UF UNIVERSITY of FLORIDA
Department of Mathematics

MAC 2313
Exam 2A
Fall 2019

A. Sign your bubble sheet on the back at the bottom in ink.

B. In pencil, write and encode in the spaces indicated:

1) Name (last name, first initial, middle initial)

2) UF ID number

3) **SKIP** Section number

C. Under "special codes" code in the test ID numbers 2, 1.

1 ● 3 4 5 6 7 8 9 0
● 2 3 4 5 6 7 8 9 0

D. At the top right of your answer sheet, for "Test Form Code", encode A.

● B C D E

E. 1) This test consists of 14 multiple choice questions worth 68 points and 2 free response questions worth 20 points. The test is counted out of 80 points, and there are 8 bonus points available.

2) The time allowed is 90 minutes.

3) You may write on the test.

4) Raise your hand if you need more scratch paper or if you have a problem with your test. DO NOT LEAVE YOUR SEAT UNLESS YOU ARE FINISHED WITH THE TEST.

F. KEEP YOUR BUBBLE SHEET COVERED AT ALL TIMES.

G. When you are finished:

1) Before turning in your test **check carefully for transcribing errors**. Any mistakes you leave in are there to stay.

2) You must turn in your scantron and tearoff sheets to your discussion leader or exam proctor. Be prepared to show your picture I.D. with a legible signature.

3) The answers will be posted in Canvas within one day after the exam. Your discussion leader will return your tearoff sheet with your exam score in discussion. Your score will also be posted in Canvas within one week of the exam.

NOTE: Be sure to bubble the answers to questions 1−14 on your scantron.

Questions 1 − 12 are worth 5 points each.

1. Find the directional derivative of $f(x, y) = xy^2 + x^2$ at $(1, 2)$ in the direction $\langle -1, 1\rangle$.

a. $-\sqrt{2}$

b. -2

c. $-\dfrac{1}{\sqrt{2}}$

d. $-\sqrt{5}$

e. -5

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2. Find the maximum value of the function $f(x, y) = x - y$ subject to the constraint $x^2 + y^2 = 4$.

a. 2

b. 4

c. $\sqrt{2}$

d. $2\sqrt{2}$

e. $4\sqrt{2}$

3. How many of the following are true for the function $f(x,y) = \dfrac{\sin(x^2y)}{x^4+y^2}$?

(i) Along the line $x = 0$, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

(ii) Along the line $y = 0$, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 0$.

(iii) Along the line $y = x$, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = \frac{1}{2}$.

(iv) Along the curve $y = x^2$, $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = \frac{1}{2}$.

(v) $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist.

a. 1 b. 2 c. 3 d. 4 e. 5

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4. Let $f(x,y) = 3x - x^3 - 3xy^2$. Classify the two critical points, $P(0,1)$ and $Q(-1,0)$, of f.

a. f has a saddle point at P and a saddle point at Q.

b. f has a local maximum at P and a local minimum at Q.

c. f has a local minimum at P and a local maximum at Q.

d. f has a saddle point at P and a local maximum at Q.

e. f has a saddle point at P and a local minimum at Q.

5. Let $f(x,y) = x^3 + y^3$ and let P be the point $(1,-1)$. Which of the following is correct?

a. The rate of change of f at P in the direction of the positive x-axis is 1 .

b. The maximum rate of increase of f at P is 3.

c. There is a unit vector $\hat{u}$ so that $D_{\hat{u}}\, f(P) = -4$.

d. f does not change in the direction $\langle -1, -1 \rangle$.

e. None of the above.

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6. Let $f(x,y) = \begin{cases} \dfrac{1 - e^{x^2+y^2}}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}.$

Which of the following statements is/are true?

P. $\lim\limits_{(x,y)\to(0,0)} f(x,y) = -1.$

Q. f is continuous at $(0,0)$.

R. f is differentiable at $(0,0)$.

a. P only

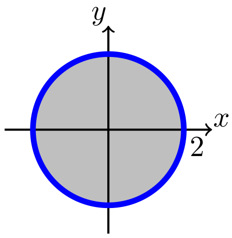
b. P and Q only

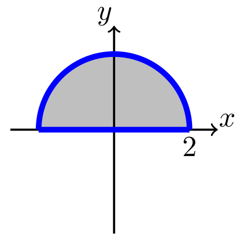
c. Q and R only

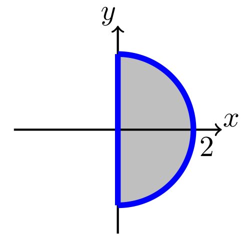
d. All of them

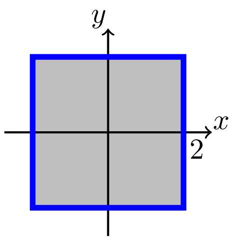
e. None of them

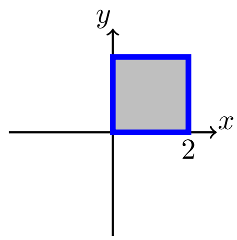
7. Which of the following is the domain of the function $f(x,y) = \sqrt{x} + \sqrt{4 - x^2 - y^2}$?

a. 

b. 

c. 

d. 

e. 

