CHAPTER 2

GEOMETRY & MEASUREMENT

SKILLS		*PAGE*
	Overview	76
IB1	*Round measurements*	78
IVB2	*Solve word problems involving the Pythagorean theorem*	82
IIB4	*Identify appropriate units of measure*	85
IB2	*Calculate distances, areas, and volumes*	89
IVB1	*Solve word problems involving geometric figures*	107
IIIB2	*Identify formulas for measuring geometric figures*	112
IIIB1	*Infer formulas for measuring geometric figures*	118
IIB1	*Identify relationships between angle measures*	122
IIB2	*Identify names of plane figures given their properties*	133
IIB3	*Recognize similar triangles and their properties*	137
PRACTICE PROBLEMS		143
PRACTICE EXPLANATIONS		157

Geometry is a branch of mathematics that deals with the measurement, properties, and/or relationships of points, lines, angles, surfaces, and solids. It is possible to describe, design, and construct visible objects using the principles of geometry. Geometry is important in many other fields of study, such as architecture, construction, engineering, science, and surveying.

The titles of the units found in this chapter are listed below in italics. Under each unit title is listed the specific geometry and measurement skill or skills covered in that unit.

Round measurements.
Skill IB1: The student will round measurements to the nearest given unit of the measuring device.

Solve word problems involving the Pythagorean theorem.
Skill IVB2: The student solves real-world problems involving the Pythagorean property.

Identify appropriate units of measure.
Skill IIB4: The student will identify appropriate units of measurement for geometric objects.

Calculate distances, areas, and volumes.
Skill IB2a: The student will calculate distances.
Skill IB2b: The student will calculate areas.
Skill IB2c: The student will calculate volumes.

Solve word problems involving geometric figures.
Skill IVB1: The student solves real-world problems involving perimeters, areas, volumes of geometric figures.

Identify formulas for measuring geometric figures.
Skill IIIB2: The student identifies applicable formulas for computing measures of geometric figures.

Infer formulas for measuring geometric figures.
Skill IIIB1: The student infers formulas for measuring geometric figures.

Identify relationships between angle measures.
Skill IIB1: The student will identify relationships between angle measures.

Identify names of plane figures given their properties.

Skill IIB2: The student will classify simple plane figures by recognizing their properties.

Recognize similar triangles and their properties.

Skill IIB3: The student will recognize similar triangles and their properties.

ROUND MEASUREMENTS

Rounding measurements is a quick method of determining a reasonable mathematical estimate.

Conversions

Some problems may involve the conversion of units within the English system of measurement before rounding can be performed. The following table gives some common relationships needed to make English-to-English conversions. Abbreviations are given within parentheses. You are expected to know the relationships and the abbreviations.

Length:	1 foot (ft.)	= 12 inches (in.)
	1 yard (yd.)	= 3 feet (ft.)
	1 yard (yd.)	= 36 inches (in.)
	1 mile (mi.)	= 5280 feet (ft.)
Weight:	1 pound (lb.)	= 16 ounces (oz.)
	1 ton (T.)	= 2000 pounds (lb.)
Capacity:	1 cup (c.)	= 8 fluid ounces (oz.)
	1 pint (pt.)	= 2 cups (c.)
	1 quart (qt.)	= 2 pints (pt.)
	1 gallon (gal.)	= 4 quarts (qt.)
Time:	1 minute (min.)	= 60 seconds (sec.)
	1 hour (hr.)	= 60 minutes (min.)
	1 day (d.)	= 24 hours (hr.)
	1 week (wk.)	= 7 days (d.)
	1 year (yr.)	= 365 days (d.)

Examples

1. Change 3 hours, 24 minutes into hours.

   1 hr. = 60 min., so the conversion ratio needed to eliminate minutes in the numerator and end up with hours in the numerator is $\frac{1 \text{ hr.}}{60 \text{ min.}}$.

   $$24 \text{ min.} = \frac{24 \text{ min.}}{1} \times \frac{1 \text{ hr.}}{60 \text{ min.}}$$

   $$24 \text{ min.} = \frac{24 \text{ hr.}}{60} = .4 \text{ hr.}$$

   Thus, 3 hours, 24 minutes = 3.4 hours.

2. Change $7\frac{1}{2}$ feet to yards.

1 yd. = 3 ft., so the conversion ratio needed to eliminate the feet units in the numerator and end up with yards in the numerator is $\frac{1 \text{ yd.}}{3 \text{ ft.}}$.

$$7\frac{1}{2} \text{ ft.} = \frac{15}{2} \text{ ft.}$$

$$7\frac{1}{2} \text{ ft.} = \frac{15 \text{ ft.}}{2} \times \frac{1 \text{ yd.}}{3 \text{ ft.}}$$

$$7\frac{1}{2} \text{ ft.} = \frac{15 \times 1}{2 \times 3} \text{ yd.} = \frac{15}{6} \text{ yd.} = \frac{5}{2} \text{ yd.}$$

Rounding a Measurement

When rounding a given measurement to a specific unit, use the following procedure:

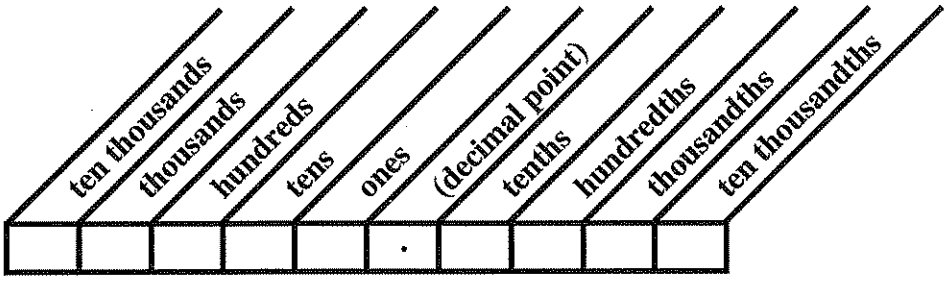
Step 1. Express the measurement in decimal form. Make any necessary conversions.

Step 2. Underline the digit to be rounded.

Step 3. If the digit to the right of the underlined digit is 5 or more, add 1 to the underlined digit. If the digit to the right of the underlined digit is less than 5, leave the underlined digit as is.

Step 4. Write the rounded measurement by dropping all digits to the right of the underlined digit. If necessary, insert zeros as placeholders.

Reminder: The chart below shows the place value of each digit in our base-ten numeration system.



Examples

3. Round 31.749 centimeters to the nearest tenth of a centimeter.

   Step 1. The measurement is given as a decimal number and conversion is not necessary.

   Step 2. Underline the digit in the tenths place.

   31.7̲49

   Step 3. If the digit to the right of the underlined digit is less than 5, leave the digit as is.

   4 is less than 5, so the underlined digit remains 7.

   Step 4. Write the rounded measurement by dropping all digits to the right of the underlined digit. Placeholders are not necessary.

   31.7 cm

4. Round 28,341 tons to the nearest ten thousand tons.

   Step 1. The measurement is given as a decimal number. (***Note:*** If a decimal point is not shown, it is assumed to be at the right of the last digit of the number.) No conversion is necessary.

   Step 2. Underline the digit in the ten thousands place.

   2̲8,341

   Step 3. If the digit to the right of the underlined digit is 5 or more, add 1 to the underlined digit.

   8 is greater than 5, so the underlined digit becomes 3.

   Step 4. Write the rounded measurement by dropping all digits to the right of the underlined digit. Insert zeros as placeholders.

   30,000 tons

5. Round 4 pounds, 6 ounces to the nearest tenth of a pound.

Step 1. Express the measurement in decimal form and make the necessary conversion. (***Caution:*** The decimal equivalent for 4 pounds, 6 ounces is NOT 4.6.)

$$6 \text{ oz.} = \frac{6 \text{ oz.}}{1} \times \frac{1 \text{ lb.}}{16 \text{ oz.}} = \frac{6 \text{ lb.}}{16} = \frac{3 \text{ lb.}}{8} = 0.375 \text{ lb.}$$

Thus, 4 pounds, 6 ounces = 4.375 pounds.

Step 2. Underline the digit in the tenths place.

4.$\underline{3}$75

Step 3. If the digit to the right of the underlined digit is less than 5, leave the digit as is.

7 is more than 5, so the underlined digit becomes 4.

Step 4. Write the rounded measurement by dropping all digits to the right of the underlined digit. Placeholders are not necessary.

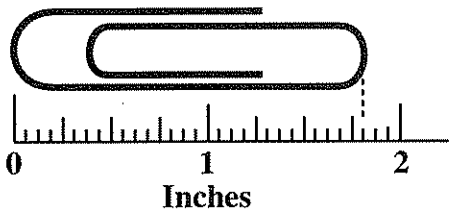
4.4 pounds

Rounding to a Specific Measurement

Some problems may require rounding to a specific measurement. If the exact measurement shown in the given diagram is less than one-half of the unit being used, then use the lower measurement. If the measurement is exactly one-half or more than one-half of the unit being used, then use the higher measurement.

Example

6. Round the measurement of the length of the pictured paper clip to the nearest $\frac{1}{4}$ inch.



The paper clip measures between $1\frac{3}{4}$ inches and 2 inches. Because it is less than halfway between these two readings, the measurement will be rounded to $1\frac{3}{4}$ inches.

SOLVE WORD PROBLEMS INVOLVING THE PYTHAGOREAN THEOREM

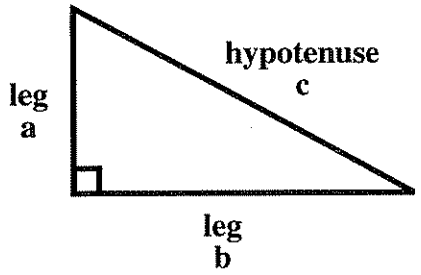
A triangle containing one right angle (90°) is called a **right triangle**. The two shorter sides (the perpendicular sides) are the **legs**, and the longest side (the side opposite the right angle) is the **hypotenuse**.

The Pythagorean theorem describes the relationship among the three sides of a right triangle and is very useful in solving geometry problems.

The **Pythagorean theorem** states

> For any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.

Given the right triangle below, the Pythagorean theorem is expressed as $a^2 + b^2 = c^2$.



The Pythagorean theorem is used to find the length of one side of a right triangle when the lengths of two sides are given.

To apply the Pythagorean theorem in solving real-world problems, use the following steps:

Step 1. Read the problem carefully.

Step 2. If a diagram is not provided, draw a right triangle and label the sides.

Step 3. Make sure the lengths are in the same unit of measure.

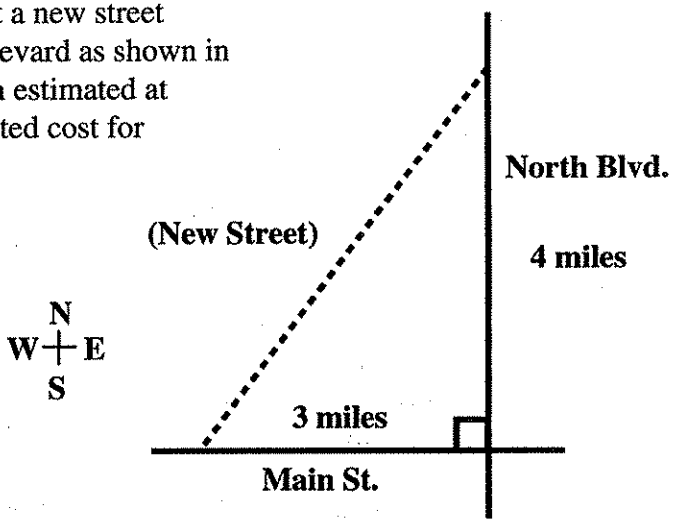
Step 4. Write the formula $a^2 + b^2 = c^2$, and substitute the information given in the diagram.

Step 5. Solve for the length of the unknown side.

Step 6. Reread the problem to identify the solution being sought, and perform any additional computation needed to find the solution.

Examples

1. The city commission wants to construct a new street connecting Main Street and North Boulevard as shown in the diagram. Construction cost has been estimated at \$100 per linear foot. What is the estimated cost for constructing the new street?



Step 1. Read the problem carefully.

Step 2. A diagram is provided. Use c to label the length of the new street.

Step 3. The lengths are in the same unit of measure.

Step 4. Write the formula $a^2 + b^2 = c^2$, and substitute the information given in the diagram.

$a^2 + b^2 = c^2$

$4^2 + 3^2 = c^2$

Step 5. Solve for c.

$c^2 = 16 + 9$ $\qquad$ $c^2 = 25$ $\qquad$ $c = 5$

Step 6. Reread the problem to identify the solution being sought, and perform any additional computation needed to find the solution.

What is the estimated cost for constructing the new street?

Because the construction cost is \$100 per linear foot, change 5 miles to feet.

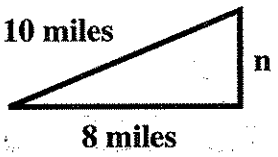
$$5 \text{ mi.} = \frac{5 \text{ mi.}}{1} \times \frac{5280 \text{ ft.}}{1 \text{ mi.}} = 26{,}400 \text{ ft.}$$

The estimated cost is \$100 x 26,400 = \$2,640,000.

2. Two soldiers have two-way radios with a maximum range of 10 miles. From the same starting point, one soldier walks 8 miles due west and the other soldier walks due north. Find the greatest distance the north-bound soldier can walk and still communicate with the west-bound soldier.

Step 1. Read the problem carefully.

Step 2. Draw a diagram and label the sides. Use n to label the maximum distance the north-bound soldier can walk.



Step 3. The lengths are in the same unit of measure.

Step 4. Write the formula $a^2 + b^2 = c^2$, and substitute the information given in the diagram.

$a^2 + b^2 = c^2$

$n^2 + 8^2 = 10^2$

Step 5. Solve for the unknown variable.

$n^2 + 64 = 100$ $\quad n^2 = 100 - 64$ $\quad n^2 = 36$ $\quad n = 6$

Step 6. Reread the problem to identify the solution being sought, and perform any additional computation needed to find the solution.

Find the greatest distance the north-bound soldier can walk and still communicate with the west-bound soldier.

There are no additional computations needed to identify the solution. The greatest distance the north-bound soldier can walk and still communicate with the west-bound soldier is 6 miles.

IDENTIFY APPROPRIATE UNITS OF MEASURE

The problems in this section include a diagram or description of a geometric object. You will be asked to identify the type of measures associated with length, area, and volume.

Length is a one-dimensional measure expressed in linear units. These units include inches, feet, yards, miles, centimeters, meters, kilometers, etc. Examples of one-dimensional measures are as follows:

- the perimeter of a rectangle
- the distance around a circular tabletop
- the amount of molding needed to frame a picture
- the distance from Gainesville, FL, to Tallahassee, FL
- the thickness of a wire
- the depth of water in a well
- the width of a desk drawer

Area is a two-dimensional measure of an enclosed surface expressed in square units. These units include square inches, square feet, square centimeters, square meters, etc. Examples of area measures are as follows:

- the amount of floor space occupied by a filing cabinet
- the amount of tissue paper needed to wrap a package
- the amount of lawn to be fertilized
- the amount of living space in a house
- the amount of wall space to be painted

Volume is a three-dimensional measure of space or capacity expressed in cubic units. These units include cubic inches, cubic feet, cubic centimeters, cubic meters, etc. Quarts, gallons, fluid ounces, liters, and milliliters are also examples of cubic measure, even though the word *cubic* is not attached to these names. Examples of volume measures are as follows:

- the amount of water in a glass
- the amount of grain in a silo
- the amount of flour needed in a bread recipe
- the capacity of an underground fuel storage tank
- the amount of milk in a carton
- the space inside a desk drawer

To identify appropriate units of measure, use the following steps:

Step 1. Read the problem carefully.

Step 2. Determine the type of measure that should be used: length, area, or volume.

Step 3. Select the unit that corresponds to this type of measure.

Examples

1. Which measurement unit could be used to give the amount of wall surface to be covered by the contents of a can of paint?

   A. liters
   B. gallons
   C. square feet
   D. cubic feet

   Step 1. Read the problem carefully.

   Step 2. Determine the type of measure that should be used: length, area, or volume.

   The amount of wall surface is a measure of area.

   Step 3. Select the unit that corresponds to this type of measure.

   Response ***C***, square feet, is correct because it is the only choice that involves area measure.

   If the question had asked for the measurement unit for the amount of paint needed, then the units in responses ***A***, ***B***, or ***D*** could be used because these units measure volume.

2. Which measurement unit could be used to report the distance around a circular racetrack?

   A. kilometers
   B. square meters
   C. cubic feet
   D. liters

Step 1. Read the problem carefully.

Step 2. Determine the type of measure that should be used: length, area, or volume.

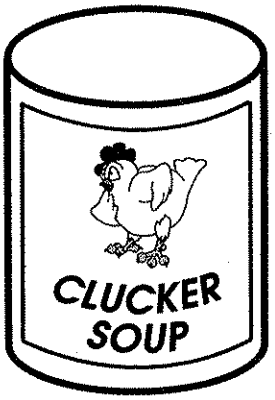
The distance around the track is a linear measure.

Step 3. Select the unit that corresponds to this type of measure.

Here, response ***A***, kilometers, is the correct choice because it is the only linear measure. Response ***B*** is a square measure of area. Responses ***C*** and ***D*** both measure volume.

3. What measurement unit would be used to record the amount of paper needed for the label on the chicken soup can shown?

A. inches
B. square inches
C. cubic inches
D. fluid ounces



Step 1. Read the problem carefully.

Step 2. Determine the type of measure that should be used: length, area, or volume.

To record the amount of paper needed will require the use of a measure of area.

Step 3. Select the unit that corresponds to this type of measure.

Response ***B***, square inches, is correct because it is the only measurement of area. Response ***A***, inches, is a linear measure, and responses ***C*** and ***D***, cubic inches and fluid ounces, are volume measures.

4. Which measurement unit would NOT be used to report the amount of water needed to fill a swimming pool?

A. cubic feet
B. liters
C. gallons
D. meters

Step 1. Read the problem carefully. Notice that the question includes "NOT."

Step 2. Determine the type of measure that should be used: length, area, or volume.

The amount of water needed to fill a swimming pool would be reported using a measure of volume.

Step 3. Select the unit that corresponds to this type of measure.

Response ***D***, meters, is correct because it is NOT a measure of volume; it is a measure of length. Responses ***A***, ***B***, and ***C*** are all measures of volume.

CALCULATE DISTANCES, AREAS, AND VOLUMES

Calculate Distances

Perimeter is the measurement of the distance around the outside of an object. This section provides instruction on calculating the distance around polygons and circles.

Problems that involve finding a perimeter may include mixed units of measurement. In such problems, finding the perimeter will require metric-to-metric or English-to-English conversion of linear measurement units.

