

# CHAPTER 1

# ARITHMETIC

<i>SKILLS</i>	<i>PAGE</i>
<i>Overview</i>	4
<i>IA1 Add, subtract, multiply, and divide fractions</i>	6
<i>IA2 Add, subtract, multiply, and divide decimals</i>	13
<i>IIA1 Recognize the meaning of exponents</i>	19
<i>IIA2 Identify place value and use expanded notation</i>	20
<i>IIA3 Identify equivalent decimals, percents, and fractions</i>	24
<i>IIA4 Determine the order relationship between two real numbers</i>	28
<i>IA4 Solve for one variable of the sentence (<math>a\%</math> of <math>b = c</math>)</i>	31
<i>IA3 Calculate percent increase and decrease</i>	34
<i>IVA1 Solve word problems without variables or percents</i>	36
<i>IVA2 Solve word problems involving percents</i>	39
<i>IVA3 Solve problems involving the structure and logic of arithmetic</i>	41
<i>IIA5 Estimate sums, averages, and products</i>	44
<i>IIIA1 Find missing numbers given a pattern</i>	46
 <i>PRACTICE PROBLEMS</i>	 49
 <i>PRACTICE EXPLANATIONS</i>	 65

---



Arithmetic is a branch of mathematics that deals with the operations of addition, subtraction, multiplication, and division. Mastering arithmetic skills is necessary for many situations people encounter in their daily lives, such as balancing a checkbook, making purchases, and measuring objects. These skills are the foundation for advanced study in many fields, such as statistics, engineering, business, and the sciences.

The titles of the units found in this chapter are listed below in italics. Under each unit title is listed the specific arithmetic skill or skills covered in that unit.

*Add, subtract, multiply, and divide fractions.*

Skill IA1a: The student will add and subtract rational numbers.

Skill IA1b: The student will multiply and divide rational numbers.

*Add, subtract, multiply, and divide decimals.*

Skill IA2a: The student will add and subtract rational numbers in decimal form.

Skill IA2b: The student will multiply and divide rational numbers in decimal form.

*Recognize the meaning of exponents.*

Skill IIA1: The student will recognize the meaning of exponents.

*Identify place value and use expanded notation.*

Skill IIA2: The student will recognize the role of the base number in determining place value in the base-ten numeration system.

*Identify equivalent decimals, percents, and fractions.*

Skill IIA3: The student will identify equivalent forms of positive rational numbers involving decimals, percents, and fractions.

*Determine the order relationship between two real numbers.*

Skill IIA4: The student will determine the order relation between real numbers.

*Solve for one variable of the sentence ( $a\%$  of  $b = c$ ).*

Skill IA4: The student will solve the sentence  $a\%$  of  $b$  is  $c$ , where values for two of the variables are given.

*Calculate percent increase and decrease.*

Skill IA3: The student will demonstrate the ability to calculate percent increase and percent decrease.

*Solve word problems without variables or percents.*

Skill IVA1: The student solves real-world problems which do not require the use of variables and which do not involve percent.

*Solve word problems involving percents.*

Skill IVA2: The student solves real-world problems which do not require the use of variables but do require the use of percent.

*Solve problems involving the structure and logic of arithmetic.*

Skill IVA3: The student solves problems that involve the structure and logic of arithmetic.

*Estimate sums, averages, and products.*

Skill IIA5: The student will identify a reasonable estimate of a sum, average, or product of numbers.

*Find missing numbers given a pattern.*

Skill IIIA1: The student infers relations between numbers in general by examining particular number pairs.

## ADD AND SUBTRACT FRACTIONS

A brief review of some key definitions may be useful.

A **fraction** is a numerical expression in the form  $\frac{a}{b}$ , where  $b$  is not equal to 0 (for example,  $\frac{2}{3}$ ).

In such an expression  $a$  is called the **numerator** and  $b$  is called the **denominator**. An **improper fraction** is a fraction with a numerator larger than its denominator. To **reduce** a fraction means to divide its numerator and denominator by a common factor greater than 1. The **least common denominator (LCD)** is the least common multiple of two or more denominators.

A **whole number** is any non-negative number not having fractional parts (for example, 0, 1, 2, 3, . . .). An **integer** is any number that is a whole number or negative number in the set { . . . -3, -2, -1, 0, 1, 2, 3, . . .}. A **mixed number** is a number composed of an integer and a fraction

(for example,  $2\frac{3}{4}$ ). Fractions that are to be added or subtracted must have a common denominator. Follow the steps below to work the problems:

- Step 1.** Express all numbers as fractions. If the problem includes mixed numbers, convert them to improper fractions.
- Step 2.** Find a common denominator.
- Step 3.** Form equivalent fractions, using the common denominator.
- Step 4.** Add or subtract numerators, keeping the same denominator.
- Step 5.** Reduce the fraction. Convert improper fractions to mixed numbers.

**Reminder:** The **absolute value** of a number is the distance the number is from 0 on a real number line; for example,  $|2| = 2$  and  $|-2| = 2$ . When subtracting a negative number, change the sign to positive and add. When adding numbers with unlike signs, subtract the smaller absolute value from the larger absolute value and use the sign of the number larger in absolute value.

## Examples

1.  $2\frac{2}{5} + 3\frac{4}{7} =$

**Step 1.** Express all numbers as fractions. Convert mixed numbers to improper fractions.

$$2\frac{2}{5} = \frac{2(5)+2}{5} = \frac{10+2}{5} = \frac{12}{5}$$



**Skill IA1****Add, subtract, multiply, and divide fractions**

$$3\frac{4}{7} = \frac{3(7)+4}{7} = \frac{21+4}{7} = \frac{25}{7}$$

*Step 2.* Find a common denominator. The LCD is the smallest number that both 5 and 7 will divide into with no remainder. The LCD for 5 and 7 is 35.

*Step 3.* Form equivalent fractions, using the LCD.

$$\frac{12}{5} = \frac{12(7)}{5(7)} = \frac{84}{35}$$

$$\frac{25}{7} = \frac{25(5)}{7(5)} = \frac{125}{35}$$

*Step 4.* Add the numerators, keeping the same denominators.

$$\frac{84+125}{35} = \frac{209}{35}$$

*Step 5.* Reduce the fraction. Convert the improper fraction to a mixed number.

Because  $\frac{209}{35}$  is already in reduced form, change to a mixed number by dividing.

$$\begin{array}{r} 5 \\ 35 \overline{)209} \\ \underline{175} \\ 34 \end{array} \quad \text{Therefore, } \frac{209}{35} = 5\frac{34}{35}$$

2.  $-\frac{2}{3} + \frac{1}{5} =$

*Step 1.* This problem does not include a mixed or whole number, so proceed to Step 2.

*Step 2.* Find a common denominator. The LCD for 3 and 5 is 15.

*Step 3.* Form equivalent fractions, using the LCD.

$$-\frac{2}{3} = -\frac{2(5)}{3(5)} = -\frac{10}{15}$$

$$\frac{1}{5} = \frac{1(3)}{5(3)} = \frac{3}{15}$$

**Skill IA1****Add, subtract, multiply, and divide fractions**

*Step 4.* Add the numerators. Because the signs are different, subtract 3 from 10 and keep the negative sign. Keep the same denominators.

$$-\frac{10}{15} + \frac{3}{15} = -\frac{7}{15}$$

*Step 5.* Reduce the fraction. Convert the improper fraction to a mixed number.

Because  $-\frac{7}{15}$  is already in reduced form,  $-\frac{2}{3} + \frac{1}{5} = -\frac{7}{15}$ .

3.  $-8 + 1\frac{3}{4} =$

*Step 1.* Express all numbers as fractions. Convert mixed numbers and integers to improper fractions.

The integer  $-8$  can be considered as the improper fraction  $-\frac{8}{1}$ .

$$1\frac{3}{4} = \frac{1(4) + 3}{4} = \frac{7}{4}$$

*Step 2.* Find a common denominator. The LCD for 1 and 4 is 4.

*Step 3.* Form equivalent fractions, using the LCD.

$$-\frac{8}{1} = -\frac{8(4)}{1(4)} = -\frac{32}{4}$$

$$1\frac{3}{4} = \frac{1(4) + 3}{4} = \frac{7}{4}$$

*Step 4.* Add the numerators, keeping the same denominators.

$$\frac{-32 + 7}{4} = -\frac{25}{4}$$

*Step 5.* Reduce the fraction. Convert improper fractions to mixed numbers.

$$\begin{array}{r} 6 \\ 4 \overline{)25} \\ \underline{24} \\ 1 \end{array} \quad \text{Therefore, } -\frac{25}{4} = -6\frac{1}{4}.$$

**Skill IA1****Add, subtract, multiply, and divide fractions**

4.  $-\frac{1}{4} - \left(-4\frac{7}{8}\right) =$       **Reminder:** Change the signs.       $-\frac{1}{4} + 4\frac{7}{8} =$

*Step 1.* Express all numbers as fractions. Convert mixed numbers to improper fractions.

$$4\frac{7}{8} = \frac{4(8)+7}{8} = \frac{39}{8} \qquad -\frac{1}{4} \text{ is already a fraction.}$$

*Step 2.* Find a common denominator. The LCD for 4 and 8 is 8.

*Step 3.* Form equivalent fractions, using the LCD.

$$-\frac{1}{4} = -\frac{1(2)}{4(2)} = -\frac{2}{8}$$

$$\frac{39}{8} = \frac{39}{8}$$

*Step 4.* Add the numerators, keeping the same denominators.

$$-\frac{2}{8} + \frac{39}{8} = \frac{-2+39}{8} = \frac{37}{8}$$

*Step 5.* Reduce the fraction. Convert improper fractions to mixed numbers.

$$\begin{array}{r} 4 \\ 8 \overline{)37} \\ \underline{32} \\ 5 \end{array} \qquad \text{Therefore, } \frac{37}{8} = 4\frac{5}{8}.$$

**MULTIPLY FRACTIONS**

To multiply fractions, use the following procedure:

*Step 1.* Convert mixed or whole numbers to improper fractions.

*Step 2.* Reduce (cancel) common factors of numerators with denominators.

*Step 3.* Multiply the numerators and multiply the denominators.

*Step 4.* Reduce the fraction. Convert an improper fraction to a mixed number.

**Reminder:** The **product** (the answer resulting from multiplication) of two negative numbers is a positive number. The product of a positive and a negative number is a negative number.

**Skill IA1****Add, subtract, multiply, and divide fractions**

---

**Examples**

5.  $4\frac{2}{3} \times \frac{3}{4} =$

*Step 1.* Convert mixed or whole numbers to improper fractions.

$$4\frac{2}{3} = \frac{4(3)+2}{3} = \frac{14}{3}$$

*Step 2.* Reduce (cancel) common factors, where possible.

$$\frac{\cancel{14}^7}{\cancel{3}_1} \times \frac{\cancel{3}_1}{\cancel{4}_2} = \frac{7}{1} \times \frac{1}{2}$$

*Step 3.* Multiply the numerators and multiply the denominators.

$$\frac{7}{1} \times \frac{1}{2} = \frac{7}{2}$$

*Step 4.* Convert the improper fraction to a mixed number.

$$2\overline{)7\frac{3}{2}} \quad \text{Therefore, } \frac{7}{2} = 3\frac{1}{2}$$

6.  $-\frac{3}{5} \times \frac{4}{6} =$

*Step 1.* This problem does not include mixed or whole numbers, so proceed to Step 2.

