Review 1 – Answers

1. (a) $\mathbf{r}(t) = \langle -1+2t, 3, 2+4t \rangle,\ t \in (-\infty, +\infty)$;
   (b) $\mathbf{r}(t) = \langle 5+2t, 1-t, -2+4t \rangle,\ t \in (-\infty, +\infty)$;
   (c) $\mathbf{r}(t) = \langle -1+3t, 4-5t, 5 \rangle,\ t \in (-\infty, +\infty)$;
   (d) $\mathbf{r}(t) = \langle 1+t, 2+2t, -1-3t \rangle,\ t \in (-\infty, +\infty)$
2. $\mathbf{r}(t) = \langle -1+3t, 4-5t, 5 \rangle,\ t \in [0,1]$
3. (a) $6\ N \cdot m$; (b) $\mathbf{F}_{\parallel} = \langle 2, 2, 1 \rangle$; (c) $|\mathbf{F}_{\parallel}| = 3$; (d) $\mathbf{F}_{\perp} = \langle -1, 0, 2 \rangle$; (e) $\theta = \arccos\left(\frac{3}{\sqrt{14}}\right)$
4. (a) neither; (b) $\theta = \arccos\left(\frac{2}{\sqrt{5}}\right)$; (c) $x = 2t,\ y = t,\ z = 5-2t;\ t \in (-\infty, +\infty)$;
   (d) $2x + y - 2z = 0$
5. (a) $19x - 11y - 2z = 33$; (b) $3x - 2y + 4z = 8$; (c) $y = 3$
6. $S = \sqrt{86}/2$
7. (a) Sphere, $(x-1)^2 + (y+2)^2 = 41$; (b) paraboloid, $(x-1)^2 + z^2 = 1$;
   (c) cylinder, $(x-2)^2 + (y+3)^2 = 37$; lines: $x = 0,\ y = -3 \pm \sqrt{33},\ z = t,\ t \in (-\infty, +\infty)$
8. (a) $(-\infty, 0) \cup (0, 1]$; (b) $\left\langle 2, -\frac{1}{2}, 3 \right\rangle$; (c) $(-\infty, 0) \cup (0, 1]$;
   (d) $\mathbf{r}'(t) = \left\langle \frac{2te^{2t} - e^{2t} + 1}{t^2}, \frac{2\sqrt{1-t} + t - 2}{2t^2\sqrt{1-t}}, \frac{3t\cos 3t - \sin 3t}{t^2} \right\rangle$
9. $\left\langle \frac{1}{2}\left(e^6 - e^2\right),\ \frac{2}{3}\left(8 - 2\sqrt{2}\right), \ln\frac{5}{3} \right\rangle$
10. $L = 8 + \ln 3$
11. (a) $\kappa = \frac{1}{5}$; (b) $\kappa = \frac{6}{\left(4t^2 + 9\right)^{3/2}}$; (c) $\kappa = 2\sqrt{2}$
12. (a) $\mathbf{T}(t) = \left\langle \frac{1}{\sqrt{1+t^2}}, 0, \frac{t}{\sqrt{1+t^2}} \right\rangle$, $\mathbf{N}(t) = \left\langle -\frac{t}{\sqrt{1+t^2}}, 0, \frac{1}{\sqrt{1+t^2}} \right\rangle$, $\mathbf{B} = \langle 0, -1, 0 \rangle$.
13. (a) $\mathbf{B}$; (b) $\mathbf{T}$; (c) $\mathbf{0}$; (d) $\mathbf{N}$; (e) 1
14. (a) $\mathbf{v}(t) = \langle -e^{-t}(\cos t + \sin t), e^{-t}(\cos t - \sin t), -e^{-t} \rangle$ and $|\mathbf{v}(t)| = e^{-t}\sqrt{3}$
    (b) $\langle 0, 0, 0 \rangle$ and $\langle 0, 0, 0 \rangle$; (c) $L = \sqrt{3}\left(1 - e^{-1}\right)$; (d) $L(t) = \sqrt{3}\left(1 - e^{-t}\right)$; (e) $\sqrt{3}$
15. (a) $a_T = \frac{t-1}{\sqrt{17 + (t-1)^2}}$, $a_N = \frac{\sqrt{17}}{\sqrt{17 + (1-t)^2}}$; (b) $a_T = 0, a_N = 1$
16. $2.82\ N \cdot m$

17. (a) $-\mathbf{i}+\mathbf{j}+\mathbf{k}$; (b) 0; (c) $\mathbf{0}$; (d) $|\mathbf{a}|^2$; (e) $\mathbf{b}\times\mathbf{a}$; (f) 0

18. (a) $x^2+y^2+(z-1)^2=1$ is a sphere of radius 1 centered at $(0,0,1)$ and $x^2+y^2=1$ is a cylinder of radius 1 whose axis is the z-axis;
(b) Circle: $x=\cos t,\ y=\sin t,\ z=1 \quad (0\le t\le 2\pi)$.

19. $\left\langle \frac{3}{20}, \frac{3\sqrt{3}}{20}, 0 \right\rangle, \frac{3}{10}$

20. (a) $\mathbf{u}\ \&\ \mathbf{v}$; (b) $\mathbf{u},\mathbf{w},\mathbf{p}$ and $\mathbf{v},\mathbf{w},\mathbf{p}$;
(c) $\alpha=\cos^{-1}\left(-\frac{1}{\sqrt{21}}\right), \beta=\cos^{-1}\left(\frac{2}{\sqrt{21}}\right), \gamma=\cos^{-1}\left(\frac{4}{\sqrt{21}}\right)$;

21. (a) $S=4\sqrt{3}$, (b) $V=24\sqrt{3}$

22. $L_1 \parallel L_2$; L_1 and L_3 intersect at $(3,4,7)$; L_2 and L_3 are skew

23. (a) $L_1: x=1+t,\ y=-1-t,\ z=1+3t$; $L_2: x=s,\ y=1-s,\ z=-1+3s$;
(b) $4x+y-z=2$; (c) $x-y+3z=8\pm 3\sqrt{11}$ (d) $(0,0,-2)$

24. (a) $P_1 \parallel P_2$; $P_1\cap P_3\neq\varnothing,\ P_2\cap P_3\neq\varnothing$; (b) $D_1=\frac{1}{2\sqrt{6}}$;
(c) $\theta=\cos^{-1}\sqrt{\frac{6}{11}}$, $\mathbf{r}(t)=\langle 6-2t, t, -11+5t\rangle, \quad -\infty<t<+\infty$; (d) $D_2=\frac{3}{\sqrt{11}}$