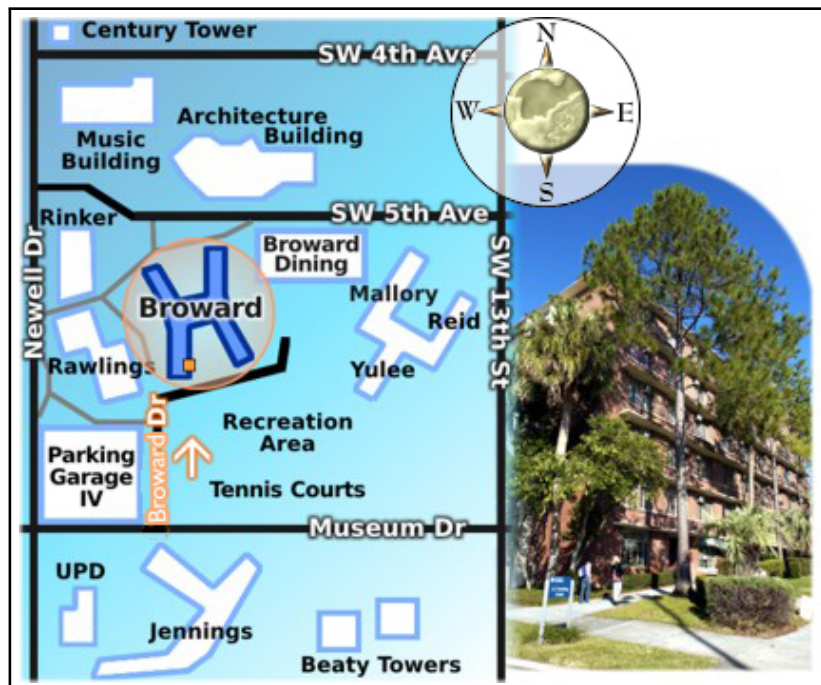




This review, produced by the Broward Teaching Center, contains a collection of questions which are representative of the type you may encounter on the exam. Other resources made available by the Teaching Center include:

- Walk-In tutoring at Broward Hall
- Private-Appointment, one-on-one tutoring at Broward Hall
- Walk-In tutoring in LIT 215
- Supplemental Instruction
- Video resources for Math and Science classes at UF
- Written exam reviews and copies of previous exams

The teaching center is located in the basement of Broward Hall:



You can learn more about the services offered by the teaching center by visiting <https://teachingcenter.ufl.edu/>

MAC 2311 Exam 1 Review

1. Consider the function $f(x) = \frac{x^2 - x - 2}{|2 - x|}$
 - (a) Calculate $\lim_{x \rightarrow 2^+} f(x)$ and $\lim_{x \rightarrow 2^-} f(x)$.
 - (b) Does $\lim_{x \rightarrow 2} f(x)$ exist? Why or why not?
 - (c) Is $f(x)$ continuous? If not, determine where $f(x)$ is discontinuous and describe the discontinuity.

2. For what real numbers A is the function $f(x) = \begin{cases} \frac{|x-1|}{x^2-1} & x < 1 \\ Ax^2 + 6 & x \geq 1 \end{cases}$ continuous at $x = 1$? Is there a choice of A which makes $f(x)$ continuous everywhere?

3. The position of a particle in one-dimensional motion is given by $s(t) = -10t^2 + 25t + 2$
 - (a) Calculate the average velocity of the particle on the time interval $[2, 7]$
 - (b) Calculate the average velocity of the particle on the time interval $[2, 2 + h]$
 - (c) Send $h \rightarrow 0$ in part (b). What is the value of the limit? What does this describe?
 - (d) Find the equation of the line tangent to $h(t)$ at $t = 2$.

4. Evaluate the limits below. If the limit does not exist, indicate why.
 - (a) $\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x - 2}$
 - (b) $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{x^2 + x} \right)$
 - (c) $\lim_{x \rightarrow -4} \frac{\sqrt{x^2 + 9} - 5}{x + 4}$

5. Suppose $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for all $x > 0$.
 - (a) Is this sufficient information to calculate $\lim_{x \rightarrow 4} f(x)$? If so, what is the value of the limit? If not, explain why.
 - (b) Is this sufficient information to calculate $\lim_{x \rightarrow 1} f(x)$? If so, what is the value of the limit? If not, explain why.