*Step 2.* Reduce (cancel) common factors, where possible.

$$-\frac{\cancel{3}_1}{5} \times \frac{4}{\cancel{6}_2} = -\frac{1}{5} \times \frac{\cancel{4}^2}{\cancel{2}_1} = -\frac{1}{5} \times \frac{2}{1}$$

*Step 3.* Multiply the numerators and multiply the denominators.

$$-\frac{1}{5} \times \frac{2}{1} = -\frac{2}{5}$$

*Step 4.* The fraction cannot be reduced.

$$-\frac{2}{5}$$

**DIVIDE FRACTIONS**

To divide fractions, use the following procedure:

- Step 1.** Convert mixed or whole numbers to improper fractions.
- Step 2.** Rewrite the problem by replacing the division symbol with a multiplication symbol and inverting the fraction on the right.
- Step 3.** Follow the steps for multiplying fractions.
- Reduce common factors, where possible.
  - Multiply the numerators and multiply the denominators.
  - Reduce the fraction. Convert an improper fraction to a mixed number.

**Examples**

7.  $\frac{3}{4} \div \frac{9}{11} =$

**Step 1.** This problem does not include mixed or whole numbers, so proceed to Step 2.

**Step 2.** Rewrite the problem by replacing the division symbol with a multiplication symbol and inverting the fraction on the right.

$$\frac{3}{4} \div \frac{9}{11} = \frac{3}{4} \times \frac{11}{9}$$

**Step 3.** Follow the steps for multiplying fractions.

a. Reduce common factors, where possible.

$$\overset{1}{\cancel{3}} \frac{\cancel{9}}{4} \times \frac{11}{\cancel{3}} = \frac{1}{4} \times \frac{11}{3}$$

b. Multiply the numerators and multiply the denominators.

$$\frac{1}{4} \times \frac{11}{3} = \frac{11}{12}$$

c. The fraction cannot be reduced.

$$\frac{11}{12}$$

8.  $-\frac{4}{5} \div -2\frac{1}{2} =$

*Step 1.* Convert mixed or whole numbers to improper fractions.

$$-2\frac{1}{2} = -\frac{2(2)+1}{2} = -\frac{5}{2}$$

*Step 2.* Rewrite the problem by replacing the division symbol with a multiplication symbol and inverting the fraction on the right.

$$-\frac{4}{5} \div -\frac{5}{2} = -\frac{4}{5} \times -\frac{2}{5} =$$

*Step 3.* Follow the steps for multiplying fractions.

a. There are no common factors.

b. Multiply the numerators and multiply the denominators.

$$-\frac{4}{5} \times -\frac{2}{5} = \frac{8}{25}$$

c. The fraction cannot be reduced.

$$\frac{8}{25}$$

**ADD AND SUBTRACT NUMBERS IN DECIMAL FORM**

To add or subtract numbers in decimal form, use the following procedure:

**Step 1.** Arrange the numbers vertically, lining up the decimal points. If necessary, add zeros to get an equal number of decimal places.

**Step 2.** Add or subtract, beginning with the right-hand column. Remember to follow the rules for combining signed numbers (see page 3).

**Examples**

1.  $8.4 + 0.02 =$

**Step 1.** Arrange the numbers vertically, lining up the decimal points. Add zeros to get an equal number of decimal places.

$$\begin{array}{r} 8.40 \\ + 0.02 \\ \hline \end{array}$$

**Step 2.** Add, beginning with the right-hand column.

$$\begin{array}{r} 8.40 \\ + 0.02 \\ \hline 8.42 \end{array}$$

2.  $-2.46 - 7.7 =$

**Step 1.** Arrange the numbers vertically, lining up the decimal points. Add zeros to get an equal number of decimal places.

$$\begin{array}{r} -2.46 \\ - 7.70 \\ \hline \end{array}$$

**Step 2.** Add, beginning with the right-hand column. Remember to follow the rules for combining signed numbers.

$$\begin{array}{r} -2.46 \\ + -7.70 \\ \hline -10.16 \end{array}$$

**Skill IA2*****Add, subtract, multiply, and divide decimals***

---

3.  $3.58 - 6 =$

*Step 1.* Arrange the numbers vertically, lining up the decimal points. Add zeros to get an equal number of decimal places.

$$\begin{array}{r} 3.58 \\ - 6.00 \\ \hline \end{array}$$

*Step 2.* Subtract, beginning with the right-hand column. Remember to follow the rules for combining signed numbers.

$$\begin{array}{r} -6.00 \\ + 3.58 \\ \hline -2.42 \end{array}$$

4.  $6.49 + 1.76 =$

*Step 1.* Arrange the numbers vertically, lining up the decimal points. If necessary, add zeros to get an equal number of decimal places.

$$\begin{array}{r} 6.49 \\ + 1.76 \\ \hline \end{array}$$

*Step 2.* Add, beginning with the right-hand column. Remember to follow the rules for combining signed numbers.

$$\begin{array}{r} 6.49 \\ + 1.76 \\ \hline 8.25 \end{array}$$

**MULTIPLY NUMBERS IN DECIMAL FORM**

To multiply numbers in decimal form, use the following procedure:

*Step 1.* Arrange the numbers vertically. It is not necessary to line up the decimal points.

*Step 2.* Multiply the numbers, ignoring the decimal points.

*Step 3.* Count the total number of digits to the right of the decimal point in both numbers, and add the total decimal places of the two numbers.

*Step 4.* Place the decimal point in the answer so that the number of digits to the right of this decimal point is the same as the total obtained in Step 3. This may require inserting zeros as placeholders.



**Skill IA2*****Add, subtract, multiply, and divide decimals***

---

**Examples**

5.  $0.007 \times 0.9 =$

*Step 1.* Arrange the numbers vertically. It is not necessary to line up the decimal points.

$$\begin{array}{r} 0.007 \\ \times 0.9 \\ \hline \end{array}$$

*Step 2.* Multiply the numbers, ignoring the decimal points.

$$\begin{array}{r} 0.007 \\ \times 0.9 \\ \hline 63 \end{array}$$

*Step 3.* Count the total number of digits to the right of the decimal point in both numbers, and add the total decimal places of the two numbers.

This total is 4.

*Step 4.* Place the decimal point in the answer 4 places from the right, inserting zeros as placeholders, as needed.

$$\begin{array}{r} 0.007 \\ \times 0.9 \\ \hline .0063 \end{array}$$

6.  $-3.2 \times 0.13 =$

*Step 1.* Arrange the numbers vertically. It is not necessary to line up the decimal points.

$$\begin{array}{r} -3.2 \\ \times 0.13 \\ \hline \end{array}$$

*Step 2.* Multiply the numbers, ignoring the decimal points.

$$\begin{array}{r} -3.2 \\ \times 0.13 \\ \hline 96 \\ \underline{32} \\ -416 \end{array}$$

*Step 3.* Count the total number of digits to the right of the decimal point in both numbers, and add the total decimal places of the two numbers.

This total is 3.

**Skill IA2*****Add, subtract, multiply, and divide decimals***

---

*Step 4.* Place the decimal point in the answer 3 places from the right, inserting zeros as placeholders, as needed.

$$\begin{array}{r} -3.2 \\ \times 0.13 \\ \hline -0.416 \end{array}$$

7.  $9 \times 0.025 =$

*Step 1.* Arrange the numbers vertically. It is not necessary to line up the decimal points.

$$\begin{array}{r} 9 \quad \text{or} \quad 0.025 \\ \times 0.025 \quad \quad \times 9 \\ \hline \end{array}$$

*Step 2.* Multiply the numbers, ignoring the decimal points.

$$\begin{array}{r} 0.025 \\ \times 9 \\ \hline 225 \end{array}$$

*Step 3.* Count the total number of digits to the right of the decimal point in both numbers, and add the total decimal places of the two numbers.

This total is 3.

*Step 4.* Place the decimal point in the answer 3 places from the right, inserting zeros as placeholders, as needed.

$$\begin{array}{r} 0.025 \\ \times 9 \\ \hline 0.225 \end{array}$$

**DIVIDE NUMBERS IN DECIMAL FORM**

To divide numbers in decimal form, use the following procedure:

*Reminder:*      $\text{divisor} \overline{) \text{dividend}}$  <sup>quotient</sup>

*Step 1.*     If the division symbol ( $\div$ ) is used, rewrite the problem as follows:  $\text{divisor} \overline{) \text{dividend}}$  <sup>quotient</sup>.

Thus,  $a \div b$  would become  $b \overline{) a}$ .

*Step 2.*     Move the decimal point of the divisor ( $b$ ) to the right to make it a whole number.  
Move the decimal point in the dividend ( $a$ ) the same number of places to the right.

*Step 3.*     Divide.

*Step 4.*     Place the decimal point in the quotient so that it is directly above the decimal place in the dividend. Insert zeros as placeholders, as needed.

**Examples**

8.      $1.4 \div 0.7 =$

*Step 1.*     If the division symbol ( $\div$ ) is used, rewrite the problem.

$$0.7 \overline{) 1.4}$$

*Step 2.*     Move the decimal point in 0.7 one place to the right to make it a whole number.  
Move the decimal point in 1.4 the same number of places to the right.

$$0.7 \overline{) 1.4} \rightarrow 0.7 \overline{) 1.4} \rightarrow 7 \overline{) 14}$$

*Step 3.*     Divide.

$$7 \overline{) 14} \begin{array}{r} 2 \\ \end{array}$$

*Step 4.*     It is not necessary to place a decimal point in this problem.

$$7 \overline{) 14} \begin{array}{r} 2 \\ \end{array}$$

**Skill IA2****Add, subtract, multiply, and divide decimals**

---

9.  $0.14 \div 7 =$

*Step 1.* If the division symbol ( $\div$ ) is used, rewrite the problem.

$$7 \overline{)0.14}$$

*Step 2.* Since the divisor, 7, is a whole number, there is no need to move the decimal point.

$$7 \overline{)0.14}$$

*Step 3.* Divide.

$$\begin{array}{r} 2 \\ 7 \overline{)0.14} \end{array}$$

*Step 4.* Place the decimal point in the quotient so that it is directly above the decimal point in the dividend. Insert zeros as placeholders.

$$\begin{array}{r} 0.02 \\ 7 \overline{)0.14} \end{array}$$

10.  $14 \div 0.07 =$

*Step 1.* If the division symbol ( $\div$ ) is used, rewrite the problem.

$$0.07 \overline{)14}$$

*Step 2.* Move the decimal point in 0.07 two places to the right to make it a whole number. Move the decimal point in 14 the same number of places to the right.

$$0.07 \overline{)14} \rightarrow 0.07 \overline{)14.00} \rightarrow 7 \overline{)1400}$$

*Step 3.* Divide.

$$\begin{array}{r} 200 \\ 7 \overline{)1400} \end{array}$$

*Step 4.* It is not necessary to place a decimal point in this problem.

$$\begin{array}{r} 200 \\ 7 \overline{)1400} \end{array}$$

**RECOGNIZE THE MEANING OF EXPONENTS**

An **exponent** indicates how many times a base number is to be used as a factor (multiplier). An expression in the form  $b^n$  is in exponential form. The  $b$  is the base. The  $n$  is the exponent.

For example, the expanded form of  $4^5$  is  $\underbrace{4 \times 4 \times 4 \times 4 \times 4}_{5 \text{ times}}$ .

