

MAC 2233: Unit 1 Exam Review
Exam covers Lectures 1 – 9,

1. Solve for x : $2(x+1)^{-1/3}x^{4/3} - (x+1)^{2/3}x^{-2/3} = 0$
2. Perform the operation and simplify the expression: $\frac{\frac{3x}{\sqrt{x^2+4}} - \sqrt{x^2+4}}{2\sqrt{x^2+4}}$
3. Solve the inequality: $\frac{x+4}{x-1} \leq 2$
4. Find and simplify $\frac{f(x+h) - f(x)}{h}$ for
 - a) $f(x) = 2x^2 - x - 3$ and b) $f(x) = \frac{x}{x+4}$.
5. Let $f(x) = \frac{x}{x-2}$ and $g(x) = \frac{2}{x} + 1$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$. Include domains.
6. Let $f(x) = \sqrt{x-1}$ and $g(x) = \frac{x}{\sqrt{x-1}}$. Find $\frac{f}{g}(x)$ and its domain.
7. Sketch the graph of $f(x) = 3 - 2x - x^2$ by using a formula to find the vertex. Show all intercepts. Confirm your work by writing your function in standard form $f(x) = a(x-h)^2 + k$ by completing the square, and using translations to graph.
8. Sketch the graph of $f(x) = 2 - \sqrt{1-x}$. Starting with $y = \sqrt{x}$, list each translation used to graph $f(x)$.
9. Use the definition of absolute value to write the function $g(x) = x|x|$ as a piecewise defined function. Then sketch its graph.
10. Find the inverse of $f(x) = \sqrt{4-x}$. Be sure to include domain.
11. Find the inverse of one-to-one function $f(x) = \frac{x+2}{x-3}$. Use that inverse function to find the range of $f(x)$. Then find the horizontal asymptote of $f(x)$ if possible.
12. Find the domain of the following functions:
 - (a) $f(x) = \sqrt{x^3 - x^2 - 6x}$
 - (b) $f(x) = \ln\left(\frac{8}{x} - 2\right)$

13. Find the solution set of each of the following equations:

(a) $\log_3(2x^2 - 5) - \log_3 x = 1$ (b) $4^{3-x^2} = \left(\frac{1}{8}\right)^{x+1}$

(c) $\ln(x+8) + \ln(x-2) = \ln(3x+2)$

14. Find the inverse of $f(x) = e^{x+3} - 4$. Sketch the graph of f and f^{-1} on the same axes. Include at least one point and any asymptotes of each function.

15. Let $f(x) = \begin{cases} x+4 & x < -2 \\ 2-|x| & -2 \leq x < 2 \\ \ln(x-1) & x > 2 \end{cases}$.

(a) Find if possible: $f(-4)$, $f(-2)$, $f(0)$, $f(2)$, $f(e+1)$.

(b) Sketch the graph of $y = f(x)$.

(c) Use your graph to evaluate the following limits if they exist:

1) $\lim_{x \rightarrow -2} f(x)$ 2) $\lim_{x \rightarrow 0} f(x)$ 3) $\lim_{x \rightarrow 2} f(x)$

16. Let $f(x) = \frac{x^2 - 4}{x^2 - 2x - 8}$. Find:

(a) domain of f

(b) all intercepts (express as ordered pairs)

(c) all vertical and horizontal asymptotes

(d) Sketch the graph of $y = f(x)$. Include the coordinates of any holes in the function.

(e) Use your graph to find $\lim_{x \rightarrow -2} f(x)$.

17. There is a linear relationship between temperature in degrees Celsius C and degrees Fahrenheit F . Water freezes at $0^\circ C$ ($32^\circ F$) and boils at $100^\circ C$ ($212^\circ F$). Write the model expressing C as function of F . What is the temperature in degrees Fahrenheit if the temperature is $30^\circ C$? What does the slope of the line tell you?

18. The financial department of a company has produced the following data, where $C(x)$ is the cost in hundreds of dollars to produce x items.

x	100	300	500	800
$C(x)$	200	450	659	750

(a) Use linear regression to calculate the least squares line and to find the correlation coefficient.

(b) According to this model, what are the fixed costs of producing this item? What are the anticipated costs of producing 1000 items?

(c) What is the marginal cost? What does that tell you?

19. The demand and supply functions for a given product are given by
 $p = D(q) = 60 - 2q^2$ and $p = S(q) = q^2 + 9q + 30$ where q is quantity in thousands and p is the unit price. Find the equilibrium quantity and price.
 How many items will the supplier provide if the unit price of the product is \$40? What will be the demand for the product when the unit price is \$40? What should happen to the price of the product?
20. A financial manager at Target has made the following observations about a certain product in one of its districts: an average of 250 units will sell in a month when the price is \$15, but an average of 50 more will sell if the price is reduced by \$1. Assuming the demand function is linear,
- Express p as a function of x .
 - Find the revenue function $R(x)$. Find the production level x that will maximize revenue. What is the maximum revenue?
 - If fixed costs are \$800 and the marginal cost is \$10 per item, find each value of x at which the company will break even. What is the profit for those values?
 - Find the profit function $P(x)$. What price should the manager charge to maximize profit on this item?
21. A farmer plans to spend \$6000 to enclose a rectangular field with two kinds of fencing. Two opposite sides will require heavy-duty fencing that costs \$3 per linear foot, while the other two sides can be constructed with standard fencing that costs \$2 per foot. Express the area of the field, A , as a function of x , the length of a side that requires the more expensive fence. Find the value of x that will maximize the area of the field, and the length of a side that uses standard fencing.
22. Rewrite the expression as the sum, difference, or multiple of logarithms:
- $\log \frac{x^2}{1000}$
 - $\ln \sqrt[3]{\frac{e^{x+1}(x-2)^4}{x^6}}$
23. Mr. Jones invested \$2500 at 5.5% compounded continuously. How long will it take his account to grow to \$4000 if he adds no new funds to the account?
24. How much money must be invested now at 3 1/4% compounded quarterly in order to have \$6000 in three years?
25. Iodine - 131 has a half-life of 8 days. Suppose some hay was contaminated with ten times the allowable amount of I-131. How long must the hay be stored before it can be fed to cattle? Hint: the hay must have one-tenth of its current amount of I-131.