6. Evaluate the limits below. If the limit does not exist, indicate why.

(a) $\lim_{x \rightarrow \infty} e^x$

(b) $\lim_{x \rightarrow \infty} e^{-x}$

(c) $\lim_{x \rightarrow \infty} \ln(x)$

(d) $\lim_{x \rightarrow 0^+} \ln(x)$

7. Evaluate the limits below. If the limit does not exist, indicate why.

(a) $\lim_{x \rightarrow \infty} \frac{7x^3 + x^2 - 5}{x^3 - 2x + 1}$

(b) $\lim_{x \rightarrow -\infty} \frac{4x^7}{3 - 2x^4}$

(c) $\lim_{x \rightarrow \infty} \frac{x - 3}{x^2 + 2x + 1}$

(d) $\lim_{x \rightarrow -\infty} \frac{2x^2 + 1}{3x - 5}$

(e) $\lim_{x \rightarrow \infty} e^{\tan^{-1}(x)}.$

8. The profit generated by selling x Florida Gators branded keychains is given by

$$P(x) = 100x + \frac{10,000}{x + 1}$$

- (a) Find the average change in profit if vendors go from selling x keychains to $x + h$ keychains.
- (b) How is profit changing when vendors are selling exactly 50,000 keychains?

9. Use (either) of the limit definition(s) of the derivative to calculate the derivatives of each function below.

(a) $f(x) = x^2$

(b) $g(x) = \sqrt{x}$

(c) $h(x) = \sin(x)$ [Hint: $\sin(x+h) = \sin(x)\cos(h) + \sin(h)\cos(x)$, $\lim_{h \rightarrow 0} \frac{\sin(h)}{h} = 1$, and $\lim_{h \rightarrow 0} \frac{\cos(h) - 1}{h} = 0$]

10. In this problem we will work with the functions from the previous problem.

- (a) For what values of x (if any) do each of the functions have horizontal tangent lines?
- (b) For what values of x (if any) do each of the functions have vertical tangent lines?

- 1) Complete the following actions given the following function.

$$f(x) = \frac{x^2 - x - 2}{|2 - x|}$$

- a) Find $\lim_{x \rightarrow 2^+} f(x)$.
- b) Find $\lim_{x \rightarrow 2^-} f(x)$.
- c) Find $\lim_{x \rightarrow 2} f(x)$.

- 2) Find the value of a for which the following piecewise function will be continuous at $x = 1$.

$$f(x) = \begin{cases} \frac{|x - 1|}{x^2 - 1} & \text{if } x < 1 \\ ax^2 + 6 & \text{if } x \geq 1 \end{cases}$$

- 3) A strange alien projectile is thrown from the ground with an initial velocity of 25 ft/s and an acceleration of 10 ft/s^2 , but the "alien-ness" provides a unique motion. Its height, $h(t)$, after t seconds is given by:

$$h(t) = -10t^2 + 25t + 5$$

- a) Find the average velocity from the time interval $[2, 6]$.
- b) Find the average velocity from the time interval $[2, 2 + h]$.
- c) Use limit and part (b) to find the instantaneous velocity after 2 seconds.
- d) What would be the equation of the tangent line to $h(t)$ at $t = 2$?

- 4) Explain why the Intermediate Value Theorem does or does not guarantee a solution of the given equation in the specified interval:

- a) $(x - 1)^4 + x - 4 = 0$ on the interval $(2, 3)$
- b) $\tan(x) = 0$ on the interval $\left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$
- c) $\sqrt[3]{x - 2} = 3 - x$ on the interval $(-2, -1)$

- 5) Let

$$f(x) = \begin{cases} x^2 - 9 & \text{if } x < -3 \\ x + 3 & \text{if } -3 \leq x < 1 \\ -\frac{x^2 - 2x - 8}{x^2 - 7x + 12} & \text{if } x \geq 1 \end{cases}$$

- a) Sketch $f(x)$.
- b) Find the following limits: $\lim_{x \rightarrow 1} f(x)$ and $\lim_{x \rightarrow 3} f(x)$
- c) Find and describe any discontinuities of $f(x)$.
- d) If possible, make $f(x)$ continuous at its points of discontinuity.
- e) Find each interval on which f is continuous (including from the left and right).

- 6) Consider the function

$$f(x) = \frac{x^2 + 2x - 8}{x^2 - 2x}$$

- | | |
|--|---|
| a) What is $\lim_{x \rightarrow 2} f(x)$? | d) What is $\lim_{x \rightarrow +\infty} f(x)$? |
| b) What is $\lim_{x \rightarrow 0^-} f(x)$? | e) What is $\lim_{x \rightarrow -\infty} f(x)$? |
| c) What is $\lim_{x \rightarrow 0^+} f(x)$? | f) Identify the removable discontinuities and vertical asymptote(s) |

- 7) Use algebra and/or other simplification techniques to evaluate the following limits:

a)

$$\lim_{x \rightarrow 3} \frac{\sqrt{x} - \sqrt{3}}{x^2 - 9}$$

b)

$$\lim_{x \rightarrow 0} e^{\frac{1}{x}}$$

c)

$$\lim_{x \rightarrow 1} \arccos\left(\frac{\sqrt{x} - 2}{1 + \sqrt{x}}\right)$$

d)

$$\lim_{x \rightarrow 4} f(x) \text{ where } f(x) = \begin{cases} \frac{x^2 - 16}{x - 4} & \text{if } x \neq 4 \\ 3 & \text{if } x = 4 \end{cases}$$

What is the type of discontinuity this function has at $x = 4$?