**Examples**

1.  $5^3 - 2^5 =$

$$5^3 - 2^5 = (5 \times 5 \times 5) - (2 \times 2 \times 2 \times 2 \times 2)$$

2.  $7^2 \div 3^3 =$

$$7^2 \div 3^3 = (7 \times 7) \div (3 \times 3 \times 3)$$

3.  $(6^2)^3 =$

$$(6^2)^3 = 6^2 \times 6^2 \times 6^2 = (6 \times 6) \times (6 \times 6) \times (6 \times 6)$$

4.  $7(2^4) =$

$$7(2^4) = 7 \times (2 \times 2 \times 2 \times 2)$$

**Skill IIA2***Identify place value and use expanded notation***IDENTIFY PLACE VALUE AND USE EXPANDED NOTATION**

The following charts illustrate place value and the use of exponential notation (in base 10).

Digits to the LEFT of the decimal point are represented as follows:

<i>trillions</i>	<i>hundred billions</i>	<i>ten billions</i>	<i>billions</i>	<i>hundred millions</i>	<i>ten millions</i>	<i>millions</i>	<i>hundred thousands</i>	<i>ten thousands</i>	<i>thousands</i>	<i>hundreds</i>	<i>tens</i>	<i>ones</i>
$10^{12}$	$10^{11}$	$10^{10}$	$10^9$	$10^8$	$10^7$	$10^6$	$10^5$	$10^4$	$10^3$	$10^2$	$10^1$	$10^0$

Digits to the RIGHT of the decimal point are represented as follows:

<i>tenths</i>	<i>hundredths</i>	<i>thousandths</i>	<i>ten thousandths</i>	<i>hundred thousandths</i>	<i>millionths</i>	<i>ten millionths</i>	<i>hundred millionths</i>	<i>billionths</i>	<i>ten billionths</i>	<i>hundred billionths</i>	<i>trillionths</i>
$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$	$\frac{1}{10^5}$	$\frac{1}{10^6}$	$\frac{1}{10^7}$	$\frac{1}{10^8}$	$\frac{1}{10^9}$	$\frac{1}{10^{10}}$	$\frac{1}{10^{11}}$	$\frac{1}{10^{12}}$

**Standard Form to Expanded Notation**

A number can be written in expanded form by first multiplying each digit in the number by its place value, then adding the results together.

**Expanded Notation to Standard Form**

A number expressed in expanded notation can be changed to standard form by performing the indicated operations.

**Skill IIA2****Identify place value and use expanded notation****Examples**

1. Find the place value of the digits in the number 1326.5874.

thousands	hundreds	tens	ones	decimal point	tenths	hundredths	thousandths	ten thousandths
$10^3$	$10^2$	$10^1$	$10^0$	.	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$
1	3	2	6		5	8	7	4

4 is in the ten thousandths place.  $\left(\frac{1}{10^4}\right)$

7 is in the thousandths place.  $\left(\frac{1}{10^3}\right)$

8 is in the hundredths place.  $\left(\frac{1}{10^2}\right)$

5 is in the tenths place.  $\left(\frac{1}{10^1}\right)$

6 is in the ones place.  $(10^0)$

2 is in the tens place.  $(10^1)$

3 is in the hundreds place.  $(10^2)$

1 is in the thousands place.  $(10^3)$

**Skill IIA2****Identify place value and use expanded notation**

2. Find the place value of the UNDERLINED digit: 4,176.029.

thousands	hundreds	tens	ones	decimal point	tenths	hundredths	thousandths
$10^3$	$10^2$	$10^1$	$10^0$	.	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$
4	<u>1</u>	7	6		0	2	9

1 is in the hundreds place.  $(1 \times 10^2)$

3. Write 25,863.4108 in expanded notation.

ten thousands	thousands	hundreds	tens	ones	decimal point	tenths	hundredths	thousandths	ten thousandths
$10^4$	$10^3$	$10^2$	$10^1$	$10^0$	.	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$	$\frac{1}{10^4}$
2	5	8	6	3		4	1	0	8

$$(2 \times 10^4) + (5 \times 10^3) + (8 \times 10^2) + (6 \times 10^1) + (3 \times 10^0) + \left(4 \times \frac{1}{10^1}\right) + \left(1 \times \frac{1}{10^2}\right) + \left(8 \times \frac{1}{10^4}\right)$$



**Skill IIA2***Identify place value and use expanded notation*

4. Find the numeral for

$$(2 \times 10^2) + (1 \times 10^0) + \left(4 \times \frac{1}{10^3}\right).$$

$10^2$	$10^1$	$10^0$	.	$\frac{1}{10^1}$	$\frac{1}{10^2}$	$\frac{1}{10^3}$
hundreds	tens	ones	decimal point	tenths	hundredths	thousandths
2		1				4

$$(2 \times 10^2) + (1 \times 10^0) + \left(4 \times \frac{1}{10^3}\right) = 200 + 1 + \frac{4}{1000} = 201.004$$

**IDENTIFY EQUIVALENT DECIMALS, PERCENTS, AND FRACTIONS**

A **percent** is a notation for a ratio with the denominator 100; for example,  $\frac{3}{100} = 3\%$ . Fractions, decimals, and percents can be used to represent the same value.

**Change Decimals to Percents**

Given a decimal, change it to a percent by moving the decimal point two places to the right and attaching the percent symbol.

**Example**

1. Change 1.382 to a percent.

Move the decimal point two places to the right and attach the percent sign.

$$1.382 = 138.2\%$$

**Change Decimals to Fractions**

Given a decimal, change it to a fraction by reading the decimal number as a fraction or mixed number; then write it. When reading the fraction, the numerator is the number with the decimal point omitted. The denominator will be 10 if there is one digit to the right of the decimal point, 100 if there are two digits, 1000 if there are three digits, and so on. Reduce the fraction, if possible.

**Examples**

2. Write 0.06 as a fraction.

0.06 is read as "6 hundredths." Write this as the fraction:  $\frac{6}{100}$ .

Reduce by dividing the numerator and denominator by 2.

$$\frac{6}{100} = \frac{3}{50}$$

3. Write 1.7 as a fraction.

1.7 is read as "one and seven tenths." Write this as the mixed number:  $1\frac{7}{10}$  or  $\frac{17}{10}$ .

**Skill IIA3*****Identify equivalent decimals, percents, and fractions***

---

**Change Percents to Decimals**

Given a percent, change it to a decimal by moving the decimal two places to the left and omitting the percent symbol. Use zeros as placeholders, if necessary.

**Examples**

4. Change 7% to a decimal.

Move the decimal two places to the left (the decimal is located at the end of the number) and omit the percent symbol.

$$7\% = 7.\% = 0.07$$

5. Change 18.2% to a decimal.

Move the decimal two places to the left and omit the percent symbol.

$$18.2\% = 0.182$$

**Change Percents to Fractions**

Given a percent, change it to a fraction by first changing the percent to a decimal and then changing the decimal to a fraction. Reduce the fraction, if possible.

**Example**

6. Change 78% to a fraction.

First, change the percent to a decimal (by moving the decimal point two places to the left).

$$78\% = 0.78$$

Next, change the decimal to a fraction. Reduce the fraction.

$$0.78 = \text{“78 hundredths.” Write this as the fraction: } \frac{78}{100}. \text{ Reduce to } \frac{39}{50}.$$

## Skill IIA3 *Identify equivalent decimals, percents, and fractions*

---

### Change Fractions to Decimals

Given a fraction  $\frac{a}{b}$ , change it to a decimal by dividing  $b$  into  $a$ .

#### Examples

7. Change  $\frac{3}{4}$  to a decimal.

$$\begin{array}{r} 0.75 \\ 4 \overline{)3.00} \end{array}$$

Therefore,  $\frac{3}{4} = 0.75$ .

8. Change  $1\frac{3}{5}$  to a decimal.

First, change  $1\frac{3}{5}$  to a mixed number.  $1\frac{3}{5} = \frac{8}{5}$

$$\begin{array}{r} 1.6 \\ 5 \overline{)8.0} \\ \underline{5} \\ 30 \\ \underline{30} \\ 0 \end{array}$$

Therefore,  $1\frac{3}{5} = 1.6$ .

### Change Fractions to Percents

Given a fraction  $\frac{a}{b}$ , change it to a percent by first changing the fraction to a decimal and then moving the decimal point two places to the right. Attach a percent symbol to the answer.

#### Example

9. Change  $\frac{3}{5}$  to a percent.

First, change  $\frac{3}{5}$  to a decimal.

$$\begin{array}{r} 0.6 \\ 5 \overline{)3.0} \end{array}$$

### **Skill IIA3**      *Identify equivalent decimals, percents, and fractions*

---

Move the decimal point two places to the right and attach the percent symbol.

$$0.6 = 0.\underline{60} = 60\%$$

The following chart shows the equivalent relationship among some common decimals, fractions, and percents. It is useful to remember these forms.

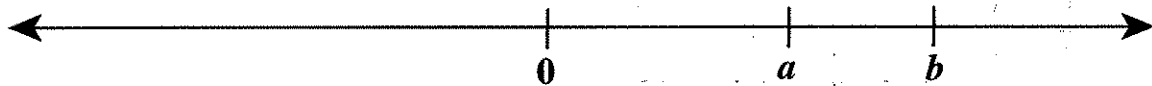
Fraction	Decimal	Percent (%)
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{10}$	0.1	10%
$\frac{2}{10}$	0.2	20%
$\frac{3}{10}$	0.3	30%
$\frac{1}{3}$	$0.\bar{3}$	$33\frac{1}{3}\%$
$\frac{2}{3}$	$0.\bar{6}$	$66\frac{2}{3}\%$

## Skill IIA4 Determine the order relationship between two real numbers

### DETERMINE THE ORDER RELATIONSHIP BETWEEN TWO REAL NUMBERS

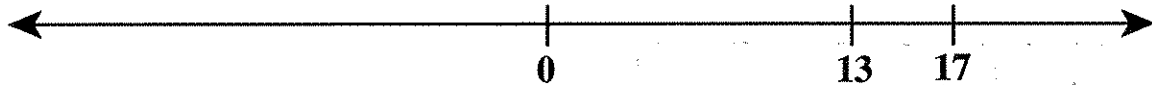
The problems in this section require a comparison of the values of two real numbers according to the relationships: = (is equal to), < (is less than), > (is greater than). A **real number** is any number that is a positive or negative number or zero. This includes numbers such as  $-\frac{1}{2}$ ,  $\pi$ ,  $\sqrt{2}$ , as well as integers. On a number line, each real number is paired with exactly one point.

Use the techniques presented in the following examples to develop some strategies for solving these problems. Remember that  $a$  is less than  $b$  means that  $a$  is to the left of  $b$  on the number line.



#### Examples

1. Compare 13 and 17 on the number line.



13 is farther to the left than 17, so the relationship is  $13 < 17$ .

2. Compare  $-17$  and  $-13$  on the number line.



$-17$  is farther to the left than  $-13$ , so the relationship is  $-17 < -13$ .

3. Compare  $\frac{3}{5}$  and  $\frac{7}{9}$ .

Since both numbers are fractions, find the LCD.

The least common denominator for 5 and 9 is 45.

Form equivalent fractions.

$$\frac{3}{5} = \frac{27}{45} \text{ and } \frac{7}{9} = \frac{35}{45}$$

## Skill IIA4 Determine the order relationship between two real numbers

---

Compare the fractions by considering the relationship between the numerators.  
This shows the relationship between the original fractions:

$$\text{Since } 27 < 35, \text{ then } \frac{27}{45} < \frac{35}{45} \text{ and } \frac{3}{5} < \frac{7}{9}.$$

4. Compare  $-\frac{3}{5}$  and  $-\frac{7}{9}$ .

The least common denominator for 5 and 9 is 45.

