MAC 2312
Integration Worksheet

This review is not designed to be comprehensive, but to be representative of the topics covered on the exam. You may also review an old MAC 2312 exams posted in Canvas and try working it in one 90 minutes sitting.

1. (a) Evaluate the integrals

   i. $\displaystyle\int \frac{1}{x^2+9}\,dx$

   ii. $\displaystyle\int \frac{x}{x^2+9}\,dx$

   iii. $\displaystyle\int \frac{x^2}{x^2+9}\,dx$

   (b) Can the following integrals be solved in a similar fashion?

   i. $\displaystyle\int \frac{e^x}{e^{2x}+9}\,dx$

   ii. $\displaystyle\int \frac{\sin(2x)}{\sin^2 x+9}\,dx$

   iii. $\displaystyle\int \frac{\sin^2 x\cos x}{\sin^2 x+9}\,dx$

2. Evaluate the integrals. What are some common features among them?

   (a) $\displaystyle\int x^8\sin(x^3)\,dx$

   (b) $\displaystyle\int_0^1 x^{11}\cos(x^4)\,dx$

3. (a) Evaluate the integrals.

   i. $\displaystyle\int \arctan(\sqrt{x})\,dx$

   ii. $\displaystyle\int \frac{1}{\sqrt{x}+x\sqrt{x}}\,dx$

   iii. $\displaystyle\int \sqrt{x}e^{\sqrt{x}}$

   iv. $\displaystyle\int e^x\cos(2x)\,dx$

   (Hint: 'Wrap around' type)

   (b) Can these integrals be solved in a similar fashion?

   i. $\displaystyle\int \ln(\sqrt{x})\,dx$

   ii. $\displaystyle\int \cos x\ln(\sin x)\,dx$

   iii. $\displaystyle\int x\ln(x+1)\,dx$

   iv. $\displaystyle\int e^x\sin(2x)\,dx$

4. (a) Evaluate the integrals.

   i. $\int \sin^3 x \, dx$

   ii. $\int \sin^2 x \, dx$

   (b) Can these integrals be solved in a similar fashion?

   i. $\int \sin^3 x \cos^3 x \, dx$

   ii. $\int \cos x \cos^3(\sin x) \, dx$

   iii. $\int \frac{\sin^3 \sqrt{x}}{\sqrt{x}}, dx$

   iv. $\int (\sin x + 1)^2, dx$

5. What's a common strategy in solving these integrals? Solve the integrals.

   (a) $\int x \sec^2(2x) \, dx$

   (b) $\int x \sin^2(2x) \, dx$

6. What's a common strategy in solving these integrals? Solve the integrals.

   (a) $\int \tan x \sec^2 x \, dx$

   (b) $\int \sec^4 x \tan^4 x \, dx$

   (c) $\int x \sec^2(x^2) \tan^4(x^2) \, dx$

7. What's a common strategy in solving these integrals? Solve the integrals.

   (a) $\int \tan^2 x \, dx$

   (b) $\int \tan^2 x \sec x \, dx$

   (c) $\int \frac{\tan^2(\sqrt{x})}{\sqrt{x}} \, dx$

8. What's a common strategy in solving these integrals? Solve the integrals.

   (a) $\int \tan^3 x \sec x \, dx$

   (b) $\int \tan^3 x \sec^9 x \, dx$

(Do problem # 9∼# 11 last.)

9. Which of the following improper integrals converge? Provide a brief explanation.

(a) $\displaystyle\int_5^\infty \frac{1}{\sqrt[4]{x}}\,dx$

(b) $\displaystyle\int_5^\infty \frac{1}{\sqrt[4]{x}\ln^2 x}\,dx$

(c) $\displaystyle\int_5^\infty \frac{1}{x^2\sqrt{\ln x}}\,dx$

(d) $\displaystyle\int_5^\infty \frac{1}{x\ln x}\,dx$

10. Use comparison tests to determine which integral converges. (do not evaluate the integrals)

(a) $\displaystyle\int_5^\infty \frac{e^x}{e^{2x}+3}\,dx$

(b) $\displaystyle\int_5^\infty \frac{\ln x}{x}\,dx$

11. Evaluate the integral if it is convergent.

(a) $\displaystyle\int_0^\infty \frac{x^2}{x^6+9}\,dx$

(b) $\displaystyle\int_5^\infty \frac{e^{-1/x}}{x^2}\,dx$

(c) $\displaystyle\int_0^\infty e^{-\sqrt{x}}\,dx$

(d) $\displaystyle\int_5^\infty \frac{\ln(x+1)}{x^2}\,dx$

12. In each trig-sub problem below, draw a triangle and label an angle and all three sides corresponding to the trig-sub you select.