The meter is the fundamental unit of length in the metric system. Multiples of the meter are given in powers of 10 as shown below.

Kilometer	Hectometer	Dekameter	Meter	Decimeter	Centimeter	Millimeter
(km)	(hm)	(dam)	(m)	(dm)	(cm)	(mm)
1000 m	100 m	10 m	1 m	.1 m	.01 m	.001 m

The following memory device can be used as an aid to recall the order of the chart above:
King **H**enry **D**ied **M**onday **D**rinking **C**hocolate **M**ilk.

Examples

1. Convert 5.2 kilometers to meters.

   According to the chart, 1 km = 1000 m.

   Therefore, $\frac{5.2 \cancel{\text{km}}}{1} \times \frac{1000 \text{ m}}{1 \cancel{\text{km}}} = 5200 \text{ m}.$

2. Convert 80 centimeters to hectometers.

   According to the chart, 1 cm = 0.01 m and 1 hm = 100 m.

   Therefore, $\frac{80 \cancel{\text{cm}}}{1} \times \frac{0.01 \cancel{\text{m}}}{1 \cancel{\text{cm}}} \times \frac{1 \text{ hm}}{100 \cancel{\text{m}}} = \frac{0.8 \text{ hm}}{100} = 0.008 \text{ hm}.$

Because the metric system is based on the powers of 10, changing from one unit to another may simply involve the movement of a decimal point. The examples that follow illustrate this process.

Examples

3. Change 5.2 kilometers to meters.

According to the chart on the previous page, converting from kilometers to meters requires moving the decimal point three places to the right.

Thus, 5.2 km = 5 2 0 0 m = 5200 m.

4. Change 80 centimeters to hectometers.

According to the chart on the previous page, converting from centimeters to hectometers requires moving the decimal point four places to the left.

Thus, 80 cm = 0 0 0 8 0 cm = 0.008 cm.

English-to-English conversions involve the use of these conversion factors.

12 inches (in.) = 1 foot (ft.)
3 feet (ft.) = 1 yard (yd.)
36 inches (in.) = 1 yard (yd.)
5280 feet (ft.) = 1 mile (mi.)

Examples

5. Change 15 inches to yards.

$$\frac{15 \text{ in.}}{1} \times \frac{1 \text{ yd.}}{36 \text{ in.}} = \frac{15 \text{ yd.}}{36} = \frac{5 \text{ yd.}}{12} = \frac{5}{12} \text{ yd.}$$

6. Change 2.1 miles to feet.

$$2.1 \text{ mi.} = \frac{2.1 \text{ mi.}}{1} \times \frac{5280 \text{ ft.}}{1 \text{ mi.}} = \frac{11{,}088 \text{ ft.}}{1} = 11{,}088 \text{ ft.}$$

A **polygon** is a closed figure composed of line segments. In a *regular polygon*, the lengths of all sides are equal. Some common polygons are named below.

sides	name	sides	name
3	triangle	7	heptagon
4	quadrilateral	8	octagon
5	pentagon	9	nonagon
6	hexagon	10	decagon

To find the perimeter (distance around) of a polygon, follow the steps below.

Step 1. Change all lengths to the same unit of measure.

Step 2. If necessary, draw the figure and label each side.

Step 3. Add the lengths of all sides.

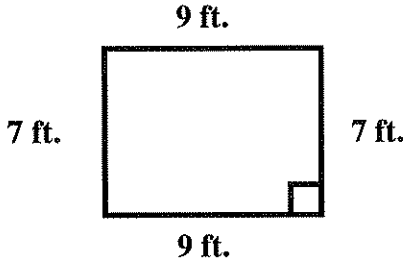
Examples

7. Find the perimeter of a rectangle that has a length of 3 yards and a width of 7 feet.

Step 1. Change all lengths to the same unit of measure. In this example, all measurements are changed to feet.

$$3 \text{ yd.} = \frac{3 \cancel{\text{yd.}}}{1} \times \frac{3 \text{ ft.}}{1 \cancel{\text{yd.}}} = 9 \text{ ft.}$$

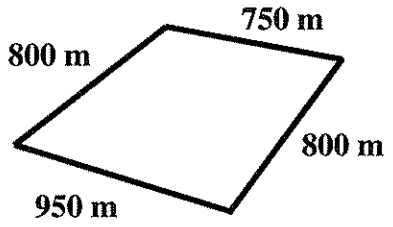
Step 2. Draw the figure and label each side.



Step 3. Add the lengths of all the sides.

7 ft. + 9 ft. + 7 ft. + 9 ft. = 32 ft.

8. What is the distance around this polygon, in kilometers?

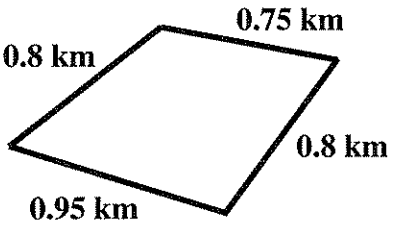


Step 1. Change all lengths to kilometers.

To convert meters to kilometers, move the decimal point three places to the left.

750 m = 0.75 km 800 m = 0.8 km 950 m = 0.95 km

Step 2. Relabel the figure with the converted measurements.



Step 3. Add the lengths of all the sides.

0.75 km + 0.8 km + 0.95 km + 0.8 km = 3.3 km

9. Find the distance around a regular hexagon with one side having a length of 8 meters.

Step 1. All measurements are expressed in meters.

Step 2. This regular hexagon refers to a geometric figure having six sides of equal length, 8 meters.

Step 3. Add the lengths of all the sides.

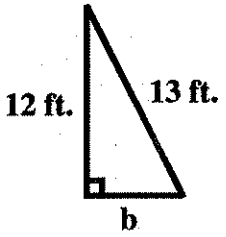
8 m + 8 m + 8 m + 8 m + 8 m + 8 m = 48 m

10. Find the distance around a right triangle whose hypotenuse is 13 feet and one leg is 12 feet.

Step 1. All measurements are expressed in feet.

Step 2. Since the length of one side of the right triangle described is not given, use the Pythagorean theorem to find it.

$12^2 + b^2 = 13^2$
$144 + b^2 = 169$
$b^2 = 25$
$b = 5$

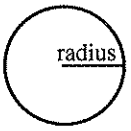


So, the length of side b is 5 ft.

Step 3. Add the lengths of the sides.

13 ft. + 12 ft. + 5 ft. = 30 ft.

The distance around a circle is called **circumference**. The following terms are used to describe circles.



The **radius**, r, of a circle is the length of a line segment from the center of the circle to a point on the circle.

The **diameter**, d, is the length of a line segment through the center of the circle with endpoints on the circle.

Reminder: The radius of a circle equals half the diameter.

Follow these steps to find the circumference of (distance around) a circular region.

Step 1. Find the radius.

Step 2. Substitute the value of the radius for r in the formula: $C = 2\pi r$.

Step 3. Multiply to find the circumference.

Examples

11. What is the distance around a circular walk that has a diameter of 8 feet?

Step 1. Find the radius.

The diameter is 8 feet; thus, the radius is 4 feet.

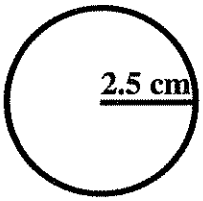
Step 2. Substitute the value of the radius for r in the formula: $C = 2\pi r$.

$C = 2\pi(4 \text{ ft.})$

Step 3. Multiply to find the circumference.

$C = 2\pi(4 \text{ ft.}) = 8\pi \text{ ft.}$

12. What is the circumference of the circle below?



Step 1. Find the radius. The radius is shown to be 2.5 cm.

Step 2. Substitute the value of the radius for r in the formula: $C = 2\pi r$.

$C = 2\pi \text{ (2.5 cm)}$

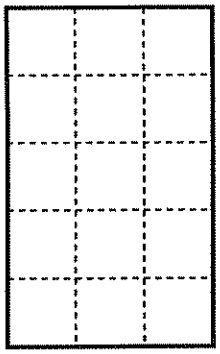
Step 3. Multiply to find the circumference.

$C = 2\pi \text{ (2.5 cm)} = 5\pi \text{ cm}$

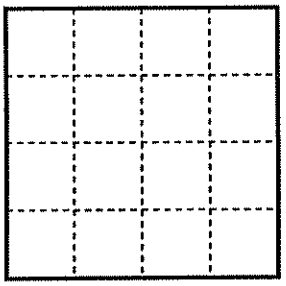
Calculate Areas

Area is a measurement of the interior surface of a closed two-dimensional figure. For example, the measurement of area can describe how much land is included in a rectangular plot or how much fabric is needed to cover a round table. The basic unit of area (one square unit) is a square whose sides are each one unit in length. Commonly used units of area are the square centimeter, square inch, square foot, square mile, square meter, etc. Hence, the area of a two-dimensional geometric figure is the number of square units it contains.

What is the area of a rectangle that is 3 cm by 5 cm?



The inside of a 3 cm by 5 cm rectangle can be divided into 15 squares that are one centimeter long on each side. The area is 15 cm^2.



What is the area of a square that is 4 cm by 4 cm?

A 4 cm by 4 cm square can be divided into 16 squares that are one centimeter long on each side. The area is 16 cm^2.

A formula for calculating the area of a rectangle is as follows: *A*rea equals *L*ength times *W*idth ($A = LW$) or *A*rea equals *b*ase times *h*eight ($A = bh$).

height (**h**)
width (**W**)

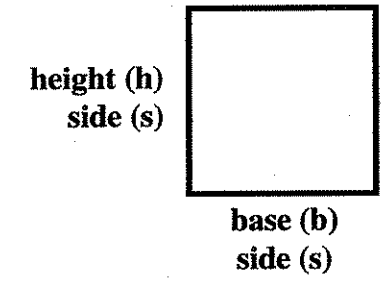
base (**b**)
length (**L**)

$A = bh$ **Area of a rectangle**

or

$A = LW$ **Area of a rectangle**

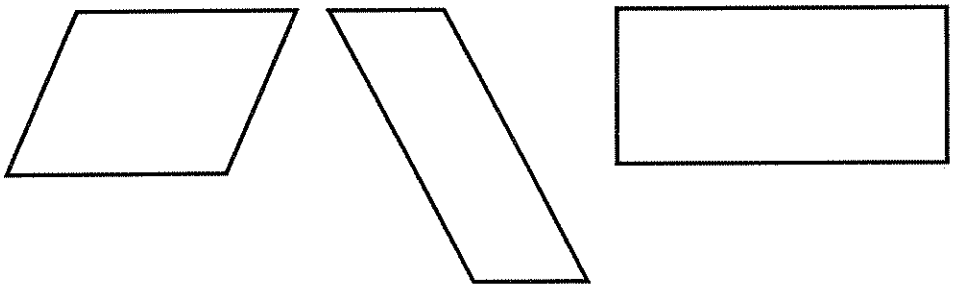
A **square** is a rectangle with four equal sides. Therefore, the base and height are the same length. A formula for calculating the area of a square with sides of length s is the same as the formula for calculating the area of a rectangle as illustrated below.



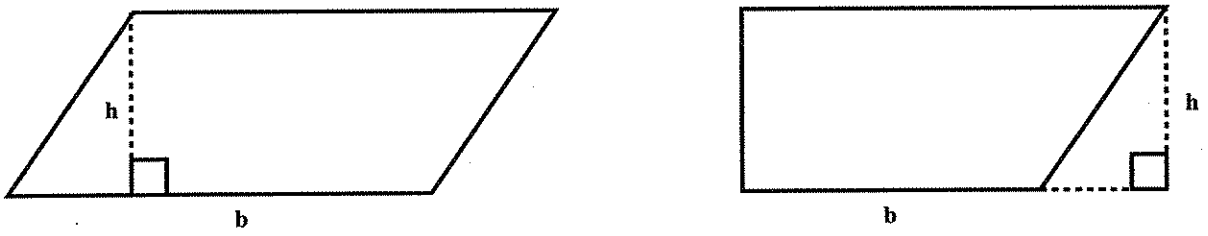
***Area* = base times height** $A = bh$
***Area* = side times side** $A = ss$
***Area* = side squared** $A = s^2$

$A = bh$ or $A = ss$ or $A = s^2$ Area of a square

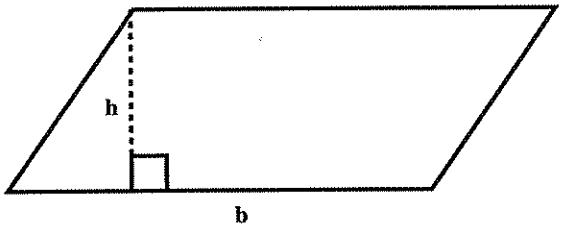
A **parallelogram** is a four-sided polygon with opposite sides that are parallel. Opposite sides of parallelograms have the same length. The following are examples of parallelograms:



Knowing how to find the area of a rectangle will be helpful in finding the area of a parallelogram. For any parallelogram, a rectangle of equal area can be constructed by cutting a right triangle from one end of the parallelogram and attaching it to the other end as follows:



Because the base and height are the same for both figures, the formula for calculating the area of the parallelogram of base (b) and height (h) is shown below.



$A = bh$ Area of a parallelogram

Reminder: Note that height (h) is measured *perpendicular* to the base. It is not necessarily a side of the parallelogram.

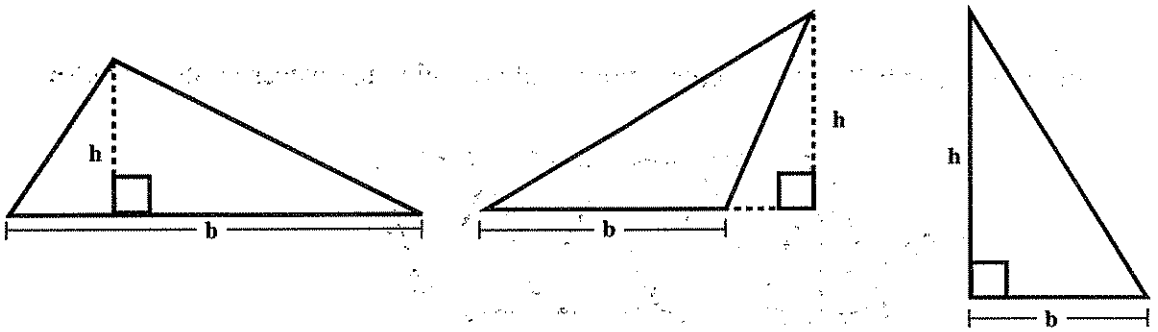
A **triangle** is a three-sided polygon.

A. A vertex of a triangle is the point where two sides of a triangle intersect. A triangle has three vertices.

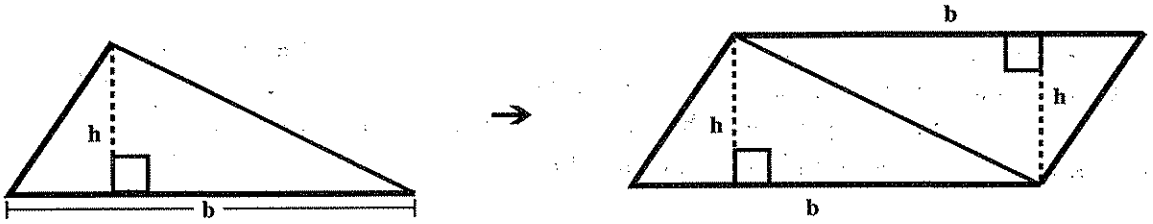
B. Any side of the triangle can be considered the base.

C. The height of a triangle is the shortest distance from the base to the opposite vertex. The shortest distance is the perpendicular distance.

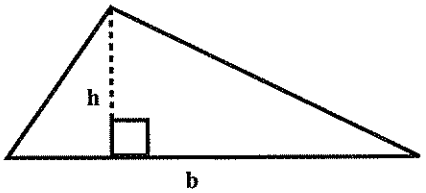
The following are examples of triangles, with b = base and h = height:



To develop a formula for finding the area of a triangle, place two identical triangles together as shown in the diagram below.

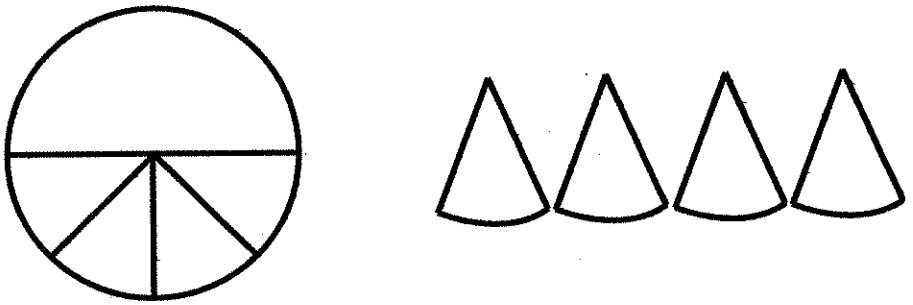


The resulting parallelogram has an area of bh. The original triangle has half the area of the parallelogram. The formula for the area of any triangle with base b and height h is shown below.

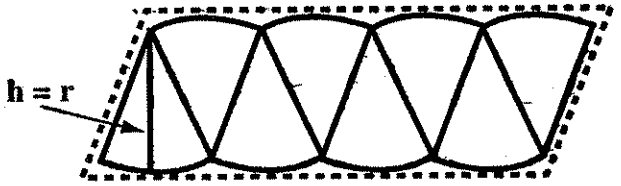


$$A = \frac{1}{2}bh$$ **Area of a triangle**

Areas can be found for two-dimensional figures that are not polygons. The formula for the area of a circle can be found by using the formula for the area of a parallelogram. Imagine cutting half of a circular region into small pieces and arranging them as shown below.

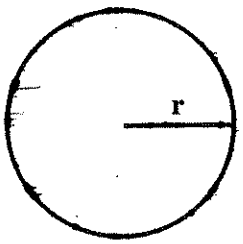


Imagine cutting the other half of the circular region and arranging the pieces as shown below.



$$b = \frac{1}{2}(2\pi r) = \pi r$$

The resulting figure is approximately a parallelogram whose base has length πr (half the circumference of the circle) and whose height is r (the radius of the circle). The area of the circle is the same as the area of a parallelogram with height, r, and base, πr, that is, $(\pi r)r$, or πr^2. The formula for the area of a circle of radius r is given below.



$A = \pi r^2$ **Area of a circle**

To find the area of a geometric figure, follow these steps.