Form equivalent fractions.

$$-\frac{3}{5} = -\frac{27}{45} \text{ and } -\frac{7}{9} = -\frac{35}{45}$$

Compare the fractions by considering the relationship between the numerators.  
This shows the relationship between the original fractions.

$$\text{Since } -27 > -35, \text{ then } -\frac{27}{45} > -\frac{35}{45} \text{ and } -\frac{3}{5} > -\frac{7}{9}.$$

5. Compare 5.72 and 5.723.

Since both numbers are decimals, each place value must be compared.

To the left of the decimals, the values are the same.

In the tenths place, the values are the same.

In the hundredths place, the values are the same.

In the thousandths place, one value is 0 (5.720) and the other value is 3.

Since  $0 < 3$ , then  $5.72 < 5.723$ .

6. Compare  $3.\overline{12}$  and  $3.\overline{1\bar{2}}$ .

The line over the digits indicates these digits are repeated.

$3.\overline{12}$  will be 3.12121212...

$3.1\overline{2}$  will be 3.1222222...

Comparing each place value shows the discrepancy in the thousandths place.

Since  $1 < 2$ , then  $3.\overline{12} < 3.1\overline{2}$ .

## Skill IIA4 Determine the order relationship between two real numbers

7. Compare  $\frac{4}{5}$  and 0.82.

To compare the two numbers, the forms should be the same. Both should be fractions or both should be decimals. A simple technique is to change the fraction  $\frac{4}{5}$  to a decimal.

$$5 \overline{)4.0}, \text{ so } \frac{4}{5} = 0.8 = 0.80 \text{ and } 0.80 < 0.82, \text{ so } \frac{4}{5} < 0.82.$$

**Reminder:** As the absolute value signs in problems 8 and 9 indicate, the comparisons to be made are with the positive square roots.

8. Compare  $|\sqrt{23}|$  and 4.

To compare a number with a square root ( $\sqrt{\quad}$ ), take the square root if feasible. Otherwise, rewrite the whole number in radical form as shown below.

$\sqrt{23}$  cannot be calculated easily. Therefore, change 4 to its equivalent square root form,  $\sqrt{16}$ .

$$4 \times 4 = 16, \text{ so } 4 = \sqrt{16}.$$

$$|\sqrt{23}| > |\sqrt{16}|, \text{ so } |\sqrt{23}| > 4.$$

Another technique is to estimate the value of the square root as shown below.

9. Compare 6.9 and  $|\sqrt{55}|$ .

$\sqrt{55}$  cannot be calculated easily. But, it falls between  $\sqrt{49}$  and  $\sqrt{64}$ , for which square roots can be easily calculated.

$$|\sqrt{49}| < |\sqrt{55}| < |\sqrt{64}|$$

$$7 < |\sqrt{55}| < 8$$

$$6.9 < 7$$

$$\text{Therefore, } 6.9 < |\sqrt{55}| \text{ or } |\sqrt{55}| > 6.9.$$



**SOLVE FOR ONE VARIABLE OF THE SENTENCE ( $a\%$  of  $b = c$ )**

A **variable** is a letter or symbol used to represent one or more numbers in a mathematical expression. In the equation  $a\%$  of  $b = c$ ,  $a$ ,  $b$ , and  $c$  are each variables. The problems in this section include the following three forms:

- What is  $a\%$  of  $b$ ?
- $c$  is  $a\%$  of what number?
- $c$  is what percent of  $b$ ?

**What is  $a\%$  of  $b$ ?**

*Step 1.* Change  $a\%$  to a decimal by moving the decimal point two places to the left.

*Step 2.* Multiply the decimal by  $b$ .

**Examples**

1. What is  $6\%$  of 200?

*Step 1.* Change  $6\%$  to a decimal by moving the decimal point two places to the left.

$$6\% = 0.06$$

*Step 2.* Multiply 0.06 by 200.

$$0.06 \times 200 = 12$$

12 is  $6\%$  of 200.

2. What is  $120\%$  of 18?

*Step 1.* Change  $120\%$  to a decimal by moving the decimal point two places to the left.

$$120\% = 1.2$$

*Step 2.* Multiply 1.2 by 18.

$$1.2 \times 18 = 21.6$$

21.6 is  $120\%$  of 18.

**Skill IA4**      *Solve for one variable of the sentence (a% of b = c)*

---

**c is a% of what number?**

*Step 1.* Change a% to a decimal.

*Step 2.* Divide the decimal into c.

**Examples**

3. 60 is 15% of what number?

*Step 1.* Change 15% to a decimal.

$$15\% = 0.15$$

*Step 2.* Divide 0.15 into 60.

$$0.15 \overline{)60} = 15 \overline{)6000} = 15 \overline{)6000}^{\text{400}} \quad 60 \text{ is } 15\% \text{ of } 400.$$

4. 7 is 140% of what number?

*Step 1.* Change 140% to a decimal.

$$140\% = 1.4$$

*Step 2.* Divide 1.4 into 7.

$$1.4 \overline{)7} = 14 \overline{)70} = 14 \overline{)70}^{\text{5}} \quad 7 \text{ is } 140\% \text{ of } 5.$$

**Skill IA4*****Solve for one variable of the sentence (a% of b = c)***

---

**c is what percent of b?**

*Step 1.* Divide  $b$  into  $c$  to obtain a decimal.

*Step 2.* Change this decimal to a percent.

**Examples**

5. 13 is what percent of 52?

*Step 1.* Divide 52 into 13.

$$\begin{array}{r} 0.25 \\ 52 \overline{)13.00} \end{array}$$

*Step 2.* Change 0.25 to a percent.

$$0.25 = 25\% \quad 13 \text{ is } 25\% \text{ of } 52.$$

6. 32 is what percent of 16?

*Step 1.* Divide 16 into 32.

$$\begin{array}{r} 2 \\ 16 \overline{)32} \end{array}$$

*Step 2.* Change 2 to a percent.

$$2 = 200\% \quad 32 \text{ is } 200\% \text{ of } 16.$$

**CALCULATE PERCENT INCREASE AND DECREASE****For a Single Amount**

Given an original amount and the percent of increase or decrease, find the new amount. To find the new amount, follow the procedure below:

- Step 1.** Calculate the amount of increase or decrease by changing the percent to a decimal (or a fraction) and multiplying by the original amount.
- Step 2.** Add the increase to the original amount or subtract the decrease from the original amount.

**Examples**

1. If you increase 500 by 25% of itself, what is the result?

**Step 1.** Calculate the amount of increase.

$$500 \times 0.25 = 125$$

**Step 2.** Add the increase to the original amount.

$$500 + 125 = 625$$

2. A television that sells for \$620 is decreased by 20%. What is the new cost?

**Step 1.** Calculate the amount of decrease.

$$\$620 \times 0.20 = \$124$$

**Step 2.** Subtract the decrease from the original amount.

$$\$620 - \$124 = \$496$$

**For More Than One Amount**

Given two amounts, find the percent of increase or decrease.

**Step 1.** Calculate the amount of increase or decrease.

**Step 2.** Divide the amount of increase or decrease by the original number.

**Step 3.** Change this result to a percent.

**Skill IA3*****Calculate percent increase and decrease***

---

**Examples**

3. If 500 is decreased to 100, what is the percent of decrease?

*Step 1.* Calculate the amount of decrease.

$$500 - 100 = 400 \quad 400 \text{ is the amount of decrease.}$$

*Step 2.* Divide the decrease (400) by the original number (500).

$$\begin{array}{r} 0.8 \\ 500 \overline{)400.0} \end{array}$$

*Step 3.* Change 0.8 to a percent.

$$0.8 = 0.80 = 80\% \quad \text{The percent of decrease is } 80\%.$$

4. A woman's salary changes from \$800 to \$1000 per week. What is the percent of increase?

*Step 1.* Calculate the amount of increase.

$$1000 - 800 = 200 \quad 200 \text{ is the amount of increase.}$$

*Step 2.* Divide the increase (200) by the original number (800).

$$\begin{array}{r} 0.25 \\ 800 \overline{)200.00} \end{array}$$

*Step 3.* Change 0.25 to a percent.

$$0.25 = 25\% \quad \text{The percent of increase is } 25\%.$$

**SOLVE WORD PROBLEMS WITHOUT VARIABLES OR PERCENTS**

The problems in this section are verbal problems. They come from business, social studies, education, economics, environmental studies, the arts, physical science, sports, or consumer situations. The solutions for these problems do not require any specific content knowledge but do require at least two applications of addition, subtraction, multiplication, or division. Conversions within the metric or English system may be necessary. Some of the problems may contain extraneous information.

**Reminder:** Read the problem very carefully. Then reread the problem and decide what steps should be taken to solve it.

**Examples**

1. A motor home rents for \$220 per week plus \$0.25 per mile. Find the rental cost for a two-week trip of 500 miles for a family of five.

Reread the problem and note that the number of family members is extraneous information and is not needed to solve the problem. The cost is figured by weeks and by mileage.

The motor home was used for two weeks at \$220 per week.  
This part of the cost is  $\$220 \times 2$  or \$440.

The motor home was driven 500 miles at a cost of \$0.25 per mile.  
This part of the cost is  $\$0.25 \times 500$  or \$125.

To find the total cost, \$440 and \$125 must be added together.  
 $\$440 + \$125 = \$565$

The rental cost is \$565.

**Skill IVA1*****Solve word problems without variables or percents***

---

2. Two classes each have an enrollment of 24 students. On a certain day,  $\frac{5}{6}$  of one class and  $\frac{7}{8}$  of the other are present. How many students are *absent* from the two classes?

Let's consider two methods for solving this problem.

**Method 1:** Determine how many students are present. Then, subtract this number from the total.

$\frac{5}{6}$  of students in one class of 24 are present. This is  $\frac{5}{6} \times 24 = 20$ .

$\frac{7}{8}$  of students in another class of 24 are present. This is  $\frac{7}{8} \times 24 = 21$ .

A total of  $20 + 21$  or 41 students are present. The total number of students is  $24 \times 2$  or 48. The number of students who are absent is  $48 - 41 = 7$ .

**Method 2:** Determine how many students are absent.

If  $\frac{5}{6}$  of the students in one class are present, then  $\frac{1}{6}$  are absent.

$$\frac{1}{6} \times 24 = 4$$

If  $\frac{7}{8}$  of the students in the other class are present, then  $\frac{1}{8}$  are absent.

$$\frac{1}{8} \times 24 = 3$$

The number of students who are absent is  $4 + 3 = 7$ .

**Skill IVA1*****Solve word problems without variables or percents***

---

3. In a town with 500 voters, candidates A and B are running for mayor. Candidate B won with 53 more votes than candidate A. If the total number of votes received by both candidates was 397, what was the number of votes received by candidate A?

Notice that the number of voters is extraneous information. The total number of votes was 397. If we subtract 53 votes from 397, the remaining votes will be shared equally by the candidates.

$$397 - 53 = 344$$

These 344 votes are equally divided between the two candidates. Therefore, divide by 2 to obtain 172 votes for each candidate.

$$344 \div 2 = 172$$

Thus, candidate A had 172 votes.

4. The scale on a map is  $1 \frac{1}{2}$  inches equals 30 miles. What is the distance between two points that are 14 inches apart on the map?