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8. Let $f(x,y) = x\sin(x - 2y)$. Calculate $f_x(2,1) + f_y(2,1)$.

a. -1

b. -2

c. 0

d. 1

e. 2

9. The point $(3, 1, c)$ is on the tangent plane to the surface $xy + yz - zx = 2$ at the point $(2, 1, 0)$. Find c.

a. 0

b. -1

c. 1

d. -2

e. 2

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10. The measurements inside a closed cylindrical tank are 20 inches high and 10 inches in radius. Use differentials to estimate the amount of metal in the tank if the metal in the top, the bottom, and the sides is 0.1 inches thick.

a. 100π in^3

b. 50π in^3

c. 90π in^3

d. 80π in^3

e. 60π in^3

11. Let $f(x,y) = y^2 e^{x^2}$. Which of the following is/are true?

P. f has exactly one critical point $(0,0)$.

Q. By definition, $f_x(1,1) = \lim_{h\to 0} \dfrac{(1+h)^2 e^{(1+h)^2} - e}{h}$.

R. $f_{xy}(1,1) = 4e$.

a. P only

b. Q only

c. R only

d. P and Q

e. P and R

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12. Evaluate $\dfrac{\partial z}{\partial x}$ at the point $\left(\dfrac{1}{e}, -1, -1\right)$ if $e^z = xyz$.

a. $1 + \dfrac{1}{e}$

b. $1 - \dfrac{1}{e}$

c. $1 + \dfrac{e}{2}$

d. $\dfrac{e}{2}$

e. $-\dfrac{e}{2}$

Bonus Questions 13 – 14 are worth 4 points each.

13. Consider the following optimization problem using Lagrange multipliers.

Find the points on the cone $x^2 + y^2 - z^2 = 0$ that are closest to the point $(2, 1, -1)$.

How many of the following equations are included in the system which must be simultaneously solved?

(i) $(x-2)^2 + (y-1)^2 + (z+1)^2 = 0$

(ii) $2x = \lambda(x+2)$

(iii) $2y = \lambda y$

(iv) $2(z+1) = \lambda(-2z)$

(v) $x^2 + y^2 - z^2 = 0$

a. 1 b. 2 c. 3 d. 4 e. 5

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14. If $z = f(x, y)$ has continuous second-order partial derivatives and $x = r^2 + t^2$ and $y = 2r$, find $\dfrac{\partial^2 z}{\partial t^2}$.

Note: $\dfrac{\partial z}{\partial t} = (2t)\dfrac{\partial z}{\partial x}$.

a. $\dfrac{\partial^2 z}{\partial t^2} = 2\,\dfrac{\partial z}{\partial x} + 4t^2\,\dfrac{\partial^2 z}{\partial x^2}$

b. $\dfrac{\partial^2 z}{\partial t^2} = 2\,\dfrac{\partial z}{\partial x} + 2t^2\,\dfrac{\partial^2 z}{\partial x^2}$

c. $\dfrac{\partial^2 z}{\partial t^2} = 2\,\dfrac{\partial z}{\partial x} + 2t\,\dfrac{\partial^2 z}{\partial x^2}$

d. $\dfrac{\partial^2 z}{\partial t^2} = 2\,\dfrac{\partial z}{\partial x}$

e. $\dfrac{\partial^2 z}{\partial t^2} = 2\,\dfrac{\partial^2 z}{\partial x^2}$

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MAC 2313 Exam 2A, Part II Free Response

Name: ______________________________

TA's Name: ____________________________ Discussion Period: ______

SHOW ALL WORK TO RECEIVE FULL CREDIT

1. (10 points) Let $f(x, y) = \ln\sqrt{x^2 + y^2}$.

(a) Find $\nabla f(1, 2)$.

$\nabla f(1, 2) =$ ____________________

(b) Find an equation of the tangent plane to the surface $z = f(x, y)$ at $(1, 2, \ln\sqrt{5})$.

$z =$ ________________________

(c) Use differentials to approximate the difference, $\Delta f = f(1.1, 1.8) - f(1, 2)$.

$df =$ ______________

2. (10 points) Let $f(x, y) = x^2 + 2y^2 - 4x - 1$.

(a) Find the critical points of f.

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(b) Find the extreme values of f on the disk $x^2 + y^2 \leq 9$.

The maximum value is ________ and it occurs at the point(s) ________________________;

The minimum value is ________ and it occurs at the point(s) ________________________

University of Florida Honor Pledge:

On my honor, I have neither given nor received unauthorized aid doing this exam.

Signature: ____________________________________