Step 1. Write the appropriate formula for the figure.

rectangle	$A = bh$ or $A = LW$
square	$A = bh$ or $A = s^2$
parallelogram	$A = bh$
triangle	$A = \frac{1}{2}bh$
circle	$A = \pi r^2$

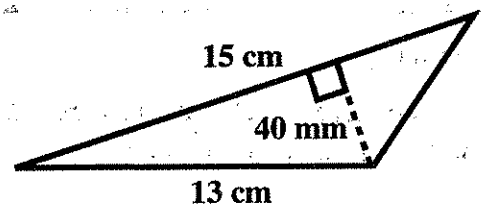
Step 2. Identify the lengths required for the formula. If necessary, convert those lengths to the same unit of measure.

Step 3. Substitute the lengths for the variables in the formula.

Step 4. Perform the computation. The result will be in square units.

Examples

13. What is the area in square centimeters of the triangle pictured below?



Step 1. Write the appropriate formula for the triangle.

$$A = \frac{1}{2}bh$$

Step 2. Identify the lengths required for the formula. $b = 15$ cm and $h = 40$ mm. Convert these lengths to the same unit of measure (centimeters).

$h = 40$ mm $= 4$ cm

Note that 13 cm is not required for the formula.

Step 3. Substitute the lengths for the variables in the formula.

$$A = \frac{1}{2}(15 \text{ cm})(4 \text{ cm})$$

Step 4. Perform the computation.

$$A = \frac{1}{\cancel{2}_1} \times \frac{15 \text{ cm}}{1} \times \frac{\cancel{24}^{2} \text{ cm}}{1} = 30 \text{ cm}^2$$

14. What is the area of a circular region whose diameter is 3 yards?

Step 1. Write the appropriate formula for a circle.

$A = \pi r^2$

Step 2. Identify the lengths required for the formula.

Since the diameter equals 3 yards, the radius (r) equals half of 3 yards, or 1.5 yards.

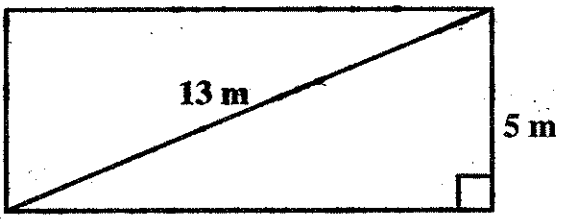
Step 3. Substitute the lengths for the variables in the formula.

$A = \pi(1.5 \text{ yd.})^2$

Step 4. **Perform the computation.**

$A = 2.25(\pi) \text{ yd.}^2$

15. Find the area of the rectangle pictured below.



Step 1. Write the appropriate formula for the rectangle.

$A = bh$

Step 2. Identify the lengths required for the formula.

The height is given as 5 m, but the base is not given. Label the base b. The length of the base can be found by using the Pythagorean theorem.

$$\begin{aligned} (a)^2 + (b)^2 &= (c)^2 \\ 5^2 + b^2 &= 13^2 \\ 25 + b^2 &= 169 \\ b^2 &= 169 - 25 \\ b^2 &= 144 \\ b &= 12 \end{aligned}$$

So, the base equals 12 meters.

Step 3. Substitute in formula.

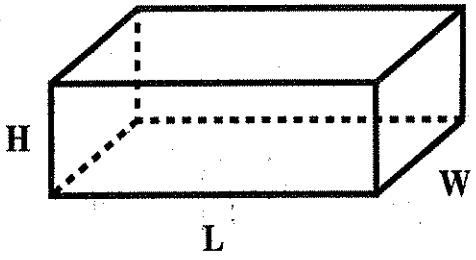
$A = (12 \text{ m})(5 \text{ m})$

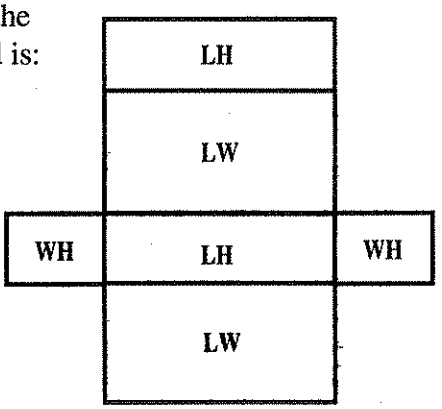
Step 4. Perform the computation to get the solution in terms of square units.

$A = 60 \text{ m}^2$

The *surface area (*SA*) of a rectangular solid* (e.g., a cardboard shoe box) can be found by using the measurements for its length (*L* units), width (*W* units), and height (*H* units).

The surface area of a rectangular solid is the sum of the area of its six faces, each of which is a rectangle. Each of the top and bottom faces has the area *LW*, each of the front and back faces has the area *LH*, and each of the faces on the left and right side has the area *HW*. The formula for the surface area of a rectangular solid is: $SA = 2(LW) + 2(LH) + 2(HW)$.





To find the surface area of a rectangular solid, follow the steps below.

Step 1. Write the formula: $SA = 2(LW) + 2(LH) + 2(HW)$.

Step 2. **Identify the lengths required for the formula. If necessary, convert those lengths to the same unit of measure.**

Step 3. **Substitute the length, width, and height of the rectangular solid into the formula.**

Step 4. Perform the computations. (The result will be in square units.)

Example

16. What is the surface area of a rectangular solid that is 5 feet long, 1 yard wide, and 4 feet high?

Step 1. Write the formula: $SA = 2(LW) + 2(LH) + 2(HW)$.

Step 2. If necessary, convert the measurements to the same unit.

$L = 5$ ft. $W = 1$ yd. $= 3$ ft. $H = 4$ ft.

Step 3. **Substitute the length, width, and height of the rectangular solid into the formula.**

SA = 2(5 ft.)(3 ft.) + 2(5 ft.)(4 ft.) + 2(4 ft.)(3 ft.)

Step 4. Perform the computations. The result will be in square units.

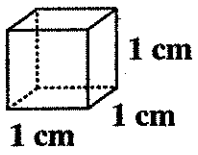
SA = 30 ft.^{2} + 40 ft.^{2} + 24 ft.^{2} SA = 94 ft.^{2}

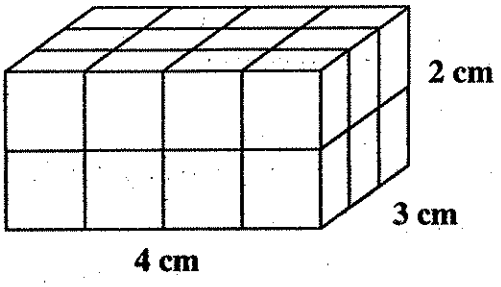
The surface area is 94 square feet.

Calculate Volumes

Volume is the measure of the amount of space in a three-dimensional object. For example, a swimming pool can hold a certain amount of water, or a box is able to hold a certain amount of sand. The volume of a three-dimensional region is measured in terms of a basic unit of volume, just as area is measured in terms of a unit of area.

The fundamental unit of volume is a **cube** that measures one unit in length on each side. Any unit of length can be used as the basis for measuring volume. Volume can be measured in cubic centimeters, cubic meters, cubic inches, cubic feet, cubic meters, and so on. Thus, volume of a solid object is the number of cubic units contained in it. The following are examples of commonly used cubic units:

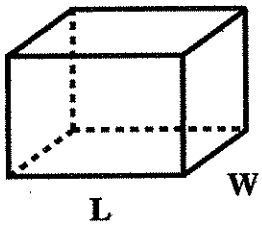




What is the volume of a rectangular solid measuring 4 cm by 3 cm by 2 cm?

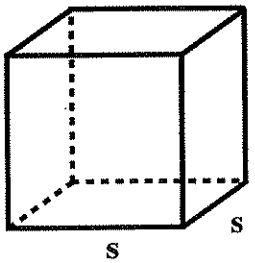
The inside of a rectangular solid measuring 4 cm by 3 cm by 2 cm can be divided into 24 cubes, each one centimeter on a side. The volume is 24 cm^3. Note that the volume can be found by multiplying the length times the width times the height.

Volume questions will require the use of the following formulas:



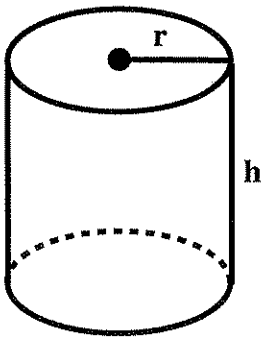
Volume of a rectangular solid (prism)

$V = LWH$



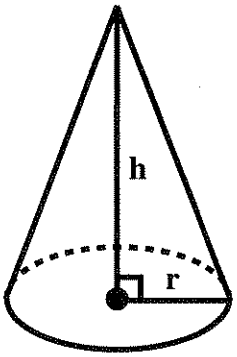
Volume of a cube

$V = LWH = s \times s \times s = s^3$



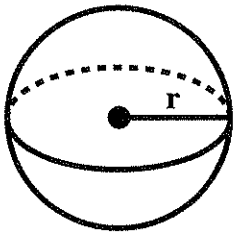
Volume of a right circular cylinder

$V = \pi r^2 h$



Volume of a right circular cone

$V = \frac{1}{3}\pi r^2 h$



Volume of a sphere

$V = \frac{4}{3}\pi r^3$

To calculate the volume of a solid figure, follow these steps.

Step 1. Write the appropriate formula for the volume of the figure.

Step 2. Identify the lengths required for the formula. If necessary, convert those lengths to the same unit of measure.

Step 3. Substitute the lengths for the variables.

Step 4. Perform the computation. (The result will be in cubic units.)

Examples

17. Find the volume of a right circular cone having a height of 5 meters and a radius of 30 centimeters.

Step 1. Write the appropriate formula.

$$V = \frac{1}{3}\pi r^2 h$$

Step 2. Identify the lengths required for the formula. Convert these lengths to the same unit of measure. In this example, the lengths are converted to centimeters.

$h = 5 \text{ m} = 500 \text{ cm}$ $\quad r = 30 \text{ cm}$

Step 3. Substitute the lengths for the variables in the formula.

$$V = \frac{1}{3}\pi(30 \text{ cm})^2 (500 \text{ cm})$$

Step 4. Perform the computation to get the solution in terms of cubic units.

$$V = \frac{1}{\cancel{3}_1}\pi(\overset{300}{\cancel{900}} \text{ cm}^2)(500 \text{ cm}) = \pi(300 \text{ cm}^2)(500 \text{ cm}) = 150{,}000\pi \text{ cm}^3$$

18. What is the volume of a rectangular solid having a length of 3 feet, width of 4 feet, and height of 9 inches?

Step 1. Write the appropriate formula.

$V = LWH$

Step 2. Identify the lengths required for the formula. Convert these lengths to the same unit of measure. In this example, all the measurements are changed to feet.

$$L = 3 \text{ ft.} \quad W = 4 \text{ ft.} \quad H = 9 \text{ in.} = \frac{9 \text{ in.}}{1} \times \frac{1 \text{ ft.}}{12 \text{ in.}} = \frac{3}{4} \text{ ft.}$$

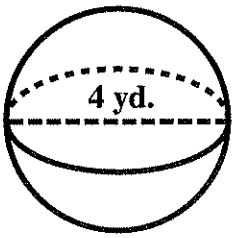
Step 3. Substitute the lengths for the variables in the formula.

$$V = (3 \text{ ft.})\,(4 \text{ ft.})\left(\frac{3}{4} \text{ ft.}\right) =$$

Step 4. Perform the computation to get the solution in terms of cubic units.

$$V = (3 \text{ ft.})\,(4 \text{ ft.})\left(\frac{3}{4} \text{ ft.}\right) = 12 \text{ ft.}^2 \left(\frac{3}{4} \text{ ft.}\right) = \frac{\overset{3}{\cancel{12}}}{1} \text{ft.}^2 \times \frac{3}{\cancel{4}} \text{ ft.} = \frac{\overset{9}{\cancel{36}}}{\cancel{4}} \text{ ft.}^3 = 9 \text{ ft.}^3$$

19. Find the volume of the sphere shown below.



Step 1. Write the appropriate formula.

$$V = \frac{4}{3}\pi r^3$$

Step 2. Identify the lengths required for the formula. If necessary, convert those lengths to the same unit of measure.

The diameter of the sphere is 4 yards. The radius of the sphere is half the diameter, 2 yards.

Step 3. Substitute the lengths for the variables in the formula.

$$V = \frac{4}{3}\pi(2 \text{ yd.})^3$$

Step 4. Perform the computation to get the solution in terms of cubic units.

$$V = \frac{4}{3}\pi(2 \text{ yd.})^3 = \frac{4}{3}\pi(8 \text{ yd.}^3) = \left(\frac{4}{3} \times \frac{8}{1}\right)\pi \text{ yd.}^3 = \frac{32}{3}\pi \text{ yd.}^3$$

Metric-to-metric conversions may be necessary to find the volumes of some geometric figures.

In the metric system, liters are used to measure the volume of liquids. The following chart is helpful in making conversions when the volume is given in terms of liters:

Kiloliter (kL)	Hectoliter (hL)	Dekaliter (daL)	Liter (L)	Deciliter (dL)	Centiliter (cL)	Milliliter (mL)
1000 L	100 L	10 L	1 L	.1 L	.01 L	.001 L

Cubic meters, cubic decimeters, and cubic centimeters are commonly used metric units when measuring the volume of solid or gaseous materials.

The liquid contained in one liter fits exactly into a cube that is 10 centimeters on a side.

Therefore, $1\text{ L} = 1000\text{ cm}^3$

$1\text{ L} = 1\text{ dm}^3 \quad [(1\text{ dm})^3 = (10\text{ cm})^3 = 1000\text{ cm}^3]$

$1000\text{ L} = 1\text{ m}^3 \quad [(1\text{ m})^3 = (10\text{ dm})^3 = 1000\text{ dm}^3]$

Examples

20. What is the volume in liters of a 950-milliliter bottle?

    Use the chart to change milliliters to liters.

    $950\text{ mL} = 0.950\text{ L} = 0.95\text{ L}$

21. What is the volume in cubic centimeters of a 23.8-liter container?

    Because this conversion involves two different units of measure, the chart cannot be used. Therefore, refer to the conversion factors above.

    $$\frac{23.8\text{ L}}{1} \times \frac{1000\text{ cm}^3}{1\text{ L}} = 23{,}800\text{ cm}^3$$

22. What is the volume in cubic meters of a 455-liter container?

    Use the conversion factor to change liters to cubic meters.

    $$\frac{455\text{ L}}{1} \times \frac{1\text{ m}^3}{1000\text{ L}} = \frac{455}{1000}\text{m}^3 = 0.455\text{ m}^3$$

23. If a box is 8 centimeters long, 6 centimeters wide, and 5 centimeters high, how many milliliters will it hold?

    Step 1. Write the appropriate formula.

    $V = LWH$

    Step 2. Identify the lengths for the formula. In this example, no conversion is necessary.

    $L = 8\text{ cm} \qquad W = 6\text{ cm} \qquad H = 5\text{ cm}$

    Step 3. Substitute the lengths for the variables in the formula.

    $V = (8\text{ cm})(6\text{ cm})(5\text{ cm})$

    Step 4. Perform the computation to get the solution in terms of cubic units.

    $V = 8 \times 6 \times 5 = 240\text{ cm}^3$

    Because, $1\text{ cm}^3 = 1\text{ mL}$, $240\text{ cm}^3 = 240\text{ mL}$.

SOLVE WORD PROBLEMS INVOLVING GEOMETRIC FIGURES

Some word problems require the use of perimeter, area, and volume formulas (see the unit on Skill IB2 on pages 89–106). English-to-English or metric-to-metric conversions may be necessary to solve these word problems (see conversions on page 78 and page 89).

To solve real-world problems involving perimeters, areas, and volumes, use the following the steps:

Step 1. Read the problem carefully. Draw a diagram of the geometric figure described. Label the geometric figure with the measurements given in the problem. Make sure the units are equivalent.

Step 2. Determine whether the problem involves perimeter, area, or volume. Write the appropriate formula.

Step 3. Substitute the values given into the formula and compute.

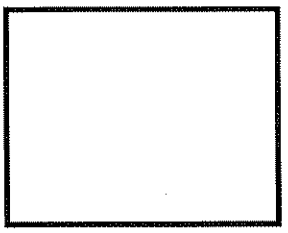
Step 4. Reread the problem to identify the solution being sought. Perform any additional computation needed to find the solution.

Step 5. Check the answer to be sure it is reasonable.

Examples

1. What will be the cost of carpeting a rectangular office that measures 12 feet by 15 feet if the carpet costs $12.50 per square yard?

   Step 1. Read the problem carefully. Draw a diagram of the rectangle described. Label the diagram with the measurements given in the problem.



Step 2. Determine whether the problem involves perimeter, area, or volume. Write the appropriate formula.

The problem involves the area of the office floor, which is in the shape of a rectangle. The area formula for a rectangle is $A = bh$.

Step 3. Substitute the values given into the formula and compute.

$A = bh = (15 \text{ ft.})(12 \text{ ft.}) = 180 \text{ ft.}^2$

Step 4. Reread the problem to identify the solution being sought.

What will be the cost of carpeting a rectangular office that measures 12 feet by 15 feet if the carpet costs $12.50 per square yard?

Perform the additional computation needed to find the solution.

The price of carpet is given in terms of square yards, so 180 ft.^{2} must be converted to square yards. First, a conversion factor must be determined.

$$\begin{aligned} 1 \text{ yd.} &= 3 \text{ ft.} \\ (1 \text{ yd.})(1 \text{ yd.}) &= (3 \text{ ft.})(3 \text{ ft.}) \\ 1 \text{ yd.}^2 &= 9 \text{ ft.}^2 \end{aligned}$$

Hence, $\dfrac{\overset{20}{\cancel{180}} \text{ ft.}^2}{1} \times \dfrac{1 \text{ yd.}^2}{\underset{1}{\cancel{9}} \text{ ft.}^2} = \dfrac{20}{1} \text{yd.}^2 = 20 \text{ yd.}^2.$

The carpet costs $12.50 per square yard; thus, the cost of carpeting the office described is $12.50 x 20 = $250.00.

Step 5. Check the answer to be sure it is reasonable.

Based on the information given, this is a reasonable amount to pay to carpet the office.

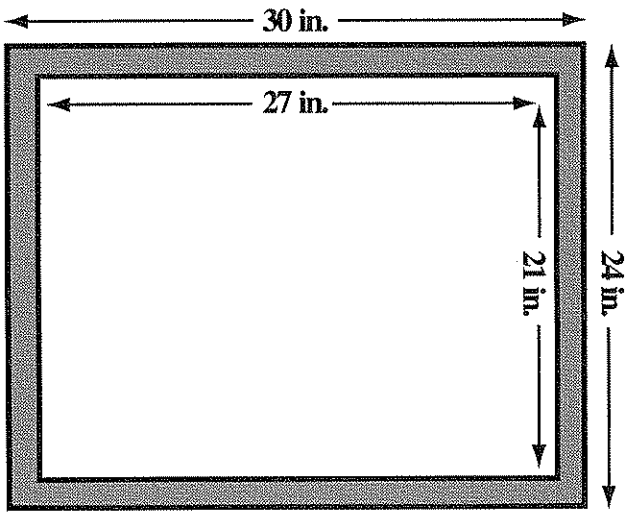
2. The outside dimensions of a picture frame are 2 feet by 30 inches. If the inside dimensions are $1\frac{3}{4}$ feet by 27 inches, what is the area of the frame?