Each 30 miles is  $1 \frac{1}{2}$  inches. Divide 30 by  $1 \frac{1}{2}$  to determine the number of miles per inch.

$$30 \div 1 \frac{1}{2} = 30 \div \frac{3}{2} = \frac{30}{1} \times \frac{2}{3} = 20$$

Each inch represents 20 miles.

Multiply 20 by the 14 inches on the map.

$$20 \times 14 = 280$$

14 inches on the map represents 280 miles.



**SOLVE WORD PROBLEMS INVOLVING PERCENTS**

The problems in this section are similar to the real-world problems found in the previous section, except that you will be required to use percents.

**Reminder:** To change a percent to a decimal, omit the percent symbol and move the decimal two places to the left.

$$17\% = 0.17 \quad 3\% = 0.03 \quad 2.8\% = 0.028 \quad 120\% = 1.20$$

**Examples**

1. A manufacturer produced 400 cars at a plant in California and 700 cars at a plant in Florida. 20% of the cars from the California plant were defective and 10% of the cars from the Florida plant were defective. How many defective cars were produced?

Change 20% to 0.20. Change 10% to 0.10.

Multiply  $400 \times 0.20 = 80$  defective cars at the California plant.

Multiply  $700 \times 0.10 = 70$  defective cars at the Florida plant.

Add 80 and 70 cars to get a total of 150 defective cars.

2. Juan was told that if he insulated his house he would use only 65% as much heating oil. The actual amount of fuel he used before insulating was 140 gallons. How much fuel will he save by insulating his house?

*Method 1:* Change 65% to 0.65.

Multiply 0.65 by 140 to get the amount of fuel he will actually use.

$$0.65 \times 140 = 91$$

Subtract 91 from 140 to obtain the savings of 49 gallons.

*Method 2:* If he uses only 65% of the fuel, he will save 35% of the fuel.  
( $100\% - 65\% = 35\%$ )

Change 35% to 0.35.

Multiply 140 by 0.35 to obtain the fuel savings of 49 gallons.

$$0.35 \times 140 = 49$$

3. A student purchases a VCR for \$480. If the VCR is purchased with cash, the student will receive a 5% discount. The tax rate is 7%. The student chooses to pay with cash. How much will the student have to pay for the VCR including tax?

Since cash was used for the purchase, subtract 5% of the original price.

$$5\% = 0.05$$

$$\$480 \times 0.05 = \$24 \text{ (off the original price).}$$

$$\text{Subtract } \$24 \text{ from } \$480 \text{ } (\$480 - \$24 = \$456).$$

Determine tax using the discounted price of \$456.

Tax is 7% or 0.07.

$$\$456 \times 0.07 = \$31.92$$

Add the tax to the discounted price.

$$\$456 + \$31.92 = \$487.92$$

The student will have to pay \$487.92 (including tax) for the VCR.

**Skill IVA3** *Solve problems involving the structure and logic of arithmetic*

**SOLVE PROBLEMS INVOLVING THE STRUCTURE AND LOGIC OF ARITHMETIC**

The problems in this section involve the structure and logic of arithmetic and require reasoning ability as well as an understanding of basic arithmetic.

**Examples**

- How many whole numbers leave a remainder of 2 when divided into 56, and a remainder of 1 when divided into 28?

**Reminder:** You are not looking for a number. You are looking for how many numbers will satisfy these conditions.

The conditions include:

*Condition 1:* whole numbers that leave a remainder of 2 when divided into 56

*Condition 2:* whole numbers that leave a remainder of 1 when divided into 28

Since 28 is a smaller number than 56, you should start with the second condition.

*Condition 2:* Look for numbers that leave a remainder of 1 when divided into 28. You may have to try 28 numbers; you may also notice a pattern before doing all the division.

$$\begin{array}{cccccccc}
 \begin{array}{r} 28 \\ 1 \overline{)28} \end{array} & \begin{array}{r} 14 \\ 2 \overline{)28} \end{array} & \begin{array}{r} 9r1 \\ 3 \overline{)28} \end{array} & \begin{array}{r} 7 \\ 4 \overline{)28} \end{array} & \begin{array}{r} 5r3 \\ 5 \overline{)28} \end{array} & \begin{array}{r} 4r4 \\ 6 \overline{)28} \end{array} & \begin{array}{r} 4 \\ 7 \overline{)28} \end{array} \\
 \begin{array}{r} 3r4 \\ 8 \overline{)28} \end{array} & \begin{array}{r} 3r1 \\ 9 \overline{)28} \end{array} & \begin{array}{r} 2r8 \\ 10 \overline{)28} \end{array} & \begin{array}{r} 2r6 \\ 11 \overline{)28} \end{array} & \begin{array}{r} 2r4 \\ 12 \overline{)28} \end{array} & \begin{array}{r} 2r2 \\ 13 \overline{)28} \end{array} & \begin{array}{r} 2 \\ 14 \overline{)28} \end{array} \\
 \begin{array}{r} 1r13 \\ 15 \overline{)28} \end{array} & \begin{array}{r} 1r12 \\ 16 \overline{)28} \end{array} & \begin{array}{r} 1r11 \\ 17 \overline{)28} \end{array} & \begin{array}{r} 1r10 \\ 18 \overline{)28} \end{array} & \begin{array}{r} 1r9 \\ 19 \overline{)28} \end{array} & \begin{array}{r} 1r8 \\ 20 \overline{)28} \end{array} & \begin{array}{r} 1r7 \\ 21 \overline{)28} \end{array} \\
 \begin{array}{r} 1r6 \\ 22 \overline{)28} \end{array} & \begin{array}{r} 1r5 \\ 23 \overline{)28} \end{array} & \begin{array}{r} 1r4 \\ 24 \overline{)28} \end{array} & \begin{array}{r} 1r3 \\ 25 \overline{)28} \end{array} & \begin{array}{r} 1r2 \\ 26 \overline{)28} \end{array} & \begin{array}{r} 1r1 \\ 27 \overline{)28} \end{array} & \begin{array}{r} 1 \\ 28 \overline{)28} \end{array}
 \end{array}$$

The numbers that satisfy Condition 2 are 3, 9, and 27.

*Condition 1:* Now determine whether 3, 9, or 27 satisfies the first condition.

$$\begin{array}{ccc}
 \begin{array}{r} 18r2 \\ 3 \overline{)56} \end{array} & \begin{array}{r} 6r2 \\ 9 \overline{)56} \end{array} & \begin{array}{r} 2r2 \\ 27 \overline{)56} \end{array}
 \end{array}$$

You will notice that all three numbers satisfy Condition 1.

Remember, the question is what number of whole numbers meet both conditions. Since there are three numbers that satisfy both conditions, the answer is 3.

### Skill IVA3 *Solve problems involving the structure and logic of arithmetic*

---

2. Find the smallest positive multiple of 3 that leaves a remainder of 3 when divided by 5 and a remainder of 5 when divided by 7.

**Reminder:** A **multiple** of 3 is any number that is divisible by 3 with no remainder.

There are *three conditions* that must be met by the number for which you are looking.

**Condition 1:** The number must be a multiple of 3.

List several multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39.

**Condition 2:** The number must leave a remainder of 3 when divided by 5.

Check the multiples of three above until one satisfies the second condition. Divide the multiples of 3 by 5. List any that have a remainder of 3.

$$\begin{array}{r} 1r1 \\ 5 \overline{)6} \end{array} \quad \begin{array}{r} 1r4 \\ 5 \overline{)9} \end{array} \quad \begin{array}{r} 2r2 \\ 5 \overline{)12} \end{array} \quad \begin{array}{r} 3 \\ 5 \overline{)15} \end{array} \quad \begin{array}{r} 3r3 \\ 5 \overline{)18} \end{array}$$

So, 18 is a number that satisfies the first and second conditions.

**Condition 3:** The number must leave a remainder of 5 when divided by 7. Determine whether 18 satisfies the third condition.

$$\begin{array}{r} 2r4 \\ 7 \overline{)18} \end{array}$$

Since the remainder is not 5, this does not satisfy the third condition. So, continue to consider Condition 2.

**Condition 2:** Continue to check the multiples of three until another number satisfies the second condition by leaving a remainder of 3 when divided by 5.

$$\begin{array}{r} 4r1 \\ 5 \overline{)21} \end{array} \quad \begin{array}{r} 4r4 \\ 5 \overline{)24} \end{array} \quad \begin{array}{r} 5r2 \\ 5 \overline{)27} \end{array} \quad \begin{array}{r} 6 \\ 5 \overline{)30} \end{array} \quad \begin{array}{r} 6r3 \\ 5 \overline{)33} \end{array}$$

So, 33 is a number that satisfies the first and second conditions. Repeat consideration of the third condition.

**Condition 3:** The number must leave a remainder of 5 when divided by 7. Determine whether 33 satisfies the third condition.

$$\begin{array}{r} 4r5 \\ 7 \overline{)33} \end{array}$$

Since the remainder is 5, 33 satisfies all three conditions. The answer is 33.

**Skill IVA3**    *Solve problems involving the structure and logic of arithmetic*

---

3. Which of the following is the smallest positive whole number that can be divided by 3, 5, and 8 with no remainder and also leaves a remainder of 3 when divided by 7?
- A. 120
  - B. 153
  - C. 180
  - D. 360

Because this question is presented in multiple-choice format, the easiest method is to test each of the four possible answers. Determine whether one of the numbers, 120, 153, 180, or 360, is divisible by 3, 5, and 8 and leaves a remainder of 3 when divided by 7.

*Condition 1:* Determine whether the choices are divisible by 3.

$$\begin{array}{r} 40 \\ 3 \overline{)120} \end{array} \quad \begin{array}{r} 51 \\ 3 \overline{)153} \end{array} \quad \begin{array}{r} 60 \\ 3 \overline{)180} \end{array} \quad \begin{array}{r} 120 \\ 3 \overline{)360} \end{array}$$

All are divisible by 3, so no answer is eliminated.

*Condition 2:* Determine whether the choices are divisible by 5.

Since 120, 180, and 360 end in zero, they are all divisible by 10, and also by 5; 153 is not, and therefore can be eliminated.

*Condition 3:* Determine whether the remaining choices are divisible by 8.

$$\begin{array}{r} 15 \\ 8 \overline{)120} \end{array} \quad \begin{array}{r} 22r4 \\ 8 \overline{)180} \end{array} \quad \begin{array}{r} 45 \\ 8 \overline{)360} \end{array}$$

The 180 answer can now be eliminated.

*Condition 4:* Determine which of the remaining choices leaves a remainder of 3 when divided by 7.

$$\begin{array}{r} 17r1 \\ 7 \overline{)120} \end{array} \quad \begin{array}{r} 51r3 \\ 7 \overline{)360} \end{array}$$

The answer is 360.

This method of elimination may also be used for example 2.

**Reminder:** Eliminating choices is an efficient method for solving these types of problems on multiple-choice exams.

**ESTIMATE SUMS, AVERAGES, AND PRODUCTS**

*Estimation* is a key skill in solving real-world problems. One way to find reasonable estimations is to round the numbers so that the calculations can be easily completed. Try to identify and eliminate any unreasonable answers.

**Examples**

1. A company employs 30 people. The lowest paid worker earns \$150.00 per week. The highest paid worker earns \$400.00 per week. What is a reasonable estimate of the total weekly payroll for the company?

Find the lowest total weekly payroll based on the lowest salary.

$$\$150 \times 30 = \$4,500$$

Find the highest total weekly payroll based on the highest salary.

$$\$400 \times 30 = \$12,000$$

The company would not employ only the lowest paid workers nor would it employ only the highest paid workers. So neither \$4,500 nor \$12,000 would be the answer.

