Exam 1 Review

Disclaimer: Exam 1 covers Chapters 6.1-6.7, 7.1, 7.3, and 7.4. This review may not cover all the material that will be on the exam. Exam 1 can cover any material from the lectures, homework, quizzes, etc. Final answers to these problems will only be released after the exam review. This is to focus your studying on understanding the process of solving them, rather than the final answer. Feel free to use the discussion boards to discuss these problems.

1. Evaluate the following integrals.

   (a) $\int_0^1 (x - e^x)^2 \, dx$

   (b) $\int_0^\infty \frac{x}{(x^2+2)^2} \, dx$

   (c) $\int_{-2}^{-1} \frac{1}{x^3} \, dx$

   (d) $\int \ln(3x) \, dx$

   (e) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

   (f) $\int_e^{e^4} \frac{\sqrt{\ln(x)}}{x} \, dx$

   (g) $\int \frac{x^2+1}{x} \, dx$

   (h) $\int_{-\infty}^0 xe^x \, dx$

2. Determine whether the statement is true or false. Give a reason for your answer.

   (a) $\int_{-1}^1 \frac{1}{x^5} dx = -\frac{1}{4x^4}\Big|_{-1}^1 = -\frac{1}{4} - \left(-\frac{1}{4}\right) = 0$

   (b) The indefinite integral is another term for the family of all antiderivatives of a function.

   (c) The definite integral of a function is always a positive value.

3. Find the area of the region between the graphs of $y = 5 - x^2$ and $y = x^2 - 3$.

4. Find the area between the x-axis and $f(x) = xe^{x^2}$ over the interval $[0, 2]$.

5. Given $\int_1^5 f(x) \, dx = 12$ and $\int_4^5 f(x) \, dx = 3.5$, calculate $\int_1^4 2f(x) \, dx$.

6. Suppose the supply function for a new brand of watch (in units q) is given (in dollars p) by

$$p = q^2 + 10q$$

and the demand function is given by

$$p = 900 - 20q - q^2.$$

Find the consumer's surplus and producer's surplus, assuming the market price is at equilibrium.

7. Approximate the value of $\displaystyle\int_1^5 \frac{2x}{x^2+1}dx$.

   (a) Using a Right Riemann Sum with $n = 4$ subintervals.

   (b) Using a Left Riemann Sum with $n = 4$ subintervals.

   (c) Using a Midpoint Riemann Sum with $n = 4$ subintervals.

   (d) Using the Trapezoidal Rule with $n = 4$ subintervals.

   (e) Using Simpson's Rule with $n = 4$ subintervals.

   (f) Can you figure out the exact value? Which method was closest to the exact value?

8. Find the cost function for the marginal cost function $C'(x) = 3\sqrt{2x-1}$ if 13 units cost \$270.

9. Find the demand function for the marginal reveune function $R'(x) = 170 - 0.02x - 0.03x^3$. (Recall: If no items are sold, the revenue is 0.)

10. Find the average value of $f(x) = x\ln(x)$ over the interval $[1, e]$.

11. Let $f(x) = x^2 + 1$. Find $\displaystyle\sum_{n=0}^{5} f(n)$.

12. If $\displaystyle\int_0^2 3x\,dx = 6$, what is the value of $\displaystyle\lim_{n\to\infty}\sum_{i=1}^{n}\left[3\left(\frac{2i}{n}\right)\left(\frac{2}{n}\right)\right]$?