Step 1. Read the problem carefully. Draw a diagram of the geometric figure described. Label the diagram with the measurements given in the problem.

Convert all measurements to the same unit. In this example, all measurements are changed to inches.

$$2 \text{ ft.} = \frac{2 \text{ ft.}}{1} \times \frac{12 \text{ in.}}{1 \text{ ft.}} = 24 \text{ in.}$$

$$1\frac{3}{4} \text{ ft.} = \frac{7}{4} \text{ ft.} = \frac{7 \text{ ft.}}{1\cancel{4}} \times \frac{\overset{3}{\cancel{12}} \text{ in.}}{1 \text{ ft.}} = 21 \text{ in.}$$



Reminder: The shaded area is the area requested.

Step 2. Determine whether the problem involves perimeter, area, or volume. Write the appropriate formula.

The problem asks for the area of the frame. This area, A_3, can be calculated by subtracting the area A_1 of the inner rectangle (21 in. by 27 in.) from the area A_2 of the outer rectangle (24 in. by 30 in.).

$$A_3 = A_2 - A_1 = b_2 h_2 - b_1 h_1$$

Step 3. Substitute the values given into the formula and compute.

$$\begin{aligned} A_3 &= (24 \text{ in.})(30 \text{ in.}) - (21 \text{ in.})(27 \text{ in.}) \\ &= 720 \text{ in.}^2 - 567 \text{ in.}^2 \\ &= 153 \text{ in.}^2 \end{aligned}$$

Step 4. Reread the problem to identify the solution being sought.

Rereading the questions reveals that the solution is *the area of the frame.*

Perform any additional computation needed to find the solution.

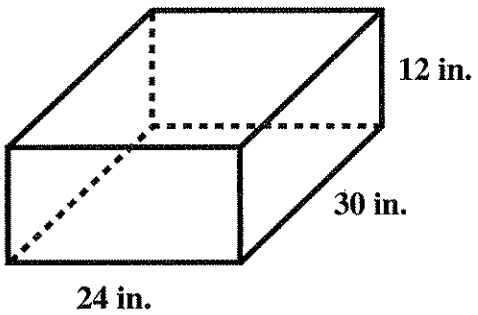
No further calculations are necessary.

Step 5. Check the answer to be sure it is reasonable.

The solution is reasonable, is smaller than the area of the largest rectangle, and is expressed in square units.

3. A rectangular aquarium measures 12 inches by 24 inches by 30 inches. The owner of Pat's Pet Paradise recommends that, in order to avoid overcrowding, there should be no more than 4 angelfish per cubic foot of the aquarium space. What is the maximum number of angelfish that this aquarium could hold without overcrowding?

Step 1. Read the problem carefully. Draw a diagram of the rectangle described. Label the geometric figure with the measurements given in the problem. Make sure the units are equivalent.



Step 2. Determine whether the problem involves perimeter, area, or volume. Write the appropriate formula.

The problem involves the volume of the fish tank that is in the shape of a rectangular solid whose volume is given by the formula $V = LWH$.

Step 3. Substitute the values given into the formula and compute.

$V = (12 \text{ in.})(24 \text{ in.})(30 \text{ in.}) = 8640 \text{ in.}^3$

Step 4. Reread the problem to identify the solution being sought.

What is the maximum number of angelfish that this aquarium could hold without overcrowding?

Perform any additional computation needed to find the solution.

To avoid overcrowding, there should be no more than 4 angelfish per cubic foot of aquarium. The aquarium has a capacity of 8640 $in.^3$, which now needs to be converted to cubic feet. A conversion factor must be found.

$$\begin{aligned} 1 \text{ ft.} &= 12 \text{ in.} \\ (1 \text{ ft.})(1 \text{ ft.})(1 \text{ ft.}) &= (12 \text{ in.})(12 \text{ in.})(12 \text{ in.}) \\ 1 \text{ ft.}^3 &= 1728 \text{ in.}^3 \end{aligned}$$

Now the conversion can be done.

$$\frac{8640 \text{ in.}^3}{1} \times \frac{1 \text{ ft.}^3}{1728 \text{ in.}^3} = \frac{8640}{1728} \text{ ft.}^3 = 5 \text{ ft.}^3$$

There can be 4 angelfish per cubic foot and the aquarium has a 5 cubic foot capacity; hence, the aquarium can sustain $4 \times 5 = 20$ angelfish.

Step 5. Check the answer to be sure it is reasonable.

This is a reasonable answer.

IDENTIFY FORMULAS FOR MEASURING GEOMETRIC FIGURES

In this section, each problem asks for the correct formula for calculating specific measures of a figure that is a composite of two or three simpler figures. The formulas for each composite figure can be derived from the standard formulas for calculating perimeters or areas of the simpler figures.

To solve these problems, use the following steps:

Step 1. Read the problem and study the diagram given to determine which two- or three-dimensional figures are used in its composition.

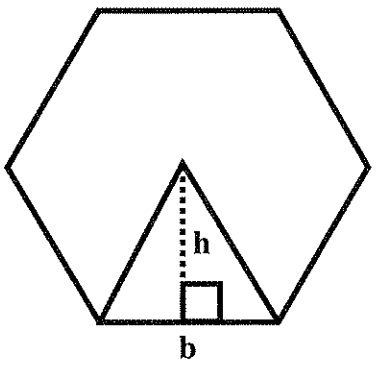
Step 2. Write the formulas necessary to find the required measure of the individual figures.

Step 3. Use the diagram to find the values for the variables in the formula. Use geometric properties to find any missing values. Substitute these values into the correct formula.

Step 4. Find the sum and/or difference of the areas or volumes.

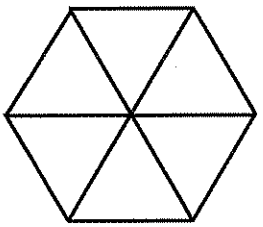
Examples

1. The figure below shows a regular hexagon. Determine the formula for computing the total area of the hexagon.



Step 1. Read the problem and study the diagram given to determine which two- or three-dimensional figures are used in its composition.

This regular hexagon consists of 6 triangles that are equal in area to the one shown.



Step 2. Write the formulas necessary to find the required measure of the individual figures.

The formula for the area of the triangle is $A = \frac{1}{2}bh$.

Step 3. Use the diagram to find the values for the variables in the formula. Use geometric properties to find any missing values. Substitute these values into the correct formula.

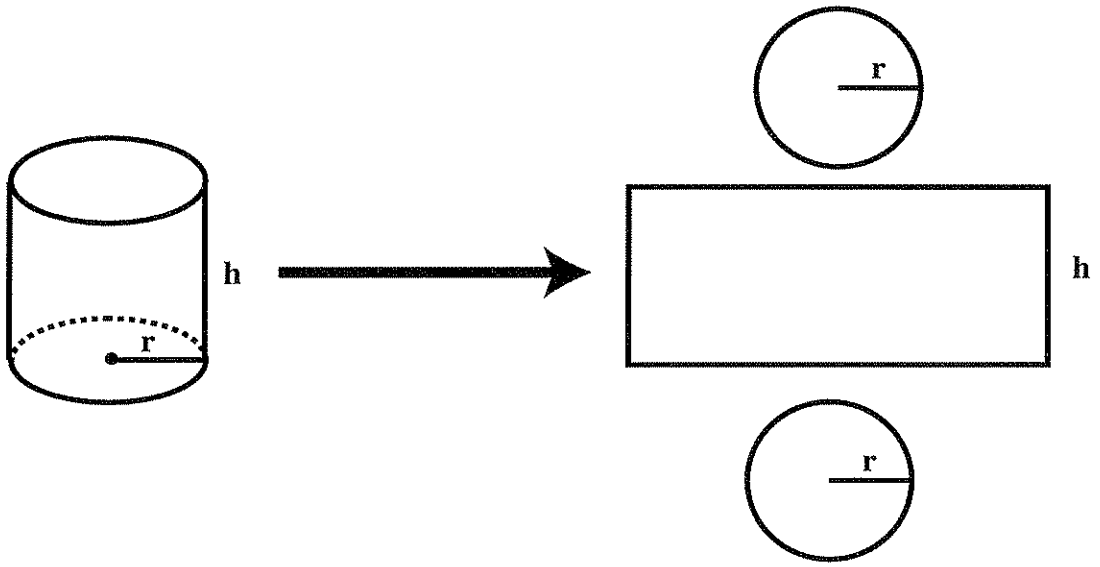
The values given in the diagram are the same as those used in the formula.

Step 4. Find the sum and/or difference of the areas or volumes.

Since there are 6 triangles, the total area is given by

$$A = \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh + \frac{1}{2}bh = 6\left(\frac{1}{2}bh\right) = 3bh.$$

2. The figure below shows the circles that form the top and bottom of a right circular cylinder and the rectangle that bends around to form the cylinder's "sides." Determine the formula needed for calculating the total surface area (SA) of a right circular cylinder.



Step 1. Read the problem and study the illustration given to determine which two- or three-dimensional figures are used in its composition.

Two circles and a rectangle are used in the composition of this figure.

Step 2. Write the formulas necessary to find the required measure of the two- or three-dimensional figures.

The area of a circle is given by the formula $A = \pi r^2$.

The area of a rectangle is given by the formula $A = bh$.

Step 3. Use the diagram to find the values for the variables in the formula. Use geometric properties to find any missing values. Substitute these values into the correct formula.

If the radius of each circle is r, the formula for the area of each circle is $A = \pi r^2$.

The rectangle bends around the circles to form the side of the cylinder. So the base of the rectangle is the same as the circumference of (distance around) the circle. The height of the rectangle is the same as the height of the cylinder.

The formula for the circumference of a circle is $C = 2\pi r$.

Therefore, the formula for the area of the rectangle is $A = bh = (2\pi r)(h) = 2\pi rh$.

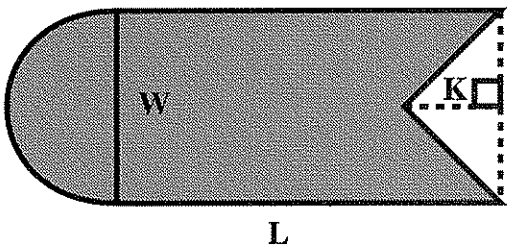
Step 4. Find the sum and/or difference of the areas or volumes.

Cylinder's surface area = Area of the circles + Area of the rectangle

$$A = \pi r^2 + \pi r^2 + 2\pi rh$$

$$A = 2\pi r^2 + 2\pi rh = 2\pi r(r + h)$$

3. The figure below consists of a rectangle, semicircle, and triangle. Determine the formula for finding the area of the shaded region.



Step 1. Read the problem and study the diagram given to determine which two- or three-dimensional figures are used in its composition.

The problem states that the figure consists of a rectangle with a triangle cut from one end and a semicircle attached to the other end.

Step 2. Write the formulas necessary to find the required measure of the individual figures.

$$\text{Area of a semicircle } (A_1) = \frac{1}{2}(\pi r^2)$$

$$\text{Area of a rectangle } (A_2) = bh$$

$$\text{Area of a triangle } (A_3) = \frac{1}{2}bh$$

Step 3. Use the diagram to find the values of the variables in the formula. Use geometric properties to find any missing values. Substitute these values into the correct formula.

The radius of the semicircle is $\frac{1}{2}W$. Hence, $A_1 = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{1}{2}W\right)^2 = \frac{1}{8}\pi W^2$.

The base of the rectangle is L and the height is W. Hence, $A_2 = bh = LW$.

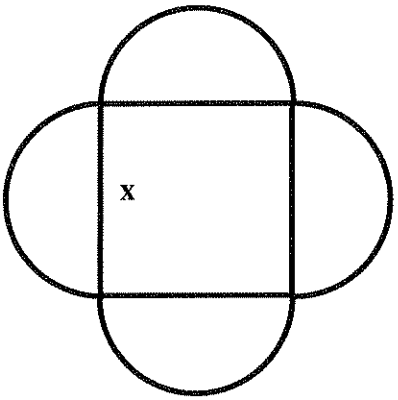
The base of the triangle is W and the height is K. Hence, $A_3 = \frac{1}{2}bh = \frac{1}{2}WK$.

Step 4. Find the sum and/or difference of the areas or volumes.

Area of the diagram = Area of the semicircle + Area of the rectangle – Area of the triangle

$$\text{Area} = \frac{1}{8}\pi W^2 + LW - \frac{1}{2}WK$$

4. The figure below consists of a square surrounded by semicircles. Determine the formula for finding the perimeter of the following figure.



Step 1. Read the problem and study the diagram given to determine which two- or three-dimensional figures are used in its composition.

The problem states that the figure consists of 4 semicircles and a square.

Step 2. Write the formulas necessary to find the required measure of the individual figures.

Since the perimeter is requested, the square is not needed. The perimeter of the semicircle is given by half the circumference of a circle or $\frac{1}{2}(2\pi r) = \pi r$.

Step 3. Use the diagram to find the values of the variables in the formula. Use geometric properties to find any missing values. Substitute these values into the correct formula.

The value of the radius is $\frac{1}{2}x$.

The perimeter of a semicircle $= \pi r = \pi\left(\frac{1}{2}x\right) = \frac{1}{2}\pi x$.

Step 4. Find the sum and/or difference of the areas or volumes.

There are 4 semicircles on the outside of the square, so the perimeter of the diagram is given by $P = \frac{1}{2}\pi x + \frac{1}{2}\pi x + \frac{1}{2}\pi x + \frac{1}{2}\pi x = 4\left(\frac{1}{2}\pi x\right) = 2\pi x.$

INFER FORMULAS FOR MEASURING GEOMETRIC FIGURES

In this section the problems present two to four figures that illustrate the calculation of certain geometric measurements and then ask you to apply the information to measure another similar figure.

To solve problems requiring a generalization, use the following steps:

Step 1. Read the problem carefully and identify the specific measurement being sought. Look at the diagrams to determine what information is given.

Step 2. Organize the information in a chart leaving a blank for the needed information.

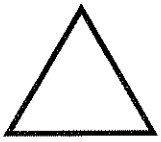
Step 3. Look for a pattern in the chart.

Step 4. Fill in the missing information.

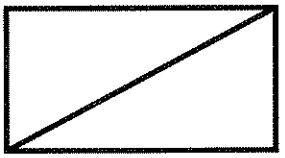
Reminder: The answers to some of these problems may be calculated from a formula. However, most of the problems involve infrequently used formulas.

Examples

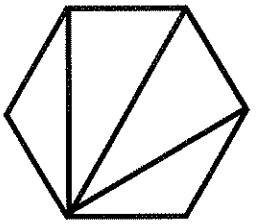
1. Study the given information. For each figure, S represents the sum of the measures of the interior angles.



3 sides → 1 triangle
$S = 180°$



4 sides → 2 triangles
$S = 360°$



6 sides → 4 triangles
$S = 720°$

Calculate S, the sum of the measures of the interior angles of an eight-sided convex polygon.

Step 1. Read the problem carefully and identify the specific measurement being sought. Look at the diagrams to determine what information is given.

The question asks for the sum of the measures of the interior angles. The diagram gives the number of sides, the number or triangles that are formed by the diagonals from a corner of each convex polygon, and the sum of the measures of the interior angles.

Step 2. Organize the information in a chart leaving a blank for the needed information.

No. of Sides	No. of Triangles	Sum of Interior Angles
3	1	180°
4	2	360°
6	4	720°
8	*?*	*?*

Step 3. Look for a pattern in the chart.

What is a pattern involving addition, subtraction, multiplication, or division?

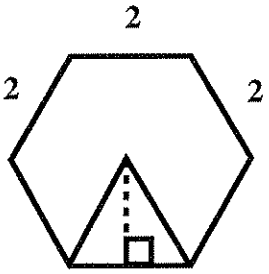
No. of Sides	No. of Triangles	Sum of Interior Angles
3 – 2 =	1 x 180° =	180°
4 – 2 =	2 x 180° =	360°
6 – 2 =	4 x 180° =	720°

Step 4. Fill in the missing information.

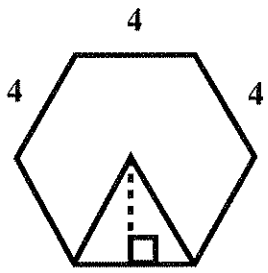
8 – 2 =	***6*** x 180° =	***1080°***

Note: There is a formula that states the sum of the interior angles of a convex polygon = $(n - 2) \times 180°$, where n = the number of sides. For an eight-sided convex polygon, $(8 - 2) \times 180° = 1080°$.

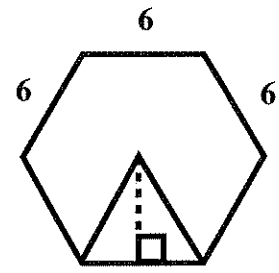
2. Study the information given with the regular hexagons.



Area = $6\sqrt{3}$



Area = $24\sqrt{3}$



Area = $54\sqrt{3}$

Calculate the area of a regular hexagon in which each side equals 8.

Reminder: The formula for finding the area of a regular hexagon is $A = 6\left(\frac{s}{2}\right)^2\sqrt{3}$, where s = the length of the side.

Step 1. Read the problem carefully and identify the specific measurement being sought. Look at the diagrams to determine what information is given.

The question asks for the area. The diagram shows six-sided figures with the length of the sides given.

Step 2. Organize the information in a chart leaving a blank for the needed information.

Length of Side	Area of Regular Hexagon
2	$6\sqrt{3}$
4	$24\sqrt{3}$
6	$54\sqrt{3}$
8	?

Step 3. Look for a pattern in the chart.

What is a pattern involving addition, subtraction, multiplication, or division?

Length of Side	Formula	Area of Regular Hexagon
2	$A = 6\left(\frac{2}{2}\right)^2\sqrt{3} = 6(1)^2\sqrt{3}$	$6\sqrt{3}$
4	$A = 6\left(\frac{4}{2}\right)^2\sqrt{3} = 6(2)^2\sqrt{3}$	$24\sqrt{3}$
6	$A = 6\left(\frac{6}{2}\right)^2\sqrt{3} = 6(3)^2\sqrt{3}$	$54\sqrt{3}$
8	A = _____	_____

Step 4. Fill in the missing information.