Therefore, the total weekly payroll must fall between \$4,500 and \$12,000.

2. An investor owns 208.45 shares of a stock fund. Each share is valued at \$47.25. What is a reasonable estimate of the value of the investor's stock fund?

First, round the number of shares to 200 and then round the value of each share to \$50.

Next, multiply 200 times \$50.

$$200 \times \$50 = \$10,000$$

A reasonable estimate is \$10,000.

3. A bank account contains \$386.50. Deposits of \$78.36 and \$100.79 are made. Withdrawals of \$272.96, \$17.92, and \$43.85 are made. What is a reasonable estimate of the amount in the bank account after these deposits and withdrawals?

Round the original amount the bank account contains.

$$\$386.50 \text{ to } \$400$$

**Skill IIA5***Estimate sums, averages, and products*

---

Round the deposits.

\$78.36 to \$80 and \$100.79 to \$100

Round the withdrawals.

\$272.96 to \$300, \$17.92 to \$20, and \$43.85 to \$40

Add the rounded deposits to the rounded original amount.

$\$400 + \$80 + \$100 = \$580$

Total the rounded withdrawals.

$\$300 + \$20 + \$40 = \$360$

Subtract the rounded withdrawals.

$\$580 - \$360 = \$220$

\$220 is a reasonable estimate of the amount in the account.

## FIND MISSING NUMBERS GIVEN A PATTERN

### Linear Relationships

Linear relationships in pairs of numbers involve operations of addition, subtraction, multiplication, or division on one number in the pair in order to obtain the other number in the pair.

Linear pairs take the form:  $(x, x + n)$ ,  $(x + n, x)$ ,  $(x, nx)$ , or  $(x, \frac{x}{n})$ .

For example, pairs in the form  $(x, nx)$  may be represented by the pairs  $(2, 4)$ ,  $(3, 6)$ ,  $(4, 8)$ ,  $(\frac{1}{4}, \frac{1}{2})$ , etc. To obtain the second number in each pair, the first number in the pair was multiplied by 2.

Pairs in the form  $(x, x + n)$  may be represented by  $(4, 7)$ ,  $(8, 11)$ ,  $(9, 12)$ ,  $(\frac{1}{2}, 3\frac{1}{2})$ , etc. To obtain the second number in each pair, the number 3 was added to the first number in each pair.

### Quadratic Relationships

Quadratic relationships in pairs of numbers involve the operation of squaring one of the numbers in the pair in order to obtain the other number in the pair. The quadratic pairs take the form  $(x, x^2)$  or  $(x^2, x)$ . For example, pairs in the form  $(x, x^2)$  may be represented by the pairs  $(2, 4)$ ,

$(5, 25)$ ,  $(0.4, 0.16)$ ,  $(\frac{1}{2}, \frac{1}{4})$ , etc. To obtain the second number in the pair, the first number was

squared. Pairs in the form  $(x^2, x)$  may be represented by the pairs  $(16, 4)$ ,  $(25, 5)$ ,  $(\frac{1}{4}, \frac{1}{2})$ ,  $(0.09, 0.3)$ , etc. To obtain the second number in the pair, note that it is a number that when squared will give the first number in the pair.

### Examples

- Look for a common linear relationship between the numbers in each pair:

$$(2, 1) \quad (0.08, 0.04) \quad (-10, -5) \quad \left(\frac{1}{10}, \frac{1}{20}\right); \text{ then, identify the missing term } \left(\frac{1}{3}, \frac{?}{6}\right).$$

Because the relationship is linear, the operation on the pair must be addition, subtraction, multiplication, or division. Notice that the second number is one half of the first in each of the pairs.

Therefore, the missing number should be one half of the first.

$$\text{So, } \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}. \text{ Therefore, the second number will be } \frac{1}{6}.$$



## Skill IIIA1

## Find missing numbers given a pattern

---

2. Look for a common quadratic relationship between the numbers in each pair:

(25, 5) (49, 7) (1, 1)  $\left(\frac{1}{4}, \frac{1}{2}\right)$ ; then, identify the missing term (81, ?).

Because the relationship is quadratic, the operation between the pair of numbers involves squaring. Notice that in each pair the second number is the square root of the first number.

Therefore, the missing number is the square root of 81.  $\sqrt{81} = 9$

3. Look for a common linear relationship between the numbers in each pair:

(10, 7) (4, 1) (-2, -5) (0, -3); then, identify the missing term (-6, ?) (see example 1).

Because the relationship is linear, the operation on the pair must be addition, subtraction, multiplication, or division. Notice that the second number is 3 less than the first.

Therefore, the missing term should be 3 less than the first.

$$-6 - 3 = -9$$

The second number is -9.

## Progressions

A **progression** is a sequence of numbers that follows some established pattern. The pattern may be established by adding the same number to obtain each following number, by multiplying by the same number to obtain each following number, or by creating a sequence of fractions that have numerators of 1 and denominators that are the result of adding the same number to obtain each following denominator.

An **arithmetic progression** is a sequence established by adding the same nonzero number (*common difference*) to obtain each following number in the sequence. In the sequence 2, 5, 8, and 11, 3 was added to each number to obtain the next number in the sequence. Therefore, 3 is the common difference in this progression. To obtain the common difference, subtract the first term from the second. For example,  $5 - 2 = 3$ .

A **geometric progression** is a sequence that is established by multiplying by the same nonzero number (*common ratio*) to determine the next number in the sequence. In the sequence 2, 6, 18, and 54, each number was multiplied by 3 to obtain the next number in the sequence. Therefore, 3 is the common ratio in this progression. To obtain the common ratio, divide the second term by the first. For example,  $6 \div 2 = 3$ .

A **harmonic progression** is a sequence of fractions having 1 as a numerator and denominators that form an arithmetic progression. In the progression  $\frac{1}{2}$ ,  $\frac{1}{5}$ ,  $\frac{1}{8}$ , and  $\frac{1}{11}$ , the numerator of each fraction is 1 and the denominators, 2, 5, 8, and 11, form an arithmetic progression with 3 as the common difference.

**Examples**

4. Identify the missing term in the following arithmetic progression:  
7, 12, 17, 22, 27, ?

Because this progression is arithmetic, find the common difference:  $12 - 7 = 5$ .  
Each term is 5 more than the previous term. To find the missing term, add 5  
to 27.

$$27 + 5 = 32$$

The missing term is 32.

5. Identify the missing term in the following geometric progression:

$$4, -2, 1, -\frac{1}{2}, \frac{1}{4}, \underline{?}$$

Because this progression is geometric, find the common ratio:  $-2 \div 4 = -\frac{2}{4} = -\frac{1}{2}$ .

The ratio between the second and the first terms is  $-\frac{2}{4}$  or  $-\frac{1}{2}$ . So the missing

term should be the previous term  $\left(\frac{1}{4}\right)$  multiplied by  $-\frac{1}{2}$ .

$$\frac{1}{4} \times \left(-\frac{1}{2}\right) = -\frac{1}{8}$$

The missing term is  $-\frac{1}{8}$ .

6. Identify the missing term in the following harmonic progression:

$$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \underline{?}$$

Because this progression is harmonic, the numerator in the missing term should  
be 1. The denominators form an arithmetic progression of 2, 5, 8, 11, . . . . The  
common difference is 3. The denominator of the missing term should be the  
previous term (11) plus the difference (3). The numerator of the missing term is 1

and the denominator is  $11 + 3$  or 14. The missing term is  $\frac{1}{14}$ .

# ARITHMETIC PRACTICE PROBLEMS

<i>SKILLS</i>	<i>PROBLEMS</i>
IA1 <i>Add, subtract, multiply, and divide fractions</i>	1–9
IA2 <i>Add, subtract, multiply, and divide decimals</i>	10–18
IIA1 <i>Recognize the meaning of exponents</i>	19–22
IIA2 <i>Identify place value and use expanded notation</i>	23–25
IIA3 <i>Identify equivalent decimals, percents, and fractions</i>	26–31
IIA4 <i>Determine the order relationship between two real numbers</i>	32–37
IA4 <i>Solve for one variable of the sentence (<math>a\%</math> of <math>b = c</math>)</i>	38–43
IA3 <i>Calculate percent increase and decrease</i>	44–47
IVA1 <i>Solve word problems without variables or percents</i>	48–51
IVA2 <i>Solve word problems involving percents</i>	52–55
IVA3 <i>Solve problems involving the structure and logic of arithmetic</i>	56–57
IIA5 <i>Estimate sums, averages, and products</i>	58–62
IIIA1 <i>Find missing numbers given a pattern</i>	63–67

## ADD, SUBTRACT, MULTIPLY, AND DIVIDE FRACTIONS

1.  $2\frac{1}{5} + 3\frac{4}{9} =$

A.  $\frac{29}{45}$

B.  $5\frac{1}{9}$

C.  $5\frac{5}{14}$

D.  $5\frac{29}{45}$

4.  $3\frac{1}{4} - 1\frac{2}{7} =$

A.  $1\frac{1}{28}$

B.  $1\frac{27}{28}$

C.  $2\frac{1}{28}$

D.  $2\frac{27}{28}$

2.  $-7 + 3\frac{2}{5} =$

A.  $4\frac{2}{5}$

B.  $3\frac{3}{5}$

C.  $-3\frac{3}{5}$

D.  $-4\frac{2}{5}$

5.  $\frac{11}{12} - \left(-\frac{2}{3}\right) =$

A.  $1\frac{7}{12}$

B.  $1\frac{5}{12}$

C.  $\frac{13}{15}$

D.  $\frac{3}{12}$

3.  $-1\frac{1}{3} + \left(-2\frac{4}{7}\right) =$

A.  $3\frac{19}{21}$

B.  $-1\frac{5}{21}$

C.  $-3\frac{11}{21}$

D.  $-3\frac{19}{21}$

6.  $1\frac{2}{3} \times 3\frac{1}{7} =$

A. 110

B.  $5\frac{5}{21}$

C.  $3\frac{2}{21}$

D.  $2\frac{13}{21}$

7.  $\left(-\frac{2}{7}\right) + \left(-\frac{6}{13}\right) =$

A.  $-\frac{13}{21}$

B.  $\frac{12}{91}$

C.  $\frac{13}{21}$

D.  $\frac{91}{12}$

8.  $\left(-\frac{5}{6}\right) \times 4\frac{2}{7} =$

A.  $3\frac{4}{7}$

B.  $-3\frac{4}{7}$

C.  $-4\frac{2}{7}$

D.  $-4\frac{5}{21}$

9.  $4\frac{1}{3} \div 3 =$

A. 13

B.  $1\frac{4}{9}$

C.  $\frac{8}{9}$

D.  $\frac{9}{13}$

**ADD, SUBTRACT, MULTIPLY, AND DIVIDE DECIMALS**

10.  $2.37 + 0.528 =$

- A. 0.02898
- B. 0.765
- C. 2.788
- D. 2.898

13.  $-3.8 - (-1.07) =$

- A. 0.69
- B. -2.73
- C. -2.83
- D. -4.87

11.  $17.1 + (-5.28) =$

- A. 22.38
- B. 12.82
- C. 11.98
- D. 11.82

14.  $2.3 \times 0.07 =$

- A. 16.1
- B. 1.61
- C. 0.161
- D. 0.141

12.  $8 - 0.291 =$

- A. 0.509
- B. 7.709
- C. 8.291
- D. 8.819

15.  $18 \div 0.06 =$

- A. 3
- B. 30
- C. 300
- D. 3000

16.  $(-0.05) \times (-3.61) =$

- A. -18.05
- B. -0.1805
- C. 0.1805
- D. 18.05

17.  $(-3.9) \times (0.21) =$

- A. -8.19
- B. -7.119
- C. -0.819
- D. 0.819

18.  $(-3.57) \div 2.1 =$

- A. 1.7
- B. -0.17
- C. -1.7
- D. -17

## RECOGNIZE THE MEANING OF EXPONENTS

19.  $(2^4)(3^2) =$

- A.  $(2 + 2 + 2 + 2)(3 + 3)$
- B.  $(2 \times 2 \times 2 \times 2)(3 \times 3)$
- C.  $(2 \times 4)(3 \times 2)$
- D.  $(6)^8$

