Review2 – Answers

1. (a) Domain: $\mathbb{R}^2$, Range: $[4,+\infty)$;
   (b) Domain: $\{(x,y,z): x^2+y^2+z^2<16\}$, Range: $(-\infty, \ln 16]$;
   (c) Domain: $\mathbb{R}^2$; Range: $\mathbb{R}$
   (d) Domain: $\mathbb{R}^2$; Range: $[1,+\infty)$
   (e) Domain: $\left\{(x,y,z): |z| \ge \sqrt{x^2+y^2}\right\}$; Range: $[0,+\infty)$
2. (a) Sphere; (b) Ellipsoid; (c) Circular Paraboloid; (d) Hyperbolic Paraboloid; (e) Circular Cylinder; (f) Hyperboloid of one sheet; (g) Hyperboloid of two sheets; (h) Cone.
3. (a) Ellipsoids: (b) Hyperboloids or one or two sheets or cone; (c) Cylinders
4. (a) $16x^2+4y^2=9$ or $\dfrac{x^2}{(3/4)^2}+\dfrac{y^2}{(3/2)^2}=1$ (ellipse); (b) $6x^2-2y^2=6+\ln 3$ (hyperbola); (c) $4x^2+y=2$ or $y=2-4x^2$ (parabola).
5. (a) 1, (b) $\dfrac{1}{2\sqrt{2}}$, (c) 0, (d) DNE, (e) 0, (f) $1/2$
6. $f(0,0)=1$
9. Yes; $f(x,y)=\dfrac{3}{2}x^2+xy-y^2+C,\ C\in\mathbb{R}$
10. (a) $dz=6dx-4dy$, (b) $dz=\dfrac{1}{\sqrt{x^2+y^2}}(xdx+ydy)$
11. $\dfrac{2}{15}$
12. (a) Yes,
    (b) $f_x(0,0)=0$ and $f_y(0,0)=0$;
    (c) $f_x(x,y)=2x\sin\dfrac{1}{x^2+y^2}-\dfrac{2x}{x^2+y^2}\cos\dfrac{1}{x^2+y^2}$, $(x,y)\neq(0,0)$
    $f_y(x,y)=2y\sin\dfrac{1}{x^2+y^2}-\dfrac{2y}{x^2+y^2}\cos\dfrac{1}{x^2+y^2}$, $(x,y)\neq(0,0)$;
    f_x, f_y are not continuous at $(0,0)$, but f is differentiable at $(0,0)$
    (d) $L(x,y)=0$
13. (1) $f_x=-\dfrac{z^{1/3}\cos x\sin x}{\sqrt{y+\cos^2 x}}$, $f_y=\dfrac{z^{1/3}}{2\sqrt{y+\cos^2 x}}$, $f_z=\dfrac{\sqrt{y+\cos^2 x}}{3z^{2/3}}$;
    (2) f is differentiable at $(0,0,1)$ since the partial derivatives are continuous there

(3) $L(x,y,z)=1+\frac{1}{2}y+\frac{1}{3}(z-1)$, $\Delta f \approx df = 0.02$;

14. $C \Rightarrow A \Rightarrow B, D, E, F$

15. (a) $8x+2y+z=0$; (2) $\mathbf{N}=\frac{1}{\left|\langle -f_x, -f_y, 1\rangle\right|}\langle -f_x, -f_y, 1\rangle = \frac{1}{\sqrt{69}}\langle 8,2,1\rangle$

16. $V'(t_0)=0$, neither increasing nor decreasing

17. $f_u(P)=2a+2b+c$, $f_v(P)=-2b$. $f_w(P)=0$

18. Normal vector: $\langle w_x, w_y, w_z\rangle\big|_{(3,-2,2)}=\langle -2,1,9\rangle$; $2x-y-9z=-10$

19. (a) $\nabla f = \langle f_x, f_y\rangle = \langle y-6x, x\rangle$, $\nabla f(1,3)=\langle -3,1\rangle$;

(b) $\mathbf{u}=\frac{1}{\sqrt{13}}\langle 2,3\rangle$, $D_{\mathbf{u}} f(1,3) = -\frac{3}{\sqrt{13}}$, the function decreases at $(1,3)$ in the direction of **u**

20. (a) the upper half of a hyperboloid of one sheet;

(b) $\nabla f(1,2)=\left\langle \frac{1}{2}, 1\right\rangle$, $\max_{\mathbf{u}}\{D_{\mathbf{u}} f\} = |\nabla f(1,2)| = \frac{\sqrt{5}}{2}$;

(c) $-\nabla f(1,2)=\left\langle -\frac{1}{2}, -1\right\rangle$, $\min_{\mathbf{u}}\{D_{\mathbf{u}} f\} = -|\nabla f(1,2)| = -\frac{\sqrt{5}}{2}$;

(d) $\left\langle -1, \frac{1}{2}\right\rangle$; it's perpendicular to $\nabla f(1,2)$ and tangent to the level curve of f passing through $(1,2)$

21. (a) Critical points: $P_1=(1,2), P_2=(-1,-2), P_3=(2,1), P_4=(-2,-1)$;

(b) saddle points: P_1 and P_2; loc.min at P_3; loc.max at P_4

(c) $f_{loc.\min}=f(2,1)=-28$ and $f_{loc.\max}=f(-2,-1)=28$;

(d) $z=-26,\ z=26,\ z=-28,\ z=28$ (horizontal tangent planes)

22. Critical point: $(0,0)$, since $\nabla f(0,0)$ dne; $(0,0)$ - point of loc. min (by definition)

23. $f_{\max}=f(-1,0)=7$; $f_{\min}=f(3,0)=-5$

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1. (a) $\max = f(1,2)=5$, $\min = f(-1,-2)=-5$; (b) $\min = f(0,\pm\sqrt{2})=-6$, $\max = f(\pm 2,0)=8$; (c) $\min = f(\pm 1,0)=-3$, max DNE; (d) $\min = -3\sqrt{14}, \max = 3\sqrt{14}$

2. $\min = f(\pm 2,0)=-7, \max = f_{loc.\max} = f(0,0)=1$