(a) $\displaystyle\int \frac{1}{\sqrt{9-x^2}}\,dx$

- State your substitution for x. What is dx?
- Based on your choice of x, draw and fill in the triangle.
- Using above information, write & solve the new integral.

(b) $\displaystyle\int x\sqrt{1-x^4}\,dx$

- State your substitution for x. What is dx?
- Based on your choice of x, draw and fill in the triangle.
- Using above information, write & solve the new integral.

(c) $\displaystyle\int \frac{x^3}{x^2-9}\,dx$

- State your substitution for x. What is dx?
- Based on your choice of x, draw and fill in the triangle.
- Using above information, write & solve the new integral.

(d) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

(e) $\displaystyle\int \sqrt{1-81x^2}\,dx$

(f) $\displaystyle\int \frac{x^6}{\sqrt{1+x^{14}}}\,dx$

13. What's a common strategy in solving these integrals? Solve the integrals.

(a) $\displaystyle\int \frac{x^2}{\sqrt{4x-x^2}}\,dx$

(b) $\displaystyle\int \frac{e^{2x}}{\sqrt{e^{2x}+2e^x}}\,dx$

(c) $\displaystyle\int \frac{x^2}{\sqrt{-4x+x^2}}\,dx$

14. Evaluate the integrals $\int f\,dx$. (Hint: Using PFD technique, (i) and (a) both are case 1; (ii) and (b) both are case 3. However, (ii), with no x term in the numerator, can be solved using just trig-sub where as (b), with a x term in the numerator, can be solved easily using the '2-step' process)

i. $\displaystyle\int \frac{1}{(x-1)(x+1)}\,dx$

a. $\displaystyle\int \frac{e^{2x}}{e^{2x}+11e^{x}+24}\,dx$

ii. $\displaystyle\int \frac{1}{x^2+2x+4}\,dx$

b. $\displaystyle\int \frac{x}{x^2+2x+4}\,dx$

15. Evaluate the integrals $f\,dx$. (Hint: Using PFD technique, (i) and (a) both are case 2; (ii) and (b): f needs to be proper before we can use PFD.)