22.  $(2 \times 5)^3 =$

- A.  $2 \times 2 \times 2 \times 5$
- B.  $(2 \times 5) + (2 \times 5)$
- C.  $6 \times 15$
- D.  $(2 \times 5) \times (2 \times 5) \times (2 \times 5)$

20.  $5^4 - 2^4 =$

- A.  $(5 - 2)^4$
- B.  $(5 \times 5 \times 5 \times 5) - (2 \times 2 \times 2 \times 2)$
- C.  $5 \times 4 - 2 \times 4$
- D.  $(5 - 2)^0$

21.  $(5^2)^3 =$

- A.  $5^5$
- B.  $5^8$
- C.  $5^2 \times 5^2 \times 5^2$
- D.  $(5 \times 5 \times 5)^6$



## IDENTIFY PLACE VALUE AND USE EXPANDED NOTATION

23. Select the place value associated with the underlined digit.

62.8129

- A.  $\frac{1}{10^2}$   
B.  $\frac{1}{10^3}$   
C.  $10^3$   
D.  $10^2$

25. Select the numeral for

$$(4 \times 10^3) + \left(8 \times \frac{1}{10}\right).$$

- A. 400.8  
B. 408  
C. 4000.08  
D. 4000.8

24. Select the correct expanded notation for 5002.07.

- A.  $(5 \times 10^3) + (2 \times 10^1) + \left(7 \times \frac{1}{10^2}\right)$   
B.  $(5 \times 10^4) + (2 \times 10^0) + \left(7 \times \frac{1}{10^2}\right)$   
C.  $(5 \times 10^3) + (2 \times 10^0) + \left(7 \times \frac{1}{10^2}\right)$   
D.  $(5 \times 10^3) + (2 \times 10^0) + \left(7 \times \frac{1}{10}\right)$

**IDENTIFY EQUIVALENT DECIMALS, PERCENTS, AND FRACTIONS**

26.  $21\% =$

- A. 21.0
- B. 2.1
- C. 0.21
- D.  $\frac{21}{1000}$

29.  $\frac{1}{8} =$

- A. 12.5%
- B. 8%
- C. 1.25%
- D. 0.0125

27.  $7\% =$

- A. 0.7
- B.  $\frac{70}{100}$
- C.  $\frac{7}{10}$
- D.  $\frac{7}{100}$

30.  $0.62 =$

- A.  $6\frac{1}{5}$
- B. 6.2%
- C.  $\frac{31}{50}$
- D.  $\frac{31}{500}$

28.  $1\frac{4}{5} =$

- A. 1.8
- B. 1.08
- C. 1.8%
- D. 0.555

31.  $0.08 =$

- A. 80%
- B. 8%
- C. 0.08%
- D. 0.0008%

**DETERMINE THE ORDER RELATIONSHIP BETWEEN TWO REAL NUMBERS**

In 32–37, identify the symbol that should be placed in the box to form a true statement.

32.  $0.25 \square \frac{1}{4}$

A. =

B. <

C. >

35.  $-7\frac{1}{5} \square -6\frac{2}{5}$

A. =

B. <

C. >

33.  $\sqrt{65} \square 6.5$

A. =

B. <

C. >

36.  $7.25 \square 7.\bar{2}$

A. =

B. <

C. >

34.  $\frac{3}{7} \square \frac{2}{3}$

A. =

B. <

C. >

37.  $4.3 \square 4\frac{3}{10}$

A. =

B. <

C. >

**SOLVE FOR ONE VARIABLE OF THE SENTENCE ( $a\%$  of  $b = c$ )**

38. What is 70% of 110?

- A. 157.14
- B. 77
- C. 7.7
- D. 0.00636

41. 78.2 is 170% of what number?

- A. 460
- B. 132.94
- C. 46
- D. 13.294

39. What is 130% of 200?

- A. 26
- B. 153.846
- C. 260
- D. 26,000

42. 105 is what percent of 300?

- A. 0.35%
- B. 2.857%
- C. 28.57%
- D. 35%

40. 4 is 8% of what number?

- A. 0.02
- B. 0.32
- C. 50
- D. 320

43. 480 is what percent of 400?

- A. 120%
- B.  $83\frac{1}{3}\%$
- C.  $8\frac{1}{3}\%$
- D. 1.2%

**CALCULATE PERCENT INCREASE AND DECREASE**

- |                                                                                                                                                           |                                                                                                                                                                                                              |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>44. If 60 is increased to 69, what is the percent of increase?</p> <p>A. 41%</p> <p>B. 15%</p> <p>C. 13%</p> <p>D. 9%</p>                              | <p>47. The weight of an object is 360 pounds. The weight is increased by 15% of itself. What is the resulting weight?</p> <p>A. 414 pounds</p> <p>B. 375 pounds</p> <p>C. 306 pounds</p> <p>D. 54 pounds</p> |
| <p>45. If you decrease 300 by 12% of itself, what is the result?</p> <p>A. 336</p> <p>B. 288</p> <p>C. 264</p> <p>D. 36</p>                               |                                                                                                                                                                                                              |
| <p>46. An item that sells for \$200 is put on sale at \$150. What is the percent of decrease?</p> <p>A. 75%</p> <p>B. 50%</p> <p>C. 25%</p> <p>D. 17%</p> |                                                                                                                                                                                                              |

**SOLVE WORD PROBLEMS WITHOUT VARIABLES OR PERCENTS**

48. An 8-ounce can of juice costs \$0.50 and a 16-ounce can costs \$0.90. How much money can be saved by purchasing 320 ounces of the larger size?

- A. \$2.00
- B. \$10.00
- C. \$18.00
- D. \$20.00

51. Two mathematics classes have a total of 329 students. The 10:00 a.m. class has 53 more students than the 8:00 a.m. class. How many students are in the 8:00 a.m. class?

- A. 111.5
- B. 138
- C. 191
- D. 276

49. A worker is paid \$12.00 per hour for the first 40 hours she works. She is paid an additional \$2.00 per hour for each hour over 40 that she works. How much will she earn if she works 52 hours in one week?

- A. \$504.00
- B. \$624.00
- C. \$648.00
- D. \$728.00

50. It costs \$120 per day for a maximum of 6 people to rent a condominium at the beach. An additional \$15 per day is charged for each person over 6. How much will it cost for 9 people to rent the condominium for 5 days?

- A. \$1,080.00
- B. \$825.00
- C. \$675.00
- D. \$645.00

## SOLVE WORD PROBLEMS INVOLVING PERCENTS

52. The main ingredient in a box of cereal is sugar. In fact, the cereal is 35% sugar. A family consumes 20 ounces of the cereal in one day. What amount of the nonsugar ingredients was consumed by the family?
- A. 7 ounces
  - B. 10 ounces
  - C. 13 ounces
  - D. 70 ounces
53. A student needs a score of at least 78% on his exam to ensure a grade of B in the course. There are 150 questions of equal value on the test. How many can he miss?
- A. 22
  - B. 33
  - C. 80
  - D. 117
54. Over a three-week period, 7% of the VCRs manufactured at plant A were defective, and 20% of the VCRs manufactured at plant B were defective. Plant A produced 300 VCRs and plant B produced 250 VCRs. How many defective VCRs were produced?
- A. 21
  - B. 50
  - C. 71
  - D. 149
55. A restaurant employs 265 people. There are 190 waiters and 75 cooks. If 76 waiters and 25 cooks receive pay raises, what percent of the waiters will receive a pay raise?
- A. 29%
  - B.  $33\frac{1}{3}\%$
  - C. 38%
  - D. 40%

**SOLVE PROBLEMS INVOLVING THE STRUCTURE AND LOGIC OF ARITHMETIC**

56. How many whole numbers leave a remainder of 3 when divided into 31 and a remainder of 4 when divided into 25?

- A. 0
- B. 1
- C. 2
- D. 3

57. Find the smallest positive multiple of 4 that leaves a remainder of 2 when divided by 3 and a remainder of 0 when divided by 5.

- A. 2
- B. 8
- C. 20
- D. 32



## ESTIMATE SUMS, AVERAGES, AND PRODUCTS

- |                                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                      |
|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p>58. A group of 100 students took a test. All of the students scored less than 82 but more than 58. Which of the following values could be a reasonable estimate of the average score of the students?</p> <p>A. 56<br/>B. 70<br/>C. 82<br/>D. 140</p>                            | <p>61. A car gets 28.72 miles per gallon. The car has been driven 42,310 miles. What is a reasonable estimate for the number of gallons of gas used?</p> <p>A. 1,260,000 gallons<br/>B. 2,100 gallons<br/>C. 1,400 gallons<br/>D. 680 gallons</p>    |
| <p>59. A basketball player plays in 20 games. In her best game she scores 18 points. In her worst game she scores 8 points. What would be a reasonable estimate of her total number of points over 20 games?</p> <p>A. 380<br/>B. 280<br/>C. 160<br/>D. 26</p>                      | <p>62. At the grocery store, Susan purchased four items costing \$2.19, \$0.89, \$2.48, and \$4.61. Which of the following is a reasonable estimate of the total cost of the items?</p> <p>A. \$15.00<br/>B. \$10.00<br/>C. \$5.00<br/>D. \$1.00</p> |
| <p>60. Ricardo has 5 items to buy at the store. He has a \$20 bill. The most expensive item he buys is \$3.79 and the least expensive item is \$1.08. Which of the following is a reasonable estimate of his bill?</p> <p>A. \$20.00<br/>B. \$10.00<br/>C. \$5.00<br/>D. \$3.00</p> |                                                                                                                                                                                                                                                      |

**FIND MISSING NUMBERS GIVEN A PATTERN**

63. Identify the missing term in the following arithmetic progression:  
5, 9, 13, 17, 21, ?

- A. 17
- B. 24
- C. 25
- D. 84

66. Look for a common linear relationship between the numbers in each pair. Then identify the missing term.

$$(7, 3) (0, -4) \left(8\frac{1}{2}, 4\frac{1}{2}\right) (2, -2) (3, \underline{?})$$

- A. -12
- B. -1
- C. 1
- D. 7

64. Identify the missing term in the following geometric progression:  
5, -10, 20, -40, 80, ?

- A. -160
- B. -120
- C. 120
- D. 160

67. Look for a common linear relationship between the numbers in each pair. Then identify the missing term.

$$(6, 18) (-2, -6) (4, 12) \left(\frac{1}{5}, \frac{3}{5}\right) (0, 0) \\ (5, \underline{?})$$

- A. -15
- B.  $\frac{5}{3}$
- C. 2
- D. 15

65. Identify the missing term in the following harmonic progression:

$$\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \frac{1}{14}, \frac{1}{18}, \underline{?}$$

