $g(4) = 2$, $f(-4) = 1$, $g(-4) = 3$, $f'(4) = -2$, $g'(4) = 12$, $f'(-4) = 6$, and $g'(-1) = -2$.

Find: (a) $h'(4)$ if $h(x) = g(f(x))$ and (b) $H'(4)$ if $H(x) = \sqrt{xf(x) + \dfrac{x^2}{2}}$.

23. Sketch a possible graph of the derivative of the function $y = f(x)$ shown below.