Length of Side	Formula	Area of Regular Hexagon
8	$A = 6\left(\frac{8}{2}\right)^2\sqrt{3} = 6(4)^2\sqrt{3}$	$96\sqrt{3}$

IDENTIFY RELATIONSHIPS BETWEEN ANGLE MEASURES

Three basic terms of geometry are point, line, and plane.

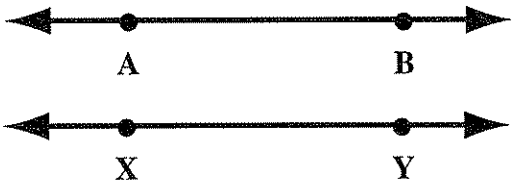
A **point** is a geometric object that has no length, width, or height. A point is symbolized by a dot.

A **line** is determined by two distinct points and extends indefinitely in both directions. The line below contains points A and B and is represented by $\overleftrightarrow{AB}$.

A **plane** is a flat surface that extends indefinitely in all directions.

When lines or line segments lie in the same plane, certain relationships exist.

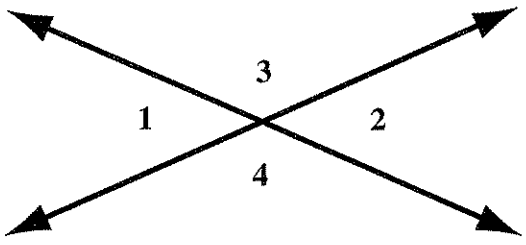
Two lines or line segments can exist in the same plane in the following ways.



Two lines in the same plane that never intersect are *parallel lines* (||). Two parallel lines are always exactly the same distance apart.

$\overleftrightarrow{AB} \parallel \overleftrightarrow{XY}$

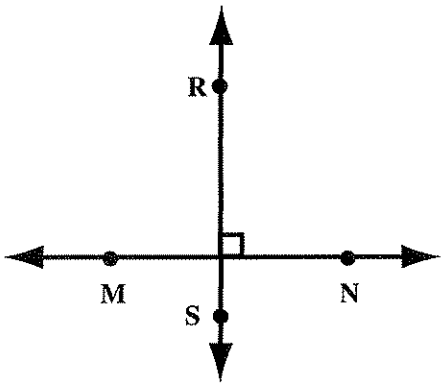
If two lines in the same plane are not parallel, they *intersect* to form angles. One unit of measure used to describe the size of an angle is the degree. The symbol for degrees is a small raised circle, °. One degree (1°) is $\frac{1}{360}$ of a circle. The measure of an angle, written $m\angle$, is the number of degrees between the two sides of an angle. Two angles having the same measure are **congruent angles**. Congruence is denoted by the symbol $\cong$.



Two lines that intersect form **vertical angles**.
$\angle 1$ and $\angle 2$ are vertical angles.
$\angle 3$ and $\angle 4$ are vertical angles.
Vertical angles have equal measures.

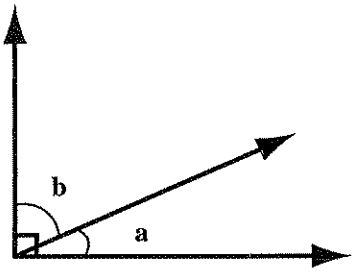
$m\angle 1 = m\angle 2$
$m\angle 3 = m\angle 4$

Two angles that share a common side are **adjacent angles**. Pairs of adjacent angles are $\angle 1$ and $\angle 3$; $\angle 3$ and $\angle 2$; $\angle 2$ and $\angle 4$; and $\angle 1$ and $\angle 4$.

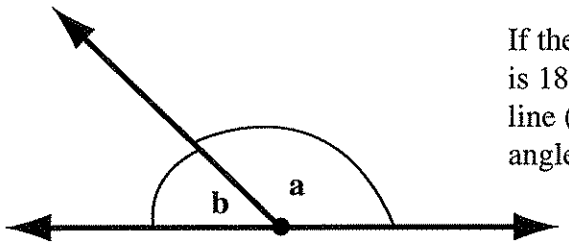


Congruent angles are angles that have the same measure. Two lines that form congruent adjacent angles when they intersect are *perpendicular lines* ($\perp$).
$\overline{RS} \perp \overline{MN}$.

When lines or line segments are perpendicular, the angles formed are right angles (90°). The symbol for a right angle is a small square drawn at the point of intersection.



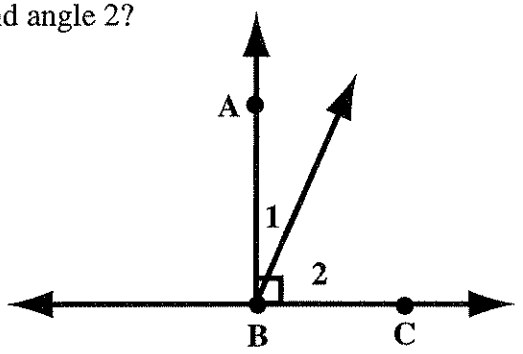
If the sum of the measures of two angles ($m\angle a + m\angle b$) is 90°, then they are **complementary angles.** A right angle is formed by two adjacent complementary angles.



If the sum of the measures of two angles ($m\angle a + m\angle b$) is 180°, then they are **supplementary angles**. A straight line (angle) is formed by two adjacent supplementary angles.

Examples

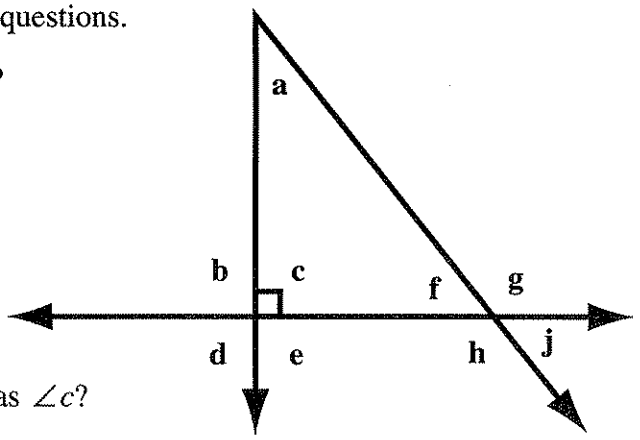
1. Given that $AB \perp BC$, what is true for angle 1 and angle 2?



$\angle ABC$ is a right angle.
$m\angle 1 + m\angle 2 = m\angle ABC$
$m\angle 1 + m\angle 2 = 90°$
$\angle 1$ and $\angle 2$ are complementary.

Caution: There is not enough information to conclude that $m\angle 1 = m\angle 2$.

2. Using this diagram, answer the following questions.



A. Which angles are supplements of $\angle g$?

B. If $m\angle f = 50°$, complete the following:

$m\angle g =$ _____

$m\angle h =$ _____

$m\angle j =$ _____

C. Which angles have the same measure as $\angle c$?

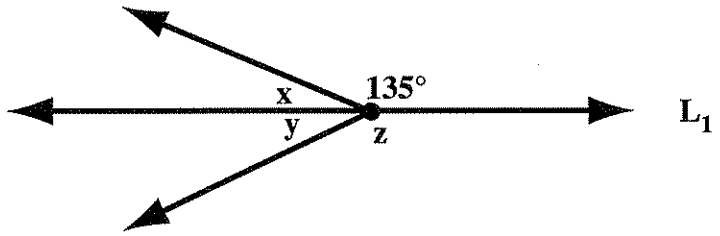
Explanation:

	Relationship	Reason
A.	$\angle f$ is a supplement of $\angle g$.	$m\angle f + m\angle g = 180°$ (form straight line)
	$\angle j$ is a supplement of $\angle g$.	$m\angle j + m\angle g = 180°$ (form straight line)

	Relationship	Reason
B.	$m\angle f + m\angle g = 180°$	$\angle f$ and $\angle g$ are supplementary angles.
	$50° + m\angle g = 180°$	$m\angle f = 50°$
	$m\angle g = 180° - 50°$	Subtract 50°.
	$m\angle g = 130°$	
	$m\angle h = m\angle g$	$\angle h$ and $\angle g$ are vertical angles.
	$m\angle h = 130°$	$m\angle g = 130°$
	$m\angle j = m\angle f$	$\angle j$ and $\angle f$ are vertical angles.
	$m\angle j = 50°$	$m\angle f = 50°$

	Relationship	Reason
C.	$m\angle c = m\angle d$	$\angle c$ and $\angle d$ are vertical angles.
	$m\angle c + m\angle b = 180°$	$\angle c$ and $\angle b$ are supplementary angles.
	$90° + m\angle b = 180°$	$m\angle c = 90°$
	$m\angle b = 180° - 90°$	Subtract 90°.
	$m\angle b = 90°$	
	$m\angle b = m\angle c$	$m\angle c = 90°$
	$m\angle e = m\angle b$	$\angle b$ and $\angle e$ are vertical angles.
	$m\angle b = m\angle c$	relationship established previously
	$m\angle e = m\angle c$	substitution

3. Given Line L_1 in the diagram below, which of the following statements is true?



A. $z = 135°$
B. $x = 55°$
C. $x = 45°$
D. y and z are complementary angles.

Explanation:

A. Incorrect. There is no relationship between angles to verify that $z = 135°$.

B. Incorrect. x is a supplement to the 135° angle. But, if $x = 55°$, then $55° + 135° = 190°$, not 180°.

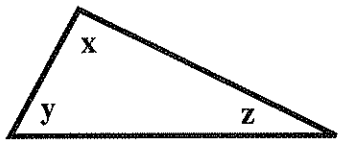
C. Correct. x is a supplement to the 135° angle. $45° + 135° = 180°$

D. Incorrect. $y + z = 180°$, which means they are supplementary angles and not complementary angles.

Other relationships hold when three lines or line segments lie in the same plane.

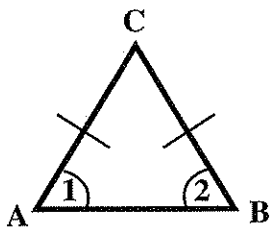
A **triangle** is a closed, three-sided figure having three angles.

Note: Congruent angles will be denoted by arcs.
Congruent sides (sides of equal measure) will be denoted by slashes.



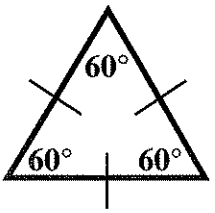
The sum of three angles of a triangle is equal to 180°.

$m\angle x + m\angle y + m\angle z = 180°$



Two sides of a triangle have equal length if and only if the angles opposite those sides have equal measure.

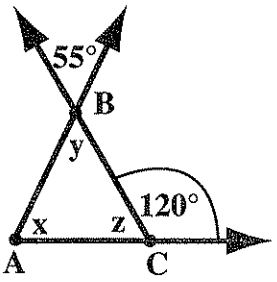
If $AC = BC$, then $m\angle 1 = m\angle 2$.
If $m\angle 1 = m\angle 2$, then $AC = BC$.



A triangle has three sides of equal length if and only if it has three equal angles (60°).

Examples

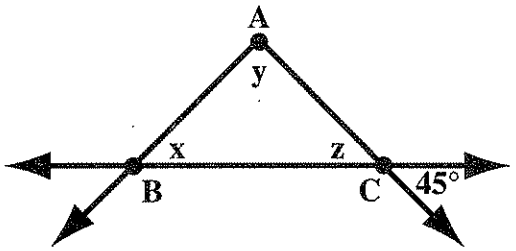
4. Study the given diagram and determine the value of x. In ΔABC, x is the measure of $\angle BAC$. (The measure of angle ABC is represented by y.)



Explanation:

Relationship	Reason
$y = 55°$	vertical angle
$z = 60°$	supplementary angle
$x + y + z = 180°$	The sum of the measures of the angles of a triangle is 180°.
$x + 55° + 60° = 180°$	substitution
$x = 65°$	Subtract 55° and 60° from 180°.

5. In ΔABC, $\overline{AB} \cong \overline{AC}$, which of the following statements is true for the figure shown? (The measure of angle ABC is represented by x.)



A. $y = x$

B. $\angle ABC$ and $\angle ACB$ are supplementary angles.

C. $\overline{AC} \cong \overline{BC}$

D. $\overline{AC} \perp \overline{AB}$

Explanation:

Relationship	Reason
$z = 45°$	vertical angle
$x = z$	angles opposite congruent sides are congruent
$x + y + z = 180°$	The sum of the measures of the angles of a triangle is 180°.
$45° + y + 45° = 180°$	substitution
$y = 180° - 90°$	Subtract 90° from each side.
$y = 90°$	

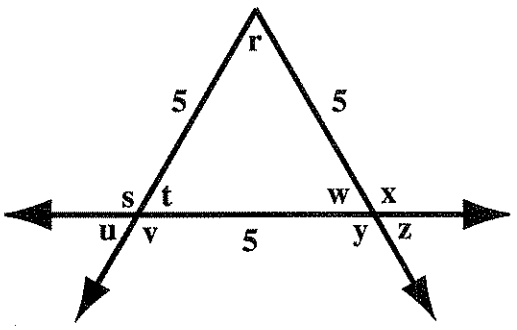
A. Incorrect. $y = 90°$ and $x = 45°$.

B. Incorrect. $\angle ABC = \angle ACB = 45°$. Hence, the two angles are complementary, not supplementary.

C. Incorrect. $y = 90°$ and $x = 45°$. If these angles are not congruent, their opposite sides are not congruent.

D. Correct. $y = 90°$. Hence, $\overline{AC} \perp \overline{AB}$.

6. Given the diagram below, which of the following statements is true?



A. $m\angle r = 70°$

B. $\angle t$ and $\angle v$ are vertical angles.

C. $m\angle x = 120°$

D. $\angle r$ and $\angle w$ are complementary angles.

Explanation:

The sides of the given triangle are of equal length.
Therefore, $m\angle r = m\angle t = m\angle w = 60°$.

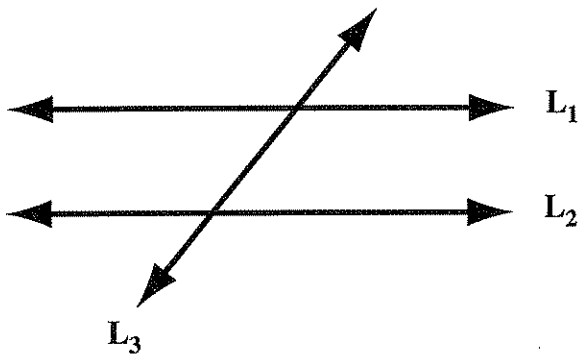
A. Incorrect. $m\angle r = 60°$, not 70°

B. Incorrect. $\angle t$ and $\angle v$ are adjacent angles, not vertical angles.

C. Correct. $m\angle w + m\angle x = 180°$ and $m\angle w = 60°$
By substitution, $60° + m\angle x = 180°$. Thus, $m\angle x = 120°$.

D. Incorrect. $m\angle r = m\angle w = 60°$
Hence, $m\angle r + m\angle w = 120°$, not 90°.

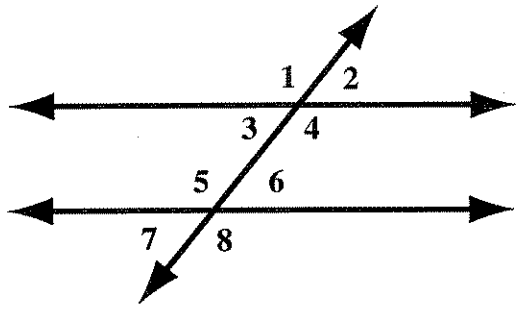
A **transversal** is a line that intersects two other lines. Special angle relationships are true when two parallel lines are cut by a transversal.



L_1 and L_2 are parallel lines.

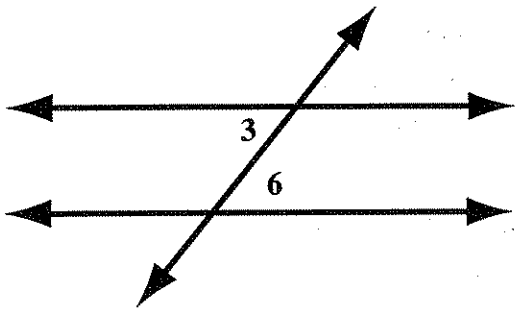
L_3 is the transversal.

Corresponding angles have equal measure. In this diagram, corresponding angles are:

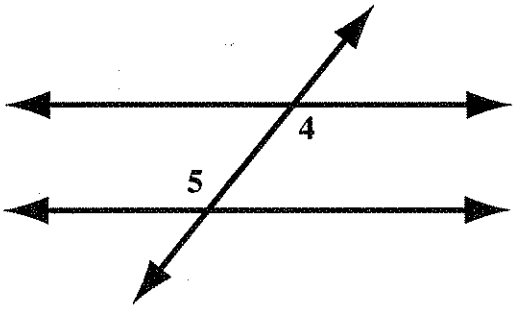


$\angle 1$ and $\angle 5$	$m\angle 1 = m\angle 5$
$\angle 2$ and $\angle 6$	$m\angle 2 = m\angle 6$
$\angle 3$ and $\angle 7$	$m\angle 3 = m\angle 7$
$\angle 4$ and $\angle 8$	$m\angle 4 = m\angle 8$

Angles that are on opposite sides of the transversal (alternate) between the two parallel lines (interior) are **alternate interior angles.** Alternate interior angles have equal measure.

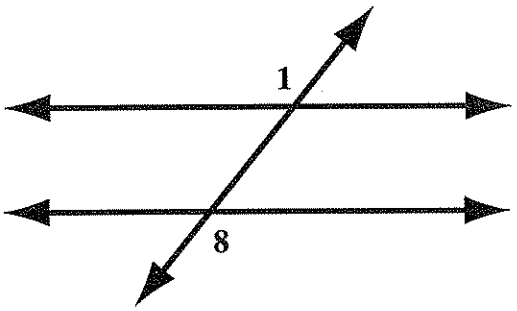


$m\angle 3 = m\angle 6$

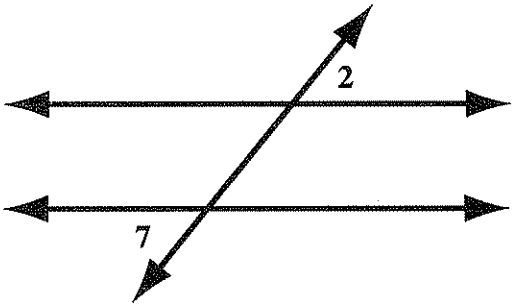


$m\angle 4 = m\angle 5$

Angles that are on opposite sides of the transversal (alternate) and are outside the parallel lines (exterior) are **alternate exterior angles.** Alternate exterior angles have equal measure.



