MAC2313, Calculus III
Exam 1 Review

1. Let $\vec{a} = \hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$, and $\vec{c} = \hat{j} - 5\hat{k}$. Find

(1) $|\vec{a}|$ (2) $\vec{a} \cdot \vec{b}$

(3) $\vec{a} \times \vec{b}$ (4) $\vec{a} \cdot (\vec{b} \times \vec{c})$

(5) the angle between $\vec{a}$ and $\vec{b}$

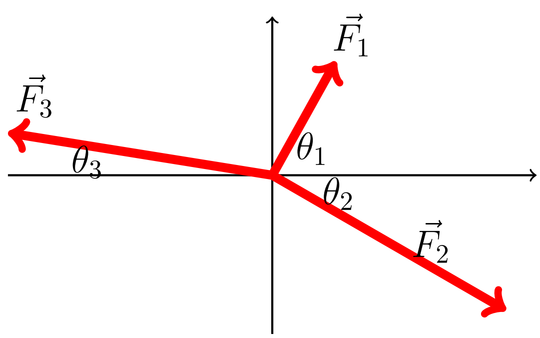
(6) the scalar projection of $\vec{b}$ onto $\vec{a}$

(7) the vector projection of $\vec{b}$ onto $\vec{a}$

(8) the area of the parallelogram determined by $\vec{a}$ and $\vec{b}$

(9) the volume of the parallelepiped determined by $\vec{a}$, $\vec{b}$, and $\vec{c}$

2. Thee forces act on a particle as given in the diagram below. Assume the system is in equilibrium.



If $\theta_1 = \dfrac{\pi}{3}$, $\theta_2 = \dfrac{\pi}{6}$, $|\vec{F}_1| = 5\text{N}$, and $|\vec{F}_2| = 10\text{N}$, find (1) $|\vec{F}_3|$ and (2) θ_3.

3. Three forces $\vec{F}_1 = \langle 2, 1, 1 \rangle$, $\vec{F}_2 = \langle -1, 5, 3 \rangle$ and $\vec{F}_3$ act on an object. Find $\vec{F}_3$ if the net force on the particle has magnitude 6 and is in the direction of $\langle 1, -2, 2 \rangle$.

4. Assume that $\vec{u} \cdot \vec{v} = -3$ and $|\vec{v}| = 2$. Find $\vec{v} \cdot (2\vec{u} - 3\vec{v})$.

5. Let $\vec{u} = \langle 3, -1, 2 \rangle$ and $\vec{v} = \langle -2, 1, -1 \rangle$. Express the vector $\vec{u}$ as the sum $\vec{u} = \vec{v}_{//} + \vec{v}_{\perp}$, where $\vec{v}_{//}$ is parallel to $\vec{v}$ and $\vec{v}_{\perp}$ is perpendicular to $\vec{v}$.

6. If $A(1, -2, 3)$, $B(-1, 4, 5)$, and $C(0, -1, 3)$ are three points in space, find

(1) the point closest to the xz-plane and the point closest to the plane $x = -2$
(2) an equation of the sphere with a diameter AB
(3) a unit vector perpendicular to the plane containing A, B, and C
(4) an equation of the plane containing A, B, and C
(5) the area of the triangle ABC

7. (1) Determine whether $A(1, 0, 1)$, $B(2, -1, 3)$, and $C(3, -2, 5)$ lie on the same line.
(2) Determine whether $P(1, 1, 1)$, $Q(2, 0, 3)$, $R(4, 1, 7)$, and $S(3, -1, -2)$ lie on the same plane.

8. Do the lines $\vec{r}_1(t) = \langle 2 + t, 1 - 2t, t + 3 \rangle$ and $\vec{r}_2(s) = \langle 1 - s, s, 2 - s \rangle$ intersect? If so, find the point of intersection.

9. Let $L_1 : \dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $L_2 : \dfrac{x+1}{6} = \dfrac{y-3}{-1} = \dfrac{z+5}{2}$ be two lines in space.

(1) Is $L_1 // L_2$? Do two lines intersect?
(2) Find the distance from the point $(1, 1, 1)$ to L_1.

10. Let $P_1 : \ x + y - z = 1$ and $P_2 : \ x - y - z = 5$.

(1) Do two planes intersect?
(2) Find the angle between P_1 and P_2
(3) Find symmetric equations of the line of intersection of the two planes.
(4) Find the distance from the point $(1, 1, -1)$ to P_1.

11. Discuss traces of the surface $x^2 - y^2 + 4z^2 + 2y = 1$ and identify the surface.

12. Find an equation of the surface consisting of all points $P(x, y, z)$ that are equidistant from P to the z-axis and from P to the plane $x = -1$. Identify the surface.

13. Consider the curve $\vec{r}(t) = \cos t\,\hat{i} + t\,\hat{j} - \sin t\,\hat{k}$. Find

(1) the unit tangent vector $\hat{T}(t)$ and the unit normal vector $\hat{N}(t)$
(2) the tangent line to the curve at $(1, 0, 0)$
(3) the arc length from $(1, 0, 0)$ to $(1, 2\pi, 0)$
(4) $\dfrac{ds}{dt}$
(5) the curvature of the curve at the point $(1, 0, 0)$

14. Find the curvature of the function $y = x^4$ at the point $(1, 1)$.

15. Let $\vec{r}(t) = \langle t^2, 2t, \ln(t)\rangle$ be a vector function describes the path of a particle with respect to t. Find the tangential and normal components of acceleration at $t = 1/2$.

16. For the curve given by $\vec{r}(t) = \langle \sin^3 t, \cos^3 t, \sin^2 t\rangle,\ 0 < t < \pi/2$, find

(1) the unit tangent vector
(2) the unit normal vector
(3) the unit binormal vector
(4) the curvature

17. Let $\vec{r}(t) = \langle t\ln(t), \sin(\pi t), \sqrt{5 - t}\rangle$ be a vector function.

(1) Find the domain of $\vec{r}(t)$.
(2) Find $\lim_{t \to 0^+} \vec{r}(t)$.
(3) Find $\int \vec{r}(t)\,dt$.
(4) Let the curve C be parametrized by $\vec{r}(t)$. Find a and b if the vector $\langle a, b, 1\rangle$ is parallel to the tangent vector of the curve C at the point $(0, 0, 2)$.