i. $\displaystyle\int \frac{6x-1}{x^2(x+1)}\,dx$

a. $\displaystyle\int \frac{2x^2-11x+19}{(2x+1)(x-2)^2}$

ii. $\displaystyle\int \frac{x^3-x-8}{x^2-x-6}\,dx$

b. $\displaystyle\int \frac{5x^3-4x-2}{x^3-x}\,dx$

16. Evaluate the integrals. Which two integrals are more similar in terms of integration strategy?

i. $\displaystyle\int \frac{e^x}{(e^x-9)(e^{2x}+16)}\,dx$

iii. $\displaystyle\int \frac{1}{(x-7)(x^2+16)}\,dx$

ii. $\displaystyle\int \frac{e^x}{(e^x-9)(e^{2x}-16)}\,dx$

b. $\displaystyle\int \frac{5x^3-4x-2}{x^3-x}\,dx$

17. Evaluate the integrals. What integration strategy is used?

a. $\displaystyle\int \frac{4x-1}{x(x^2+1)^2}\,dx$

b. $\displaystyle\int \frac{1}{(x-1)(x^2+1)^2}\,dx$

18. Evaluate the integrals.

a. $\displaystyle\int \frac{x}{x^4 + x^2 + 4}\,dx$

b. $\displaystyle\int \frac{1}{e^{3x} - e^x}\,dx$

c. $\displaystyle\int \frac{1}{e^x + 18}\,dx$

d. $\displaystyle\int \frac{6x + 10}{x^2 + 4x + 24}\,dx$

e. $\displaystyle\int \frac{1}{\sqrt{x} - \sqrt[3]{x}}\,dx$

f. $\displaystyle\int \frac{7x + 1}{x^2 + 6x + 13}\,dx$

1. Evaluate the improper integral (if convergent). $\int_0^\infty e^{-2x}\cos(3x)\,dx$

2. Evaluate: $\int \frac{dx}{2\sqrt{x-1}+x}$, $\int e^{\sqrt[3]{x}}\,dx$, $\int_1^\infty \frac{\sqrt{x}}{x^3+9}\,dx$

3. Evaluate: $\int x^3\cos(3x)\,dx$, $\int (x+5)\ln(x+1)\,dx$, $\int x^3 e^{x^2}\,dx$, $\int x^8 e^{x^3}\,dx$

4. Evaluate: $\int \sin^3 x\cos x + \cos^2 x\,dx$, $\int \sec^4 x\tan x\,dx$, $\int \tan^4 x + \tan^3 x\,dx$

5. Find the correct form of PFD of $\frac{-x^2+8x+9}{(x-1)(x^2+x+2)}$ and determine the constant coefficients.

6. To perform a partial fraction decomposition, we set

$$\frac{1}{x(x-1)(x+2)^2(x-2)(x+1)^2} =$$

$$\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+2}+\frac{D}{(x+2)^2}+\frac{E}{x-2}+\frac{F}{x+1}+\frac{G}{(x+1)^2}.$$

Find $A - D$.

7. Evaluate the limit: $\lim_{x\to\infty}(1+\frac{1}{x})^{3x} = $ ____.

8. Evaluate the limit: $\lim_{x\to\infty} 2x\sin\left(\frac{1}{x}\right)$.

9. Evaluate the indefinite integral $\int \frac{1}{\sqrt{-4x+x^2}}\,dx$.

10. Evaluate the indefinite integral $\displaystyle\int \frac{dx}{x(16x^2+1)^{3/2}}$.

11. Evaluate the indefinite integral $\displaystyle\int \frac{x\ dx}{\sqrt{-x^2+2x}}, \quad \int \frac{1}{\sqrt{3-2x-x^2}}\ dx.$

12. Evaluate the integral $\displaystyle\int_0^3 x^7 \cos(x^4)\ dx.$

13. Evaluate the integral $\displaystyle\int \frac{e^{2x}\ dx}{e^{2x}+1}.$

14. Evaluate the integral $\displaystyle\int_0^1 \arctan(\sqrt{x})\ dx$ by first $u-$sub.

15. Evaluate the integral $\displaystyle\int \arctan\left(\frac{1}{x+3}\right)\ dx.$

16. Which of the following improper integrals <u>**converge(s)**</u>?

$$\int_2^\infty x^{\frac{5}{2}}\ dx, \qquad \int_4^\infty \frac{9}{\sqrt{x}}\ dx, \qquad , \int_4^\infty x^{-0.9}\ dx,$$

$$\int_4^\infty e^{-\frac{x}{10}}\ dx, \qquad \int_3^\infty \frac{\ln x}{x^{1.4}}\ dx, \qquad \int_5^\infty \frac{\ln x}{\sqrt{x}}\ dx$$

$$\int_6^\infty \frac{1}{x \ln x}\ dx, \qquad \int_3^\infty \frac{1}{x(\ln x)^{1.001}}\ dx, \qquad \int_3^\infty \frac{1}{x^{1.01}(\ln x)}\ dx,$$

17. Evaluate: $\displaystyle\int \frac{x+1}{(x^4-x^3)}\ dx, \quad \int \frac{x}{(x^4-1)}\ dx, \quad \int \frac{2x+3}{(x-1)(x-2)}\ dx,$

$$\int \frac{2x+3}{(x^2+2)}\ dx, \qquad \int_5^\infty \frac{2x+3}{(x-1)(x-2)^2}\ dx$$

18. Evaluate the integrals. (hint: try the 2-step process by first $u-$ substitution and then a trig-sub. (of course, you will need to complete

the square first $\cdots$))

$$\int \frac{2x-3}{(x^2-2x+5)}\, dx$$

$$\int \frac{x+7}{(x^2+4x+8)}\, dx$$

MULTIPLE CHOICE

1. Evaluate the integral.

$$\int \frac{1-2x}{x^2(x-1)}\,dx = \_\_\_\_\_\_\_\_ + c$$

A. $\frac{1}{2}\ln|x| - \frac{2}{x} - 4\ln|x-1|$

B. $\frac{2}{x} - 4\ln|x-1|$

C. $6\ln|-3+x| + 6\ln|-1+x| + 2\ln|x|$

D. $\ln|x| - \frac{2}{x} - 4\ln|x-1|$

E. $\ln|x| + \frac{1}{x} - \ln|x-1|$

2. Evaluate the definite integral.

$$\int_0^{\pi/4} \tan^2 x \sec^2 x\,dx$$

A. -4 B. -2 C. 0 D. $\frac{1}{3}$ E. ∞

3.