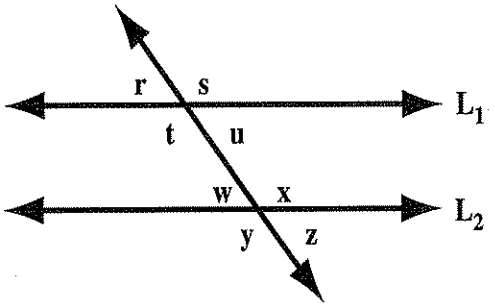
$m\angle 1 = m\angle 8$



$m\angle 2 = m\angle 7$

Examples

7. In the diagram below, lines L_1 and L_2 are parallel.



A. Name all angles having the same measure as $\angle r$.

B. Name all angles having the same measure as $\angle x$.

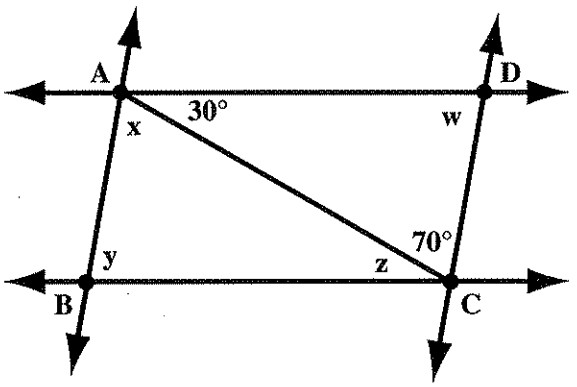
C. Given that $m\angle r = 32°$, find the measure of all the other angles.

Explanation:

	Relationship	Reason
A.	$m\angle r = m\angle u$	vertical angles
	$m\angle r = m\angle w$	corresponding angles
	$m\angle r = m\angle z$	alternate exterior angles
B.	$m\angle x = m\angle y$	vertical angles
	$m\angle x = m\angle s$	corresponding angles
	$m\angle x = m\angle t$	alternate interior angles
C.	$m\angle s = 180° - 32° = 148°$	supplementary angles
	$m\angle t = 148°$	$\angle s$ and $\angle t$ are vertical angles.
	$m\angle u = 32°$	$\angle r$ and $\angle u$ are vertical angles.
	$m\angle w = 32°$	$\angle u$ and $\angle w$ are alternate interior angles.
	$m\angle x = 148°$	$\angle t$ and $\angle x$ are alternate interior angles.
	$m\angle y = 148°$	$\angle x$ and $\angle y$ are vertical angles.
	$m\angle z = 32°$	$\angle w$ and $\angle z$ are vertical angles.

Note: Other reasons could be used to obtain solutions.

8. Given that $\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$, find the values of w, x, y, and z. (The measure of angle BAC is represented by x.)



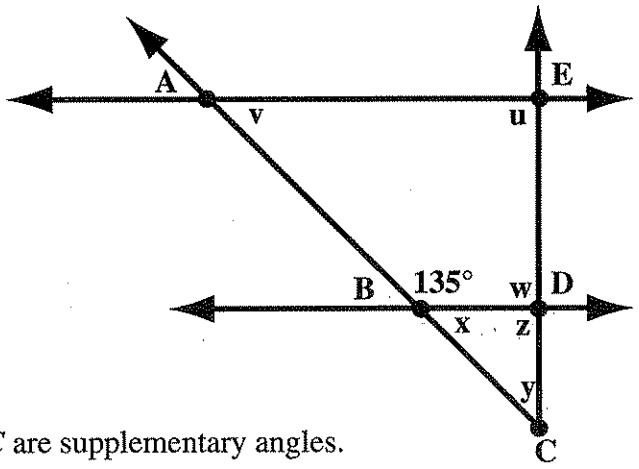
I hate MATH

Explanation:

Relationship	Reason
$x = 70°$	alternate interior angles
$z = 30°$	alternate interior angles
$30° + 70° + w = 180°$	The sum of the measures of the angles of a triangle is 180°.
$w = 180° - 100°$	Subtract 100° from each side.
$w = 80°$	

A similar argument can be made to show $y = 80°$.

9. Given that $\overline{AE} \parallel \overline{BD}$ and $\overline{CE} \perp \overline{AE}$, which of the following statements is true for the figure shown? (The measure of EAB is represented by v.)



A. $x = 55°$

B. $\overline{BD} \cong \overline{CD}$

C. $\angle EAB$ and $\angle DBC$ are supplementary angles.

D. $u = y$

Explanation:

Relationship	Reason
$\angle u = 90°$	$\overline{CE} \perp \overline{AE}$
$\angle z = 90°$	$\angle z$ and $\angle u$ are corresponding angles.
$\angle w = 90°$	$\angle z + \angle w = 180°$
	$90° + \angle w = 180°$
	$\angle w = 180° - 90° = 90°$
$\angle x = 45°$	$135° + \angle x = 180°$
	$\angle x = 180° - 135° = 45°$
$\angle v = 45°$	$\angle x$ and $\angle v$ are corresponding angles.
$\angle y = 45°$	The sum of the measures of the angles of a triangle is 180°.

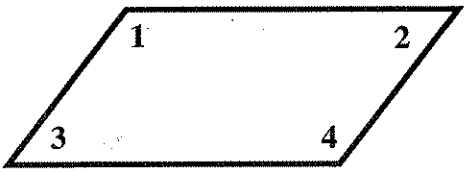
A. Incorrect. $x = 45°$, not 55°.

B. Correct. $x = y = 45°$. Because the angles opposite $\overline{BD}$ and $\overline{CD}$ are congruent, $\overline{BD}$ and $\overline{CD}$ are also congruent.

C. Incorrect. $\angle EAB = 45°$ and $\angle DBC = 45°$. Hence, these angles are complementary, not supplementary.

D. Incorrect. $u = 90°$ and $y = 45°$.

A **parallelogram** is a four-sided polygon that has two pairs of parallel sides.



Its *opposite angles* have the same measure.

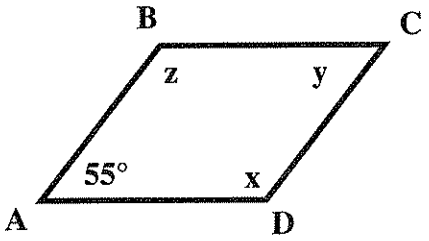
$m\angle 1 = m\angle 4$
$m\angle 3 = m\angle 2$

Its *adjacent angles* are supplementary.

$m\angle 1 + m\angle 2 = 180°$
$m\angle 2 + m\angle 4 = 180°$
$m\angle 4 + m\angle 3 = 180°$
$m\angle 3 + m\angle 1 = 180°$

Example

10. Given parallelogram *ABCD*, which of the following statements is true? (The measure of angle *ABC* is represented by *z*.)



A. $z = y$
B. x and z are supplementary.
C. $z = 125°$
D. $x = 55°$

Explanation:

Relationship	Reason
$y = 55°$	$\angle C$ is opposite $\angle A$.
$z + 55^0 = 180°$	$\angle B$ and $\angle A$ are adjacent angles. Hence, $z = 125°$.
$z = x = 125°$	$\angle B$ and $\angle D$ are opposite angles.

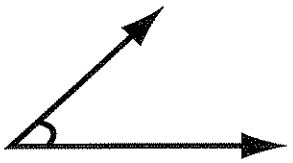
A. Incorrect.	$z = 125°$ and $y = 55°$
B. Incorrect.	$x + z = 125° + 125° = 250°$, NOT 180°
C. Correct.	$z = 125°$
D. Incorrect.	$x = 125°$

Skill IIB2 *Identify names of plane figures given their properties*

IDENTIFY NAMES OF PLANE FIGURES GIVEN THEIR PROPERTIES

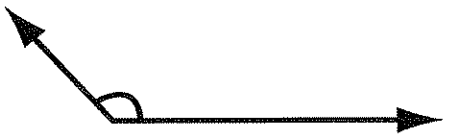
This skill requires the knowledge of basic facts about types of angles, triangles, and quadrilaterals. Recall the following angle pairs: vertical, supplementary, complementary, alternate interior, alternate exterior, and corresponding.

Angles may be classified according to their measure.



An **acute angle** is an angle of measure greater than 0° and less than 90°.

A **right angle** is an angle of measure 90°.

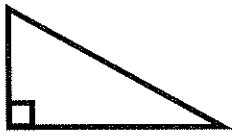


An **obtuse angle** is an angle of measure greater than 90° and less than 180°.

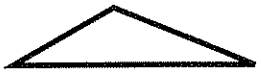
A **straight angle** is an angle of measure 180°.

Triangles may be classified by the measures of their angles.

An **acute triangle** has three acute angles.

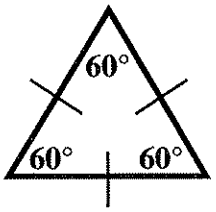


A **right triangle** has one right angle.



An **obtuse triangle** has one obtuse angle.

Triangles may also be classified according to the length of their sides.



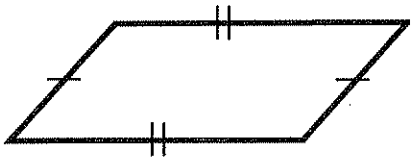
An **equilateral triangle** has three sides of equal length, and all angles measure 60°.

An **isosceles triangle** has two congruent sides and the angles opposite these sides are of equal measure.

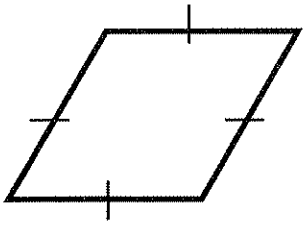
A **scalene triangle** has three sides of unequal length.

Quadrilaterals (four-sided figures) are classified by distinguishing characteristics, also.

A **trapezoid** has one pair of parallel sides.

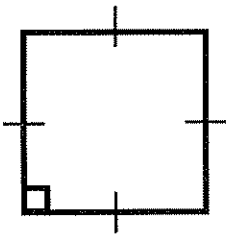


A **parallelogram** has two pairs of parallel sides. Opposite sides are equal in length. Opposite angles are equal in measure. Diagonals bisect each other.

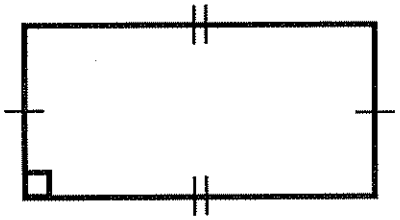


A **rhombus** is a parallelogram with all sides congruent. Diagonals are perpendicular to each other.

Skill IIB2 *Identify names of plane figures given their properties*



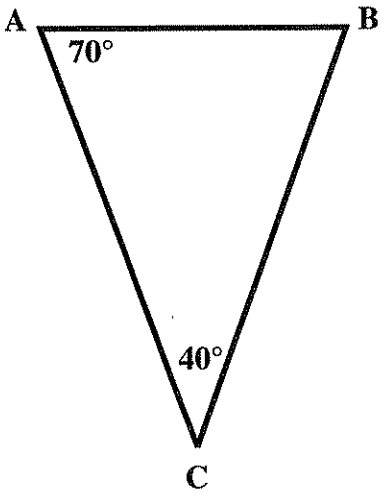
A **square** is a parallelogram with all sides equal in length and four right angles (90°). Diagonals are perpendicular and equal in length.



A **rectangle** is a parallelogram with four right angles. Opposite sides are equal in length. Diagonals are equal in length.

Examples

1. What type of triangle is ΔABC?



First, find the measure of the third angle.

$70° + 40° + m\angle ABC = 180°$ Measures of angles of a triangle sum to 180°.

$110° + m\angle ABC = 180°$

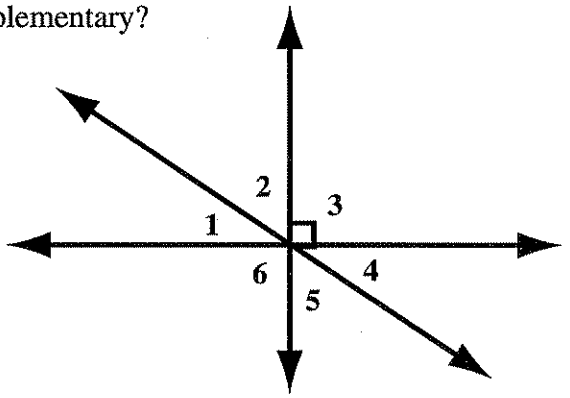
$m\angle ABC = 180° - 110°$

$m\angle ABC = 70°$

ΔABC is acute because the three angles are all less than 90°.

ΔABC is isosceles because the two base angles of equal measure infer that the two sides opposite them are of equal length.

2. Which of the following pairs of angles are complementary?



A. 3 and 4
B. 1 and 4
C. 6 and 3
D. 1 and 5

Explanation:

A. Incorrect. $m\angle 3 = 90°$, so $m\angle 3 + m\angle 4$ is more than $90°$.

B. Incorrect. $m\angle 1 = m\angle 4$ since they are vertical angles. There is not enough information to determine whether or not these angles are complementary.

C. Incorrect. $m\angle 3 = 90°$
$m\angle 6 = m\angle 3$ because they are vertical angles.
So, $m\angle 6 + m\angle 3 = 180°$.
Hence, they are supplementary, NOT complementary.

D. Correct. $m\angle 1 + m\angle 2 = 90°$
$m\angle 2 = m\angle 5$ because they are vertical angles.
So, $m\angle 1 + m\angle 5 = 90°$.

3. Select the geometric figure that possesses all of the following characteristics.

i. quadrilateral
ii. opposite sides are parallel
iii. diagonals are not equal

A. rhombus
B. trapezoid
C. square
D. rectangle

Explanation:

A. Correct. The solution is rhombus.

B. Incorrect. A trapezoid has only one pair of parallel sides.

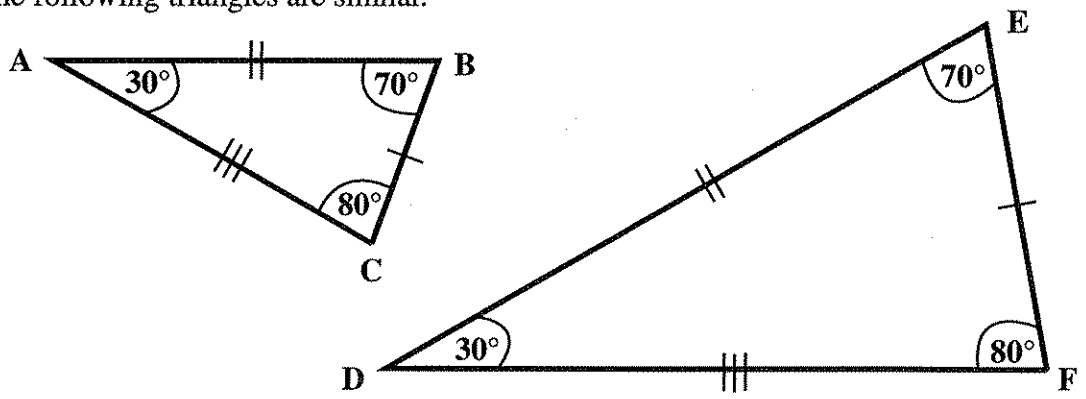
C. Incorrect. Squares have diagonals of equal length.

D. Incorrect. Rectangles have diagonals of equal length.

RECOGNIZE SIMILAR TRIANGLES AND THEIR PROPERTIES

In **similar triangles**, the measures of corresponding angles are equal, and the lengths of corresponding sides are in proportion.

The following triangles are similar.



Corresponding angles are angles that have the same measure. In the diagram above,

$\angle A$ corresponds to $\angle D$
$\angle B$ corresponds to $\angle E$
$\angle C$ corresponds to $\angle F$

When naming similar triangles it is necessary to put congruent angles in corresponding positions.

For example, ΔABC is similar to ΔDEF. The notation for similarity is $\Delta ABC \sim \Delta DEF$.

Corresponding sides are the sides directly across from corresponding angles.

$\overline{BC}$ corresponds to $\overline{EF}$
$\overline{AC}$ corresponds to $\overline{DF}$
$\overline{AB}$ corresponds to $\overline{DE}$

Corresponding sides are proportional, thus:

$$\frac{BC}{EF}=\frac{AC}{DF};\ \frac{BC}{EF}=\frac{AB}{DE};\ \frac{AC}{DF}=\frac{AB}{DE};$$

$$\frac{BC}{AC}=\frac{EF}{DF};\ \frac{BC}{AB}=\frac{EF}{DE};\ \frac{AC}{AB}=\frac{DF}{DE}$$

Reminder: Congruent angles are often marked by arcs, and proportional sides are often labeled with slashes. $\overline{AB}$ represents the side joining A to B, while AB represents the length of that side.

Another way to match corresponding sides is by using the notation for similar triangles obtained after matching corresponding angles.

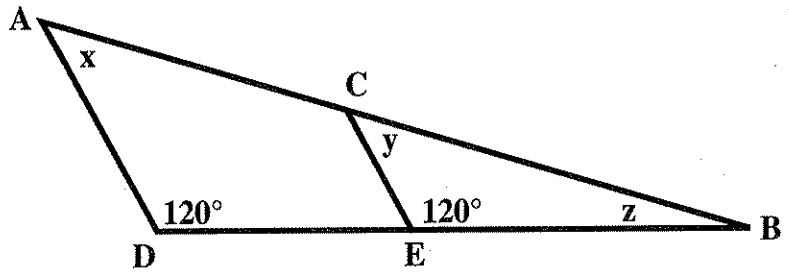
$\Delta \underline{ABC} \sim \Delta \underline{DEF}$ $\overline{AB}$ is matched with $\overline{DE}$.

$\Delta A\underline{BC} \sim \Delta D\underline{EF}$ $\overline{BC}$ is matched with $\overline{EF}$.

$\Delta \underline{A}B\underline{C} \sim \Delta \underline{D}E\underline{F}$ $\overline{AC}$ is matched with $\overline{DF}$.

Examples

1. Refer to the figure below. (The measure of angle A is represented by x.)



Complete each statement:

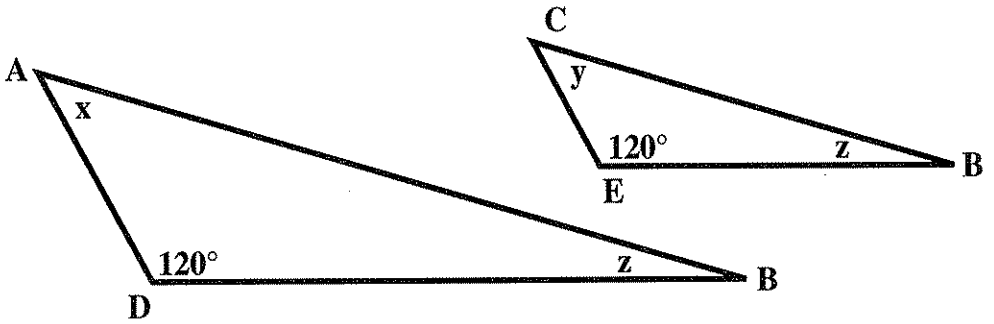
A. $x =$ *?*

B. $\frac{CE}{AD} = \frac{?}{AB}$

C. $\frac{BC}{BE} = \frac{?}{?}$

Explanation:

For clarity, the triangles can be separated and redrawn.



