MAC2313, Calculus III
Exam 2 Review

This review is **not** designed to be comprehensive, but to be representative of the topics covered on the exam.

1. Find and sketch the domain of the function.

(1) $f(x, y) = \ln(x + y + 1)$
(2) $f(x, y) = \sqrt{4 - x^2 - y^2} + \sqrt{1 - x^2}$

2. Show that the limit does not exist.

(1) $\lim_{(x,y)\to(1,1)} \frac{xy^2 - 1}{y - 1}$ (2) $\lim_{(x,y)\to(0,0)} \frac{x^3 + y^3}{xy^2}$

3. Evaluate the following limits.

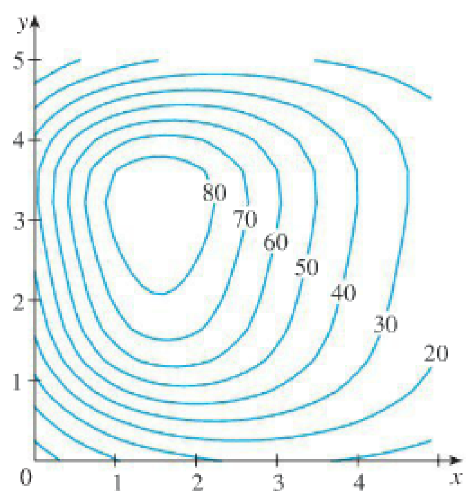
(1) $\lim_{(x,y)\to(1,1)} \frac{x^3y^3 - 1}{xy - 1}$ (2) $\lim_{(x,y)\to(2,2)} \frac{x + y - 4}{\sqrt{x + y} - 2}$

(3) $\lim_{(x,y)\to(0,0)} \frac{e^y \sin(2x)}{x}$ (4) $\lim_{(x,y)\to(0,0)} (x^2 + y^2)\ln(x^2 + y^2)$

4. The contour map of a function f is shown.

(1) Is $f_x(3, 2)$ positive or negative?

(2) Which is greater, $f_y(2, 1)$ or $f_y(2, 2)$?



5. Consider the function $f(x,y) = \begin{cases} \dfrac{\sin(xy)}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$

(1) Is f continuous at $(0,0)$?
(2) Is f differentiable at $(0,0)$?

6. Consider the function $f(x,y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2} & (x,y) \neq (0,0) \\ 1 & (x,y) = (0,0) \end{cases}.$

(1) Is f continuous at $(0,0)$?
(2) Can you redefine the function so that f continuous at $(0,0)$?

7. Find all the first and second order partial derivatives of $f(x,y) = x^y$.

8. Find the linear approximation of the function $f(x,y,z) = x^3\sqrt{y^2+z^2}$ at the point $(2,3,4)$ and use it to estimate the number $(1.98)^3\sqrt{(3.02)^2+(4.01)^2}$.

9. Use differentials to estimate the amount of metal in a closed cylindrical can that is 30 cm high and 5 cm in radius if the metal in the top and the bottom is 0.3 cm thick and the metal in the sides is 0.05 cm thick.

10. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ at $(0,1,2)$ if $x - yz + \cos(xyz) = 2$.

11. Find an equation of the tangent plane to the surface $z = x\sin(x+y)$ at the point $(-1,1,0)$.

12. Let $z = \sqrt{x^2+y^2}$. Show that $\dfrac{\partial^2 z}{\partial x^2}\dfrac{\partial^2 z}{\partial y^2} = \left(\dfrac{\partial^2 z}{\partial x \partial y}\right)^2$.

13. Find $\dfrac{\partial w}{\partial r}$ and $\dfrac{\partial w}{\partial \theta}$ when $r = 2$ and $\theta = \pi/2$ if $w = xy + yz + zx$, and $x = r\cos\theta,\ y = r\sin\theta,\ z = r\theta$.

14. Find the directional derivative of $f(x, y) = x^2e^{-y}$ at the point $(-2, 0)$ in the direction toward the point $(2, -3)$.

15. Let $f(x, y) = \ln(1 + xy)$.

(1) Find the unit vectors that give the direction of steepest ascent and steepest descent at $(1, 2)$.
(2) Find a unit vector that points in a direction of no change at $(1, 2)$.

16. Find equations of (1) the tangent plane and (2) the normal line to the surface $xy + yz + zx = 5$ at the point $(1, 2, 1)$.

17. Where does the normal line to the paraboloid $z = x^2 + y^2$ at the point $(1, 1, 2)$ intersect the paraboloid a second time?

18. The plane $y+z = 3$ intersects the cylinder $x^2+y^2 = 5$ in an ellipse. Find parametric equations for the tangent line to this ellipse at the point $(1, 2, 1)$.

19. Find the points on the surface $2x^3 + y - z^2 = 5$ at which the tangent plane is parallel to the plane $24x + y - 6z = 3$.

20. Let $f(x, y) = 3x^2 - 3xy^2 + y^3 + 3y^2$. Find the critical points of f and classify each critical point.

21. Find the local maximum and minimum values and saddle point(s) of the function $f(x, y) = (x^2 + y^2)e^{-x}$.

22. Find the absolute maximum and minimum values of
(1) $f(x, y) = x^2 + y^2 - 2x$ on the closed triangular region with vertices $(2, 0)$, $(0, 2)$, and $(0, -2)$
(2) $f(x, y) = (x^2 + 2y^2)e^{-x^2-y^2}$ on the disk $\{(x, y) \mid x^2 + y^2 \leq 4\}$
(3) $f(x, y) = e^{-xy}$ on $\{(x, y) \mid x^2 + 4y^2 \leq 1\}$

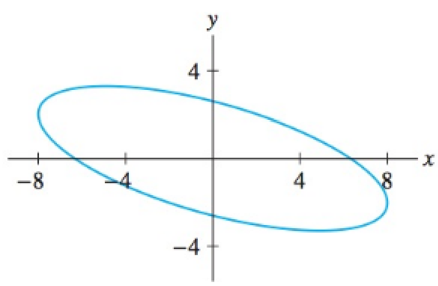
23. Find the maximum and minimum values of

(1) $f(x, y, z) = x + y + z$ subject to $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z} = 1$

(2) $f(x, y, z) = x^2 + y^2 + z^2$ subject to $x - y = 1$ and $y^2 - z^2 = 1$

24. Find the point(s) on the surface $x^2 - yz = 1$ that are closest to the origin.

25. Find the point on the ellipse $x^2 + 6y^2 + 3xy = 40$ with the largest x coordinate.



26. True or False:

(1) There exists a function f with continuous second partial derivatives such that $f_x = x + y^2$ and $f_y = x - y^2$.

(2) If $f_x(a, b)$ and $f_y(a, b)$ both exist, then f is differentiable at (a, b).

(3) If $f(x, y)$ is differentiable, then the rate of change of f at the point (a, b) in the direction of $\vec{w}$ is $\nabla f(a, b) \cdot \vec{w}$.

(4) If $f_x(a, b) = 0$ and $f_y(a, b) = 0$, then f must have a local maximum or minimum at (a, b).

(5) If $f(x, y)$ is differentiable and f has a local minimum at (a, b), then $D_{\vec{u}} f(a, b) = 0$ for any unit vector $\vec{u}$.