Use trig-sub to transform the integral $I = \int \frac{dx}{(x^2+6x+5)^2}$ into a trig integral.

A. $I = \frac{1}{8}\int (\tan\theta - \sin\theta\cos\theta)\, d\theta$

B. $I = \frac{1}{8}\int \frac{\cos\theta}{\sin^2\theta}\, d\theta$

C. $I = \frac{1}{8}\int (\sec\theta + \sec^3\theta)\, d\theta$

D. $I = \frac{1}{8}\int \tan^2\theta\sec^2\theta\, d\theta$

E. $I = \frac{1}{8}\int (\csc^3\theta - \csc\theta)\, d\theta$

4. Evaluate:

$\lim_{x\to\infty}\left(\left(1+\frac{3}{x}\right)^x\right)$

A. $1/e^3$ B. 1 C. e D. e^3 E. infinity

5. Partial Fraction decomposition of $\frac{8x+3}{(x-2)(x^2+4)^2}$ has the form

A. $\frac{A}{x-2}+\frac{B}{x^2+4}$ B. $\frac{A}{x-2}+\frac{B}{x^2+4}+\frac{C}{(x^2+4)^2}$ C. $\frac{A}{x-2}+\frac{Bx+C}{x^2+4}$ D. $\frac{A}{x-2}+\frac{Bx+C}{x^2+4}+\frac{Dx+E}{(x^2+4)^2}$

6.

Integral $\int_1^e \frac{\tan^3(\ln x)}{x}\,dx = A\tan^2 1 + B\ln(\sec 1)$. Find $A+B$.

A. $\ln 2$ B. $-3/2$ C. $e/2$ D. $-1/2$ E. $2/e$

7.

Evaluate the integral $\int_0^{\pi^2/4} \cos(\sqrt{x})\, dx$.

A. $\sqrt{\pi/2} - 1$

B. $\pi - 2$

C. $\pi + 2$

D. $\sqrt{2}(\pi^2/4 + 1)$

E. $\pi/4 + 1$

FREE RESPONSE

8.

Evaluate the indefinite integral.

$$\int \frac{e^{2x}}{e^{2x} + 4e^{x} + 8}\, dx$$

(Hint: Begin with an appropriate u-sub)

9.

Evaluate and simplify the integral $\displaystyle\int \frac{\sec^3 x}{\tan^2 x}\,dx.$

10.

(8 pts)(a) Complete the square and write $x^2 + 4x + 8$ as a sum of two squares.

$x^2 + 4x + 8 =$ ______________________

(b) Evaluate: $\displaystyle\int \frac{\sec^2\theta(6\tan\theta - 11)}{\tan^2\theta + 4\tan\theta + 8}\,d\theta.$

substitution: $x =$ ________________

$dx =$ ______________________

$$\int \frac{\sec^2\theta(6\tan\theta - 11)}{\tan^2\theta + 4\tan\theta + 8}\,d\theta = \rule{8cm}{0.4pt}$$

Exam 1 Review

1. Evaluate

a. $\int \frac{6x+10}{x^2+4x+24}dx$ b. $\int \frac{x+5}{6x^2-5x-6}dx$ c. $\int \frac{x}{\sqrt{3-2x-x^2}}dx$

d. $\int \frac{x^2}{(3+4x-4x^2)^{\frac{3}{2}}}dx$ e. $\int \frac{1}{\sqrt{3-2x-x^2}}\,dx$

2. Evaluate

a. $\int \frac{dx}{x\sqrt{x-1}}$ b. $\int \frac{dx}{1+\sqrt[3]{x}}$ c. $\int \frac{dx}{\sqrt{x}-\sqrt[3]{x}}$

d. $\int \frac{dx}{2\sqrt{x+3}+x}$ e. $\int \frac{dx}{x\sqrt{4x+1}}$ f. $\int \frac{dx}{x\sqrt{4x^2+1}}$