The angle at D is congruent to the angle at E, and the angle at B is congruent to the angle at B. Thus, the angle at A is congruent to the angle at C, because in either case, the angle is the one that when combined with the other two makes a total of 180°.

Hence, $\Delta ABD \sim \Delta CBE$.

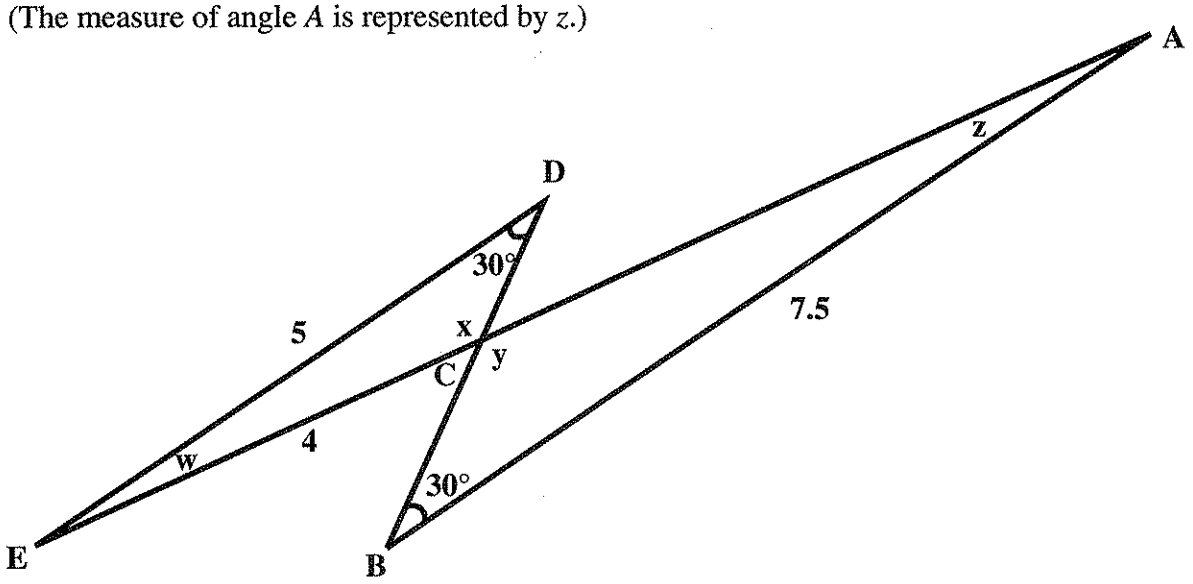
A. x is the measure of the angle at A on the large triangle. A corresponds to C on the small triangle. Therefore, $x = y$.

B. The use of notation for similar triangles reveals that $\overline{AB}$ corresponds to $\overline{CB}$ ($\Delta \underline{AB}D \sim \Delta \underline{CB}E$).

C. The use of notation for similar triangles reveals that $\overline{BC}$ is matched with $\overline{BA}$ ($\Delta \underline{ABD} \sim \Delta \underline{CBE}$) and $\overline{BE}$ is matched with $\overline{BD}$ ($\Delta \underline{ABD} \sim \Delta \underline{CBE}$).

Therefore, $\frac{BC}{BE} = \frac{BA}{BD}$.

2. Which of the statements is true for the pictured triangles?
(The measure of angle A is represented by z.)



A. $z \neq w$

B. $x = 30°$

C. $AC = 6$

D. $\frac{CE}{CA} = \frac{CB}{CD}$

Explanation:

$\angle CDE \cong \angle ABC$ because they both measure 30°. $\angle DCE \cong \angle ACB$ because they are vertical angles. Hence, $\angle CAB \cong \angle CED$.

Therefore, $\Delta ECD \sim \Delta ACB$.

A. Incorrect. $\angle CAB$ and $\angle DEC$ are corresponding angles. Hence, $z = w$.

B. Incorrect. There is not enough information given to determine the value of x using geometric properties.

C. Correct AC can be determined by setting up a proportion involving AC and three other known lengths of sides.

$\Delta \underline{ECD} \sim \Delta \underline{ACB}$ $\overline{AC}$ having length AC corresponds to $\overline{EC}$ having length 4.

$\Delta \underline{ECD} \sim \Delta \underline{ACB}$ $\overline{AB}$ having length 7.5 corresponds to $\overline{DE}$ having length 5.

Thus, $\frac{AC}{EC} = \frac{AB}{DE} \rightarrow \frac{AC}{4} = \frac{7.5}{5}$.

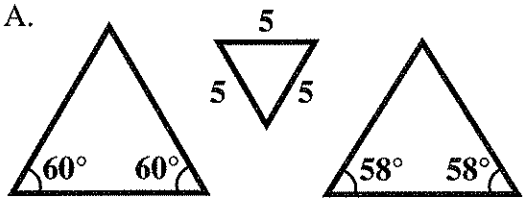
Cross multiply. $5(AC) = 30$

Divide by 5. $AC = 6$

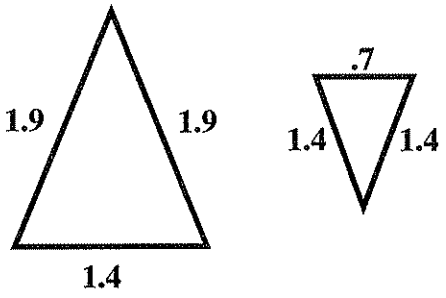
D. Incorrect. $\overline{CE}$ corresponds with $\overline{CA}$ ($\Delta \underline{ECD} \sim \Delta \underline{ACB}$), and NOT with $\overline{CB}$.

3. Study figures A, B, C, and D. Select the figure in which all triangles are similar.

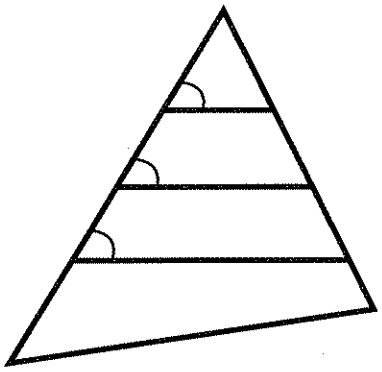
A.



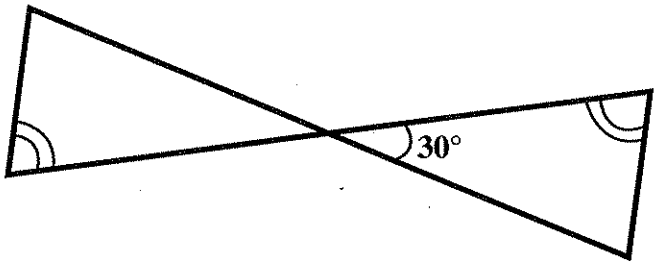
B.



C.

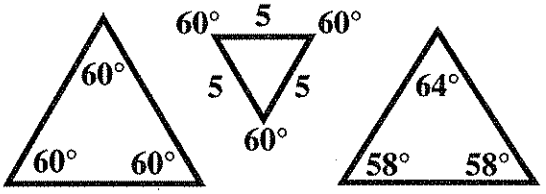


D.



Explanation:

A. Incorrect. Solve for the remaining angles.

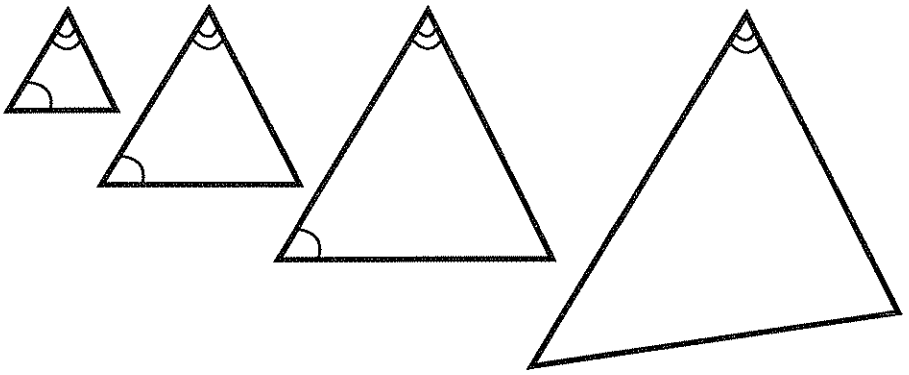


The first two triangles are similar, but the third one is not.

B. Incorrect Ratios of lengths of corresponding sides reveal that $\frac{1.4}{.7} \neq \frac{1.9}{1.4}$.

The triangles are not similar, since ratios of corresponding side lengths are not equal.

C. Incorrect. When the triangles are separated, they share a common vertex angle. The outside triangle does not share the third angle.



D. Correct. The vertical angles are congruent and the two marked angles are congruent. Hence, the third angles are congruent.

GEOMETRY & MEASUREMENT PRACTICE PROBLEMS

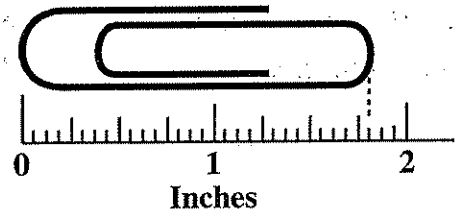
SKILLS		*PROBLEMS*
IB1	*Round measurements*	1–2
IVB2	*Solve word problems involving the Pythagorean theorem*	3–4
IIB4	*Identify appropriate units of measure*	5–8
IB2	*Calculate distances, areas, and volumes*	9–20
IVB1	*Solve word problems involving geometric figures*	21–24
IIIB2	*Identify formulas for measuring geometric figures*	25–27
IIIB1	*Infer formulas for measuring geometric figures*	28–29
IIB1	*Identify relationships between angle measures*	30–31
IIB2	*Identify names of plane figures given their properties*	32–34
IIB3	*Recognize similar triangles and their properties*	35–36

ROUND MEASUREMENTS

1. Round the measure 27.58 centimeters to the nearest centimeter.

   A. 27 centimeters
   B. 27.6 centimeters
   C. 28 centimeters
   D. 276 centimeters

2. Round the measurement of the length of the paper clip to the nearest $\frac{1}{4}$ inch.



   A. $1\frac{1}{4}$ inches

   B. $1\frac{5}{8}$ inches

   C. $1\frac{3}{4}$ inches

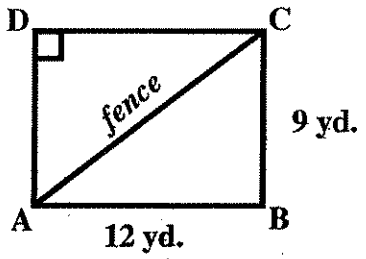
   D. 2 inches

SOLVE WORD PROBLEMS INVOLVING THE PYTHAGOREAN THEOREM

3. A tent is being set up for a group of people. A 12-foot pole is to be placed in the center. Four pieces of heavy-duty rope are to be attached to the top of the center pole and also attached to points on the ground that are 9 feet from the base of the pole. What is the total length of the four pieces of rope?

A. 15 feet
B. 45 feet
C. 50 feet
D. 60 feet

4. The owner of a rectangular piece of land 12 yards in length and 9 yards in width wants to divide it into two parts. He plans to join two opposite corners with a fence as shown in the diagram below. The cost of the fence will be approximately $40 per linear foot. What is the estimated cost for the fence needed by the owner?



A. $1260
B. $1800
C. $2520
D. $27,000

IDENTIFY APPROPRIATE UNITS OF MEASURE

5. Which of the following could be used to report the amount of carpet needed to cover a bedroom floor?

A. degrees
B. kilometers
C. square meters
D. cubic feet

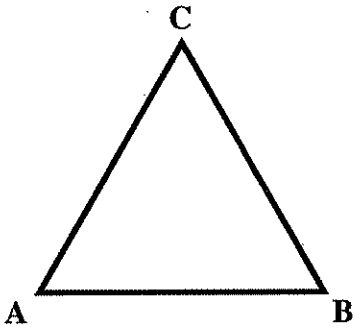
6. Which of the following would NOT be used to describe the amount of liquid contained in a can?

A. centimeters
B. gallons
C. liters
D. cubic feet

7. What measure could be used to report the distance traveled in running around a track?

A. degrees
B. square meters
C. kilometers
D. cubic feet

8. Which of the following would be used to report the measure of $\angle ACB$ in the following figure?



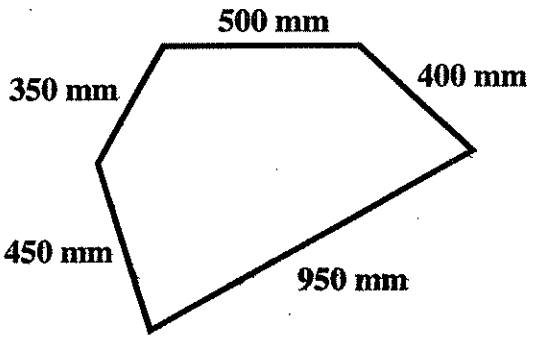
A. meters
B. degrees
C. liters
D. square feet

CALCULATE DISTANCES, AREAS, AND VOLUMES

9. What is the distance around a circular swimming pool that has a 9-foot radius?

A. 81π feet
B. 18π feet
C. 18π square feet
D. 16π feet

10. What is the distance around this polygon, in meters?



A. 2650 meters
B. 265 meters
C. 26.5 meters
D. 2.65 meters

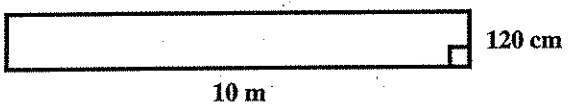
11. What is the area of a circular region whose diameter is 12 centimeters?

A. 12π square centimeters
B. 24π square centimeters
C. 36π square centimeters
D. 144π square centimeters

12. What is the area of a square whose side is 8 feet?

A. 32 feet
B. 32 square feet
C. 64 feet
D. 64 square feet

13. What is the area of this figure, in square meters?



A. 12 square meters
B. 22.4 square meters
C. 260 square meters
D. 1200 square meters

14. What is the area in square feet of a triangle whose base is 18 feet and whose height is 20 inches?

A. 15 square feet
B. 30 square feet
C. 180 square feet
D. 360 square feet

15. What is the surface area of a rectangular solid that is 15 inches long, 10 inches wide, and 5 inches high?

A. 550 inches
B. 550 square inches
C. 750 square inches
D. 750 cubic inches

16. What is the volume of a rectangular solid that is 15 inches long, 10 inches wide, and 5 inches high?

A. 750 cubic inches
B. 750 square inches
C. 720 cubic inches
D. 550 square inches

17. What is the volume of a right circular cylinder that has a radius of 6 inches and is 8 inches high?

A. 92π cubic inches
B. 288π cubic inches
C. 288π square inches
D. 368π cubic inches

18. What is the volume of a circular cone that has a radius of 12 inches and is 10 inches high?

A. 1440π cubic inches
B. 480π square inches
C. 480π cubic inches
D. 400π cubic inches

19. What is the volume of a sphere that has a radius of 6 feet?

A. 144π square feet
B. 144π cubic feet
C. 288π cubic feet
D. 288π square feet

20. What is the volume in centiliters of a 5.25-liter bottle?

A. .525 centiliters
B. 52.5 centiliters
C. 525 centiliters
D. 5250 centiliters

SOLVE WORD PROBLEMS INVOLVING GEOMETRIC FIGURES

21. A patio is to be built of concrete. The base of the patio is to be a slab of concrete 15 feet long by 12 feet wide by 6 inches thick. If one cubic yard of concrete costs $39, how much will the concrete for the patio cost?

A. $65
B. $130
C. $1560
D. $3510

22. A fence that costs $6.50 per yard is to be placed around a rectangular yard that is 90 feet by 120 feet. What is the total cost of the fence?

A. $ 910
B. $1365
C. $2730
D. $7800

23. A rectangular flower bed measures 5 feet by 54 inches. The outside dimensions of a path around the bed are $7\frac{1}{4}$ feet by 81 inches. What is the area of the path?

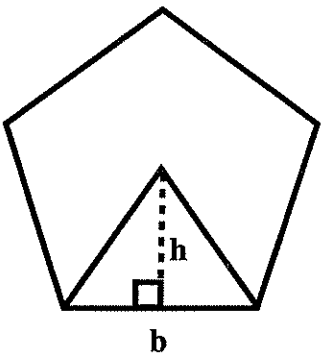
A. 29 square feet
B. 317 square feet
C. 317.25 square feet
D. 3807 square inches

24. The trunk of a tree has a 1.2-meter diameter. What is its circumference?

A. 0.36π square meters
B. 0.6π meters
C. 1.2π meters
D. 1.44π square meters

IDENTIFY FORMULAS FOR MEASURING GEOMETRIC FIGURES

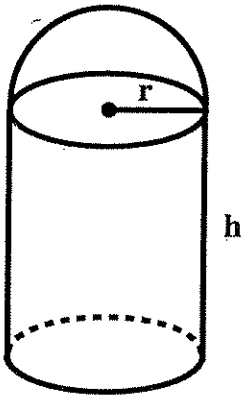
25. The figure below shows a regular pentagon.



Select the formula for computing the total area of the pentagon.

A. Area = $5bh$
B. Area = $2.5bh$
C. Area = $2.5h + b$
D. Area = $5(h + b)$

26. The figure below shows a water tank in the shape of a cylinder with a hemisphere on top.



Select the formula for calculating the volume of the tank.

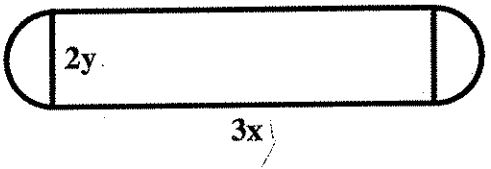
A. $\frac{4}{3}\pi r^3 + \pi r^2 h$

B. $\frac{2}{3}\pi r^3 + \pi r^2 h$

C. $\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

D. $4\pi r^3 + \pi r^2 h$

27. The figure below shows a running track having the shape of a rectangle with semicircles at each end.



Calculate the distance around the track.