3. Evaluate

a. $\int_0^{\frac{\pi}{4}} \tan^3 x \sec^2 x\, dx$ b. $\int (\tan^2 x + \tan^4 x)\, dx$

c. $\int_0^{\frac{\pi}{4}} 12\tan^4 x \sec^4 x\, dx$ d. $\int_0^{\pi} \sin^4 x \cos^2 x\, dx$

e. $\int_0^{\frac{\pi}{2}} \sec^4(\frac{x}{2})\, dx$ f. $\int \frac{\cos^5 x}{\sqrt{\sin x}}\, dx$

4. Determine whether the integral is convergent or divergent. Use the comparison test if possible.

a. $\displaystyle\int_1^\infty \frac{1+e^{-x}}{x}dx$ b. $\displaystyle\int_1^\infty \frac{\ln x}{x^4}\,dx$ c. $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^2}\,dx$

d. $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^2-0.1}}\,dx$ e. $\displaystyle\int_\pi^\infty \frac{2+\cos x}{x}\,dx$ f. $\displaystyle\int_1^\infty \frac{\ln x}{x^3}\,dx$

g. $\displaystyle\int_1^\infty \frac{dx}{x^4+\cos^2 x}$ h. $\displaystyle\int_0^\infty (5+x)^{-\frac{1}{3}}dx$ i. $\displaystyle\int_0^\infty e^{-x}\cos x\,dx$

5. Evaluate the limits. It may be helpful to recall the heirarchy of growth rates: $\ln^p x << x^q << a^x << x! << x^x$, where p, q, and a are positive real numbers

a. $\displaystyle\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)$ b. $\displaystyle\lim_{x\to 0^+} \sin x^{\tan x}$ c. $\displaystyle\lim_{x\to 0}(e^x+x)^{\frac{1}{x}}$

d. $\displaystyle\lim_{x\to-\infty}(\sqrt{4x^2+3x}+2x)$ e. $\displaystyle\lim_{x\to 0^+}\sqrt{x}e^{\sin(\frac{\pi}{x})}$ f. $\displaystyle\lim_{x\to\infty}(14xe^{\frac{1}{x}}-14x)$

g. $\displaystyle\lim_{x\to-\infty} e^x\cos(3x)$ h. $\displaystyle\lim_{x\to\infty}\frac{(\ln x)^{837}}{\sqrt[12]{x}}$ i. $\displaystyle\lim_{u\to\infty}\frac{e^{\frac{u}{10}}}{u^3}$

j. $\displaystyle\lim_{x\to 0}\frac{e^x-1-x}{x^2}$ k. $\displaystyle\lim_{t\to\infty}\frac{8^t-5^t}{t}$ l. $\displaystyle\lim_{x\to\infty}\frac{e^x}{(3x)!}$

6. Partial Fraction Decomposition

a. As a partial fraction decomposition, we set $\frac{-x+7}{x(x+7)(x-1)^2(x+1)}$ equal to $\frac{A}{x}+\frac{B}{x+7}+\frac{C}{x-1}+\frac{D}{(x-1)^2}+\frac{E}{x+1}$. Find $A-D$.

b. As a partial fraction decomposition, we set $\frac{3x^2-8x+1}{(x-3)(x+1)(x-2)}$ equal to $\frac{A}{x-3}+\frac{B}{x+1}+\frac{C}{x-2}$. Find $A+2B-C$.

c. Suppose the partial fraction decomposition of $\frac{5x^4-9x^3+9x^2+4x+8}{(x-1)(x-2)(x^2)(x+1)}$ has the form $\frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{x-1}+\frac{D}{x}+\frac{E}{x^2}$. Find the value of $B+E$.

7. Evaluate

a. $\displaystyle\int e^x \cos(2x)dx$ b. $\displaystyle\int x^5(e^{x^2+4})^2\,dx$ c. $\displaystyle\int \frac{\ln x\,dx}{x\sqrt{1+(\ln x)^2}}$

d. $\displaystyle\int \frac{\sec^2(\ln x)}{x}dx$ e. $\displaystyle\int \tan x \ln(\sec^2 x)dx$ f. $\displaystyle\int_0^\pi e^{\cos x}\sin(2x)\,dx$

g. $\displaystyle\int e^{\sqrt{x}}dx$ h. $\displaystyle\int \cot^4 x dx$ i. $\displaystyle\int e^{-x}\cos(\frac{x}{2})\,dx$