A. $\pi y^2 + 6x$

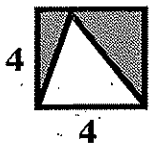
B. $\pi y^2 + 3x$

C. $2\pi y + 6x$

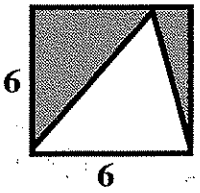
D. $6x + 4y + 2\pi y$

INFER FORMULAS FOR MEASURING GEOMETRIC FIGURES

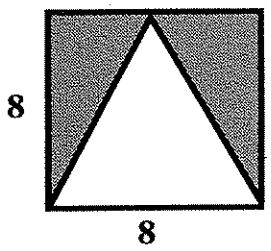
28. Study the given information. For each figure, *S* represents the shaded area when a triangle is removed from a square.



Side = 4
S = 8



Side = 6
S = 18

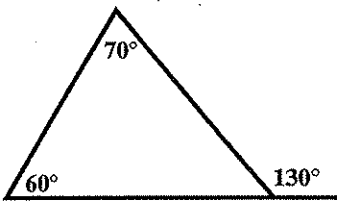


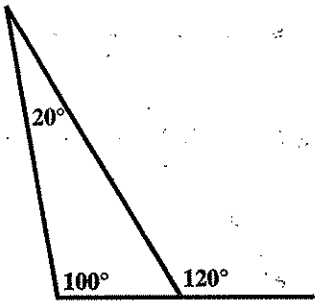
Side = 8
S = 32

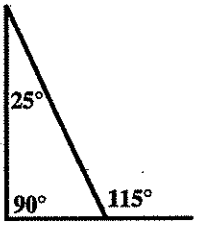
Calculate *S* if the side of the square is 10.

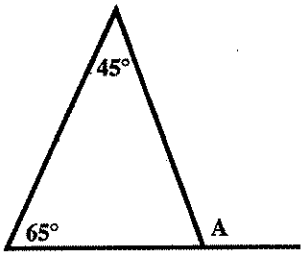
A. 40
B. 50
C. 75
D. 100

29. Study the information given in the figures.







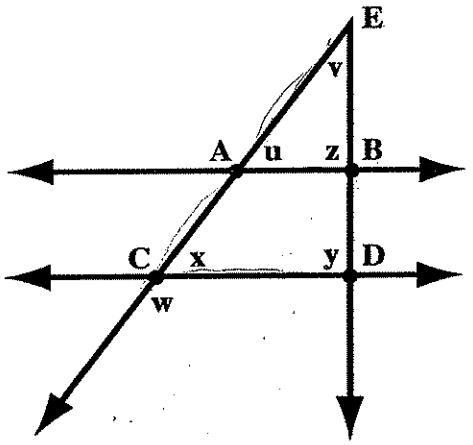


Calculate *A*, the exterior angle in the bottom figure.

A. 65°
B. 110°
C. 115°
D. 135°

IDENTIFY RELATIONSHIPS BETWEEN ANGLE MEASURES

30. Given that $\overline{AB} \parallel \overline{CD}$ and $\overline{BD} \perp \overline{CD}$, which of the following statements is true for the figure shown? (The measure of angle ACD is represented by x.)



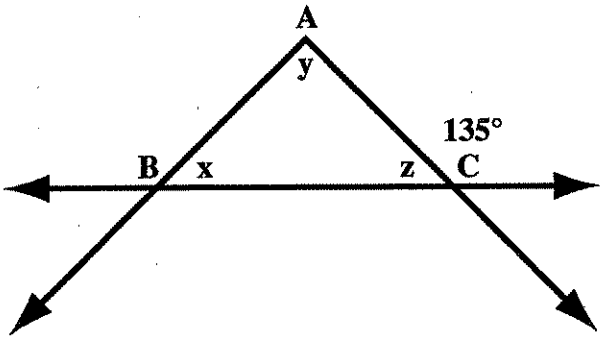
A. $w = v$

B. $\angle EAB$ and $\angle EBA$ are supplementary angles.

C. $\angle AEB$ and $\angle ECD$ are complementary angles.

D. $z = x$

31. In ΔABC, $\overline{AB} \cong \overline{AC}$, which of the following statements is true for the figure shown? (The measure of angle ABC is represented by x.)



A. ΔABC is equilateral.

B. $z = y$

C. $\angle ABC$ and $\angle ACB$ are supplementary angles.

D. $\overline{AB} \perp \overline{AC}$

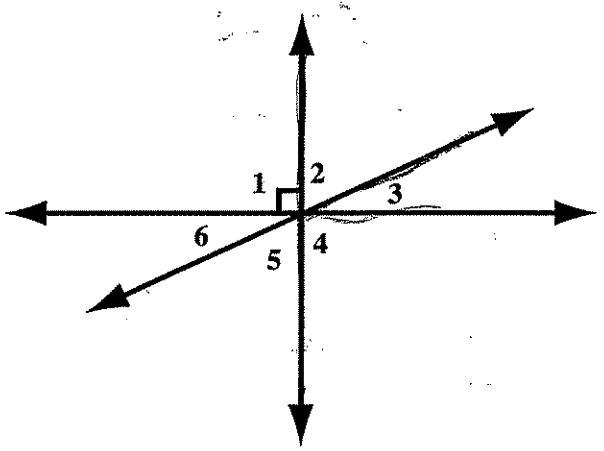
IDENTIFY NAMES OF PLANE FIGURES GIVEN THEIR PROPERTIES

32. Select the geometric figure that possesses all of the following characteristics.

i. four-sided
ii. adjacent sides equal and opposite sides parallel
iii. right angle

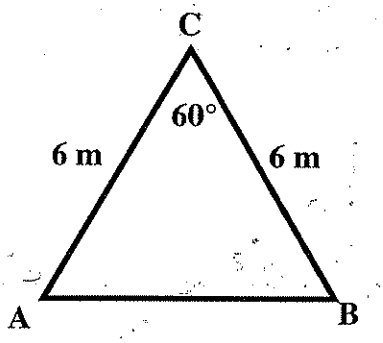
A. rectangle
B. square
C. trapezoid
D. rhombus

33. Which of the following pairs of angles are vertical?



A. 2 and 3
B. 5 and 6
C. 3 and 5
D. 2 and 5

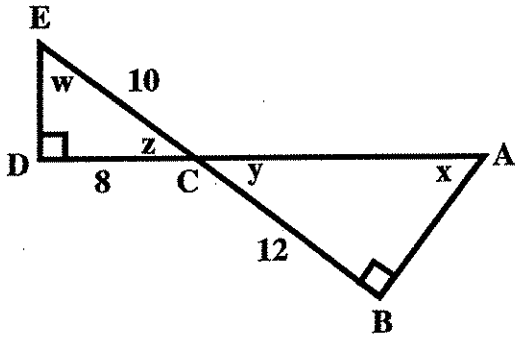
34. What type of triangle is ΔABC?



A. right
B. equilateral
C. scalene
D. obtuse

RECOGNIZE SIMILAR TRIANGLES AND THEIR PROPERTIES

35. Which of the statements is true for the pictured triangles? (The measure of angle A is represented by x.)



A. $AB = 9$

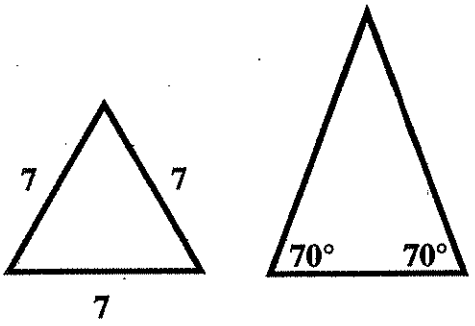
B. $x \neq w$

C. $\frac{ED}{DC} = \frac{BC}{AB}$

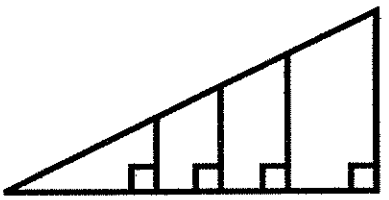
D. $\frac{CA}{EC} = \frac{CD}{BC}$

36. Study figures A, B, C, and D. Select the letter in which all triangles are similar.

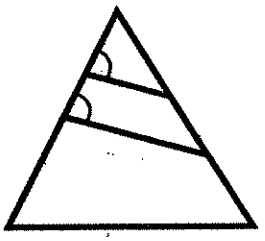
A.



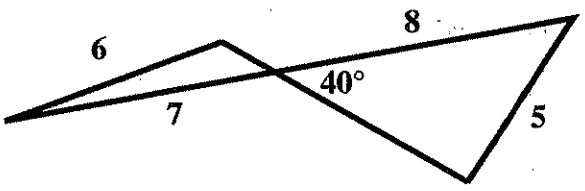
B.



C.



D.



GEOMETRY & MEASUREMENT PRACTICE EXPLANATIONS

SKILLS		*PROBLEMS*
IB1	*Round measurements*	1–2
IVB2	*Solve word problems involving the Pythagorean theorem*	3–4
IIB4	*Identify appropriate units of measure*	5–8
IB2	*Calculate distances, areas, and volumes*	9–20
IVB1	*Solve word problems involving geometric figures*	21–24
IIIB2	*Identify formulas for measuring geometric figures*	25–27
IIIB1	*Infer formulas for measuring geometric figures*	28–29
IIB1	*Identify relationships between angle measures*	30–31
IIB2	*Identify names of plane figures given their properties*	32–34
IIB3	*Recognize similar triangles and their properties*	35–36

Geometry & Measurement Practice Explanations — Skill IB1

ROUND MEASUREMENTS

1. ***C*** is the correct response. In ***A***, 27.58 was rounded to 27 when it is actually closer to 28. In ***B***, 27.6 was the result of rounding to the nearest tenth of a centimeter instead of the nearest centimeter. In ***D***, the decimal point was misplaced.

2. ***C*** is the correct response. In ***A***, the diagram was not read correctly. In ***B***, the answer was not rounded to the specified unit of $\frac{1}{4}$ inch. In ***D***, the answer was not rounded to the specified unit of $\frac{1}{4}$ inch.

Geometry & Measurement Practice Explanations — Skill IVB2

SOLVE WORD PROBLEMS INVOLVING THE PYTHAGOREAN THEOREM

3. ***D*** is the correct response. In ***A***, only one of the lengths was found. In ***B***, only three of the lengths were found instead of four. In ***C***, the estimation was incorrect and an error was made in multiplication.

4. ***B*** is the correct response. In ***A***, 12 and 9 were added and then divided by 2; the sum was multiplied by 3 and then the product multiplied by $40. In ***C***, 12 and 9 were added, the sum multiplied by 3, and the product multiplied by $40. In ***D***, the square of 9 and the square of 12 were added. The sum was then multiplied by 3 and the product was multiplied by $40.

Geometry & Measurement Practice Explanations — Skill IIB4

IDENTIFY APPROPRIATE UNITS OF MEASURE

5. ***C*** is the correct response. Square meters are used to measure area. In ***A***, degrees are used to measure angles. In ***B***, kilometers are used to measure distances. In ***D***, cubic feet are used to measure volume.

6. ***A*** is the correct response. Centimeters are used to measure distance, NOT volume. In ***B***, gallons are used to measure volume. In ***C***, liters are used to measure volume. In ***D***, cubic feet are used to measure volume.

7. ***C*** is the correct response. Kilometers are used to measure distance. In ***A***, degrees are used to measure angles. In ***B***, square meters are used to measure area. In ***D***, cubic feet are used to measure volume.

8. ***B*** is the correct response. Degrees are used to measure angles. In ***A***, meters are used to measure distance or length of line segments. In ***C***, liters are used to measure capacity or volume. In ***D***, square feet are used to measure area.

CALCULATE DISTANCES, AREAS, AND VOLUMES

9. ***B*** is the correct response. In ***A***, the formula for area was used to get the numerical value. In ***C***, an incorrect unit of measurement (square feet) was used with the correct numerical value. In ***D***, a computational error was made.

10. ***D*** is the correct response. In ***A***, the sum of one side was obtained using the incorrect measurement (mm). In ***B***, the conversion was applied incorrectly and the decimal point was not moved enough places. In ***C***, the conversion was applied incorrectly and the decimal point was not moved enough places.

11. ***C*** is the correct response. In ***A***, the wrong formula was used. In ***B***, the wrong formula was used and the diameter, not the radius, was used. In ***D***, the diameter, not the radius, was used.

12. ***D*** is the correct response. In ***A***, the wrong formula was used and the wrong unit was given. In ***B***, the wrong formula was used even though the correct unit was given. In ***C***, the correct formula was used but the incorrect unit was given.

13. ***A*** is the correct response. In ***B***, the wrong formula was used. In ***C***, the wrong formula was used and centimeters were not converted to meters. In ***D***, the correct formula was used but the conversion was not made.

14. ***A*** is the correct response. In ***B***, the $b \times h$ was not multiplied by $\frac{1}{2}$. In ***C***, the correct formula was used but the inches were not converted to feet. In ***D***, the formula was used incorrectly and the conversion was not made.

15. ***B*** is the correct response. In ***A***, the correct formula was used but the wrong units were given. In ***C***, the wrong formula was used even though the correct units were used. In ***D***, the wrong formula was used.

16. ***A*** is the correct response. In ***B***, the correct formula was used but the wrong units were used. In ***C***, a computational error was made. In ***D***, the incorrect formula was used.

17. ***B*** is the correct response. In ***A***, the wrong formula was used. In ***C***, the wrong units were used. In ***D***, the wrong formula was used.

18. ***C*** is the correct response. In ***A***, the wrong formula was used. In ***B***, the wrong units were used. In ***D***, a computational error was made.

19. ***C*** is the correct response. In ***A***, the wrong formula was used and the wrong units were used. In ***B***, the wrong formula was used. In ***D***, the wrong units were used.

20. ***C*** is the correct response. In ***A***, the wrong formula was used. In ***B***, the wrong formula was used. In ***D***, the wrong formula was used.

SOLVE WORD PROBLEMS INVOLVING GEOMETRIC FIGURES

21. ***B*** is the correct response. In ***A***, the wrong formula was used. In ***C***, 6 inches should have been changed to yards, not feet. In ***D***, the measurements should have been changed to yards.

22. ***A*** is the correct response. In ***B***, the wrong formula for the perimeter of a rectangle was used (210 feet) and the units were not changed to yards. In ***C***, the units were not changed to yards. In ***D***, the area was found instead of the perimeter.

23. ***D*** is the correct response. In ***A***, estimating was incorrect. In ***B***, estimating was incorrect. In ***C***, units were not converted to feet.

24. ***C*** is the correct response. In ***A***, the area was found. In ***B***, the wrong formula for circumference was used. In ***D***, the wrong formula was used.

IDENTIFY FORMULAS FOR MEASURING GEOMETRIC FIGURES

25. ***B*** is the correct response. The regular pentagon has 5 equal triangles. Therefore, $A = 5\left(\frac{1}{2}bh\right) = (2.5)bh$. In ***A***, ***C***, and ***D***, the wrong formula was used for the area of a triangle.

26. ***B*** is the correct response. The volume of the hemisphere on the top is one-half the volume of a sphere or $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$. The volume of the cylinder is $\pi r^2 h$. Therefore, the total volume is $\frac{2}{3}\pi r^3 + \pi r^2 h$. In ***A***, the volume of the sphere was not halved to find the volume of a hemisphere. In ***C***, the wrong formula was used for the cylinder. In ***D***, the wrong formula was used for the hemisphere.

27. ***C*** is the correct response. The two semicircles form a circle of circumference $\pi d = \pi(2y) = 2\pi y$. The two sides of the rectangle measure $3x + 3x = 6x$. So, the total perimeter of the track is $2\pi y + 6x$. In ***A***, the formula for the circle's area (πr^2) was used instead of the formula for its circumference. In ***B***, the formula for the circle's area was used and only one side of the rectangle was considered. In ***D***, the length of the ends of the rectangle (4y) was incorrectly included. The perimeter of the track does not include the ends of the rectangle.

INFER FORMULAS FOR MEASURING GEOMETRIC FIGURES

28. ***B*** is the correct response. $S = \frac{1}{2}(\text{side})^2 = \frac{1}{2}(10)\,(10) = 50$. In ***A***, the formula for the perimeter of the square was used without subtracting the area of the triangle. In ***C***, the area of the triangle was assumed to be $A = \frac{1}{2}(5)(10)$ and was subtracted from the area of the square. In ***D***, the area of the triangle was not subtracted from the area of the square.

29. ***B*** is the correct response. The exterior angle equals the sum of the two given angles. In ***A***, $\angle A$ was incorrectly assumed to be the alternate exterior angle of the 65° angle of the triangle. In ***C***, $\angle A$ was incorrectly assumed to be a supplementary angle of the 65° angle of the triangle and 65° was subtracted from 180°. In ***D***, $\angle A$ was incorrectly assumed to be a supplementary angle of the 45° angle of the triangle and 45° was subtracted from 180°.

IDENTIFY RELATIONSHIPS BETWEEN ANGLE MEASURES

30. ***C*** is the correct response. In ***A***, w is an obtuse angle and v is an acute angle; therefore, $w \neq v$. In ***B***, $\angle EBA$ is a right angle and $\angle EAB$ is an acute angle. Therefore, the sum is less than 180°. In ***D***, $z = 90°$ since $\overline{BD} \perp \overline{CD}$ and x is an acute angle.

31. ***D*** is the correct response. In ***A***, ΔABC is an isosceles triangle. In ***B***, $z = 45°$, $x = 45°$, and $y = 90°$, so $z \neq y$. In ***C***, $\angle ABC$ and $\angle ACB$ are complementary angles.

IDENTIFY NAMES OF PLANE FIGURES GIVEN THEIR PROPERTIES

32. ***B*** is the correct response. In ***A***, the adjacent sides in a rectangle are not necessarily equal. In ***C***, a trapezoid has only one pair of opposite sides parallel. In ***D***, a rhombus does not necessarily have a right angle.

33. ***D*** is the correct response. In ***A***, these angles are complementary and adjacent. In ***B***, these angles are complementary and adjacent. In ***C***, these angles are complementary.

34. ***B*** is the correct response. In ***A***, the triangle has no right angle. In ***C***, the triangle has all sides the same measure. In ***D***, there is no angle larger than 90°.

RECOGNIZE SIMILAR TRIANGLES AND THEIR PROPERTIES

35. ***A*** is the correct response. In ***B***, $x = w$. In ***C***, $\frac{ED}{DC} = \frac{AB}{BC}$. In ***D***, $\frac{EC}{CA} = \frac{CD}{BC}$.

36. ***B*** is the correct response. In ***A***, the first triangle is equilateral and the second is isosceles but not equilateral. In ***C***, the base of the largest triangle is not parallel to the bases of the other two triangles. In ***D***, based on the given information we can only be sure of one pair of congruent angles.