8. Which integral do you have after the most appropriate u-sub followed by trig sub for evaluating the integral below?

$$\int \frac{e^{2x}}{\sqrt{-e^{2x}+2e^x}}dx$$

a. $\displaystyle\int \frac{\sec^3\theta}{\tan^2\theta}\,d\theta$

b. $\displaystyle\int \frac{\sin^2\theta}{\cos\theta}\,d\theta$

c. $\displaystyle\int \frac{\sec^2\theta-1}{\tan^2\theta}\,d\theta$

d. $\displaystyle\int (\sin\theta + 1)\, d\theta$

e. $\displaystyle\int (\tan\theta + 1)\, d\theta$

9. Put the seven functions below in ascending order of their growth rates. See problem 5 for a hint.

a. e^x
b. $\frac{x}{27}$
c. $x^3 - 3x$
d. $\sqrt{x}$
e. $\ln x$
f. $x \ln x$
g. $0.03x^x$

Indefinite Integrals

1. Evaluate $\int x^4 \sin(x)\, dx$.

2. Evaluate $\int \frac{\cos x}{1 - \cos x}\, dx$.

3. Evaluate $\int \frac{x^2}{x^2 + 4}\, dx$.

4. Evaluate $\int \sin^4 x\, dx$.

5. Evaluate $\int \frac{1 + e^x}{e^x - e^{2x}}\, dx$.

6. Evaluate $\int \tan^3 \theta \sec \theta\, d\theta$.

7. Evaluate $\int \frac{x^3 + x^2}{x^2 + 9}\, dx$.

8. Evaluate $\int \sin(\ln(z))\, dz$.

9. Evaluate $\int \frac{5}{(x-2)(x^2+1)}\, dx$.

10. Evaluate $\int e^{-t} \cos t\, dt$.

Definite Integrals

11. Evaluate $\int_9^{16} \frac{\sqrt{x}}{x-4}\,dx$.

12. Evaluate $\int_0^1 \tan^{-1}(x)\,dx$.

13. Evaluate $\int_{-1}^1 \frac{1}{4+x^2}\,dx$.

14. Evaluate $\int_0^\pi \sin^2 x \cos^4 x\,dx$.

15. Evaluate $\int_0^{\pi/2} \cos(x)\cos(4x)\,dx$.

16. Evaluate $\int_0^{pi/2} \sec\theta\,d\theta$.

17. Evaluate $\int_0^{\pi/2} \sec^3\theta\,d\theta$.

Limits, The Squeeze Theorem, & L'Hôpital's Rule

18. Suppose $f(x)$ satisfies the inequality $e^{-x} \leq f(x) \leq \frac{1}{x}$.
    (a) Is this sufficient to calculate $\lim_{x\to\infty} f(x)$? Explain.
    (b) Is this sufficient to calculate $\lim_{x\to-\infty} f(x)$? Explain.
    (c) Calculate the limits above where possible.

19. Evaluate $\lim_{x\to 0} x^{1/3}\cos\left(\frac{1}{x}\right)$.

20. Suppose n is a fixed positive integer.
    (a) Evaluate $\lim_{x\to\infty} \frac{\ln(x)}{x^n}$.
    (b) Evaluate $\lim_{x\to\infty} \frac{x^n}{e^x}$.

21. Evaluate $\lim_{x\to 0^+} x^x$.

22. Evaluate $\lim_{x\to\infty} (1+x)^{1/x}$

23. Evaluate $\lim_{x\to\infty} \sqrt{4x^2-x}-2x$.

24. Evaluate $\lim_{x\to-\infty} e^x\cos(3x)$.

25. What happens if you apply L'Hôpital's rule to $\lim_{x\to\infty} \frac{x}{\sqrt{x^2+1}}$? Find the limit.

Improper Integrals

26. Evaluate $\int_0^\infty \frac{1}{(3x+1)^2}\,dx.$

27. Evaluate $\int_2^\infty \frac{1}{x(\ln(x))^2}\,dx.$

28. Evaluate $\int_0^\infty xe^{-x}\,dx$

29. Evaluate $\int_0^\infty e^{-t}\cos(t)\,dt$
[Hint: use your previous work from problem 10]

30. For what values of p does the improper integral $\int_2^\infty \frac{1}{x(\ln(x))^p}\,dx$ converge?
[Hint: make a substitution]