- A.  $\frac{1}{54}$
- B.  $\frac{1}{22}$
- C.  $\frac{1}{6}$
- D. 22

# ARITHMETIC PRACTICE EXPLANATIONS

## SKILLS

IA1	<i>Add, subtract, multiply, and divide fractions</i>	1–9
IA2	<i>Add, subtract, multiply, and divide decimals</i>	10–18
IIA1	<i>Recognize the meaning of exponents</i>	19–22
IIA2	<i>Identify place value and use expanded notation</i>	23–25
IIA3	<i>Identify equivalent decimals, percents, and fractions</i>	26–31
IIA4	<i>Determine the order relationship between two real numbers</i>	32–37
IA4	<i>Solve for one variable of the sentence (<math>a\%</math> of <math>b = c</math>)</i>	38–43
IA3	<i>Calculate percent increase and decrease</i>	44–47
IVA1	<i>Solve word problems without variables or percents</i>	48–51
IVA2	<i>Solve word problems involving percents</i>	52–55
IVA3	<i>Solve problems involving the structure and logic of arithmetic</i>	56–57
IIA5	<i>Estimate sums, averages, and products</i>	58–62
IIIA1	<i>Find missing numbers given a pattern</i>	63–67

## PROBLEMS

## ADD, SUBTRACT, MULTIPLY, AND DIVIDE FRACTIONS

1. **D** is the correct response. In **A**, the fractions were correctly added, but the whole numbers were omitted. In **B**, the numerators were not changed. In **C**, the numerators and the denominators were both added and no common denominator was found.
2. **C** is the correct response. In **A**, there was no regrouping. In **B**, the wrong sign was used for the answer. In **D**, an error was made in regrouping.
3. **D** is the correct response. In **A**, the wrong sign was used. In **B**, the numbers were subtracted. In **C**, an error was made in forming equivalent fractions.
4. **B** is the correct response. In **A**, the error was made by not regrouping the 3. In **C**, the error was made in failing to regroup. In **D**, the error was made in failing to change the numerators.
5. **A** is the correct response. In **B**, an error was made changing  $\frac{19}{12}$  to a mixed number. In **C**, a common denominator was not found. In **D**,  $-\left(-\frac{2}{3}\right)$  was not changed to  $+\frac{2}{3}$ .
6. **B** is the correct response. In **A**, the error was made by finding a common denominator and multiplying only numerators. In **C**, the error was made by multiplying the whole numbers and fractions separately. In **D**, the error was made by changing  $3\frac{1}{7}$  to  $\frac{11}{7}$  instead of  $\frac{22}{7}$ .
7. **C** is the correct response. In **A**, the wrong sign was used. In **B**, the second term was not inverted. In **D**, both terms were inverted.
8. **B** is the correct response. In **A**, the wrong sign was used. In **C**, a mistake was made in canceling. In **D**, whole numbers and fractions were incorrectly multiplied separately.
9. **B** is the correct response. In **A**, there was an inversion error. In **C**, an error was made in changing  $4\frac{1}{3}$  to an improper fraction. In **D**, the wrong term was inverted.

**ADD, SUBTRACT, MULTIPLY, AND DIVIDE DECIMALS**

10. *D* is the correct response. In *A*, the decimal was misplaced. In *B*, the decimals were improperly aligned. In *C*, an addition error was made.
11. *D* is the correct response. In *A*, the quantities were mistakenly added. In *B*, an error was made in regrouping. In *C*, the 8 was brought down and not subtracted.
12. *B* is the correct response. In *A*, an error was made in aligning the decimals. In *C*, the quantities were incorrectly added. In *D*, an error was made in regrouping.
13. *B* is the correct response. In *A*, the decimal points were improperly aligned. In *C*, an error was made in regrouping. In *D*, an error was made by adding the quantities.
14. *C* is the correct response. In *A*, the decimal point was misplaced. In *B*, the decimal point was misplaced. In *D*, an error was made in multiplication.
15. *C* is the correct response. In *A*, the decimal point was misplaced. In *B*, the decimal point was misplaced. In *D*, the decimal point was misplaced.
16. *C* is the correct response. In *A*, the wrong sign was used and the decimal point was misplaced. In *B*, the wrong sign was used. In *D*, the decimal point was misplaced.
17. *C* is the correct response. In *A*, the decimal point was misplaced. In *B*, an error was made in regrouping. In *D*, the wrong sign was used.
18. *C* is the correct response. In *A*, the wrong sign was used. In *B*, the decimal point was misplaced. In *D*, the decimal point was misplaced.

## RECOGNIZE THE MEANING OF EXPONENTS

19. **B** is the correct response. In **A**, the bases were added instead of multiplied. In **C**, the base and the exponent were multiplied. In **D**, the bases were multiplied and the exponents were multiplied.
20. **B** is the correct response. In **A**, the bases were subtracted. In **C**, the exponents were multiplied by the bases. In **D**, bases and exponents were subtracted.
21. **C** is the correct response. In **A**, the exponents were added. In **B**, the exponent was cubed. In **D**, the base was cubed.
22. **D** is the correct response. In **A**, the exponent was used only with the 2. In **B**, the bases were added. In **C**, the bases were multiplied by the exponent.

## IDENTIFY PLACE VALUE AND USE EXPANDED NOTATION

23. **B** is the correct response. In **A**, the wrong power of 10 was used. In **C**, thousandths was used instead of thousands. In **D**, an error was made with power of 10.
24. **C** is the correct response. In **A**, **B**, and **D**, errors were made in associating the incorrect power of 10 with a digit in the expanded notation.
25. **D** is the correct response. In **A**, the error was made in choosing the value of  $(4 \times 10^3)$ . In **B**, the error was made in choosing the value for  $\left(8 \times \frac{1}{10}\right)$  and  $(4 \times 10^3)$ . In **C**, the error was made in choosing the value for  $\left(8 \times \frac{1}{10}\right)$ .

## IDENTIFY EQUIVALENT DECIMALS, PERCENTS, AND FRACTIONS

26. *C* is the correct response. In *A*, the percent symbol was omitted but the decimal point was not moved. In *B*, the decimal point was incorrectly moved. In *D*, the percent was changed to an incorrect fraction.
27. *D* is the correct response. In *A*, the decimal point was moved one place instead of two. In *B*, the wrong numerator was used. In *C*, the wrong denominator was used.
28. *A* is the correct response. In *B*, the conversion was incorrect. In *C*, a percent symbol was incorrectly added. In *D*, the numerator was divided into the denominator.
29. *A* is the correct response. In *B*, the denominator was presented as the answer. In *C* and *D*, the decimal point was misplaced.
30. *C* is the correct response. In *A*, the decimal point was moved only one place and 6.2 was changed to a fraction. In *B*, the decimal point was misplaced. In *D*, the decimal was changed to  $\frac{62}{1000}$  and then reduced.
31. *B* is the correct response. In *A*, the decimal point was moved three places to the right. In *C*, the decimal point was not moved but the percent symbol was attached. In *D*, the decimal point was moved two places to the left.

## DETERMINE THE ORDER RELATIONSHIP BETWEEN TWO REAL NUMBERS

32. *A* is the correct response.  $0.25 = \frac{25}{100} = \frac{1}{4}$
33. *C* is the correct response.  $\sqrt{65} > \sqrt{64} = 8$ , so  $8 > 6.5$
34. *B* is the correct response.  $\frac{3}{7} = \frac{9}{21}$ ;  $\frac{2}{3} = \frac{14}{21}$        $\frac{9}{21} < \frac{14}{21}$

35. **B** is the correct response.  $-7\frac{1}{5} = -\frac{36}{5}$ ;  $-6\frac{2}{5} = -\frac{32}{5}$        $-\frac{36}{5} < -\frac{32}{5}$
36. **C** is the correct response.  $7.\bar{2} = 7.22\dots$ . Since  $5 > 2$ ,  $7.25 > 7.22$ .
37. **A** is the correct response.  $4.3 = 4\frac{3}{10}$

**SOLVE FOR ONE VARIABLE OF THE SENTENCE ( $a\%$  of  $b = c$ )**

38. **B** is the correct response. In **A**, 110 was divided by 0.7. In **C**, 70% was incorrectly changed to 7%. In **D**, 0.7 was divided by 110.
39. **C** is the correct response. In **A**, 200 was multiplied by .13. In **B**, 200 was divided by 1.3. In **D**, 130% was not changed to a decimal.
40. **C** is the correct response. In **A**, 0.08 was divided by 4. In **B**, 0.08 was multiplied by 4. In **D**, 4 was multiplied by 80 instead of 0.08.
41. **C** is the correct response. In **A**, 78.2 was divided by 0.17. In **B**, 78.2 was multiplied by 1.7. In **D**, 78.2 was multiplied by 1.7 and the decimal point was misplaced.
42. **D** is the correct response. In **A**, the decimal point is not removed when changing to a percent. In **B**, 300 was divided by 105. In **C**, 300 is divided by 105 and the decimal point was misplaced.
43. **A** is the correct response. In **B**, 480 was divided by 400. In **C**, 480 was divided by 400 and the decimal point was misplaced. In **D**, the decimal point was misplaced.



**CALCULATE PERCENT INCREASE AND DECREASE**

44. **B** is the correct response. In **A**, 60% of 69 was calculated. In **C**, 69 was used as the base. In **D**, the amount of increase was expressed as a percent.
45. **C** is the correct response. In **A**, the percent of decrease was added instead of subtracted. In **B**, the 12% was changed to a whole number and then subtracted from 300. In **D**, the amount of change was computed but not added to the original amount.
46. **C** is the correct response. In **A**, 150 was divided by 200. In **B**, the \$50 decrease was expressed as a percent. In **D**, the wrong number was used as the base.
47. **A** is the correct response. In **B**, the percent was changed to pounds and added. In **C**, the percent was subtracted instead of added. In **D**, the increase was not added to the original weight.

**SOLVE WORD PROBLEMS WITHOUT VARIABLES OR PERCENTS**

48. **A** is the correct response. In **B**, a computation error was made. In **C**, only 20 of the 16-ounce cans would need to be purchased for a total cost of \$18. In **D**, 320 ounces of the 8-ounce cans would require 40 cans for a total cost of \$20. To find the savings, subtract answer **C** from answer **D**.
49. **C** is the correct response. To arrive at this solution, multiply  $40 \times \$12 = \$480$ , subtract  $52 - 40 = 12$ . Next multiply  $12 \times (\$12 + \$2)$  and then add 480 and 168. The answer is \$648. In **A**, the solution was obtained by calculating  $40 \times 12 = \$480$ ,  $52 - 40 = 12$ ,  $12 \times 2 = 24$ ,  $480 + 24 = 504$ . The error was made in computing the overtime. In **B**, 52 was simply multiplied by 12. In **D**, \$14 was multiplied by the 52 hours.
50. **B** is the correct response. To arrive at this solution, subtract  $9 - 6 = 3$  people over 6. At \$15 per day, 3 additional people will cost \$45 extra per day.  $\$120 + \$45 = \$165$  per day. Multiply the  $\$165 \times 5$  days = \$825. In **A**, \$120 was multiplied by 9. In **C**, the \$120 was added to \$15 and multiplied by 5. In **D**, \$120 was multiplied by 5 and then \$45 was added.
51. **B** is the correct response. To arrive at this solution, subtract 53 from 329 and divide by 2. In **A**, 329 was divided by 2, then 53 was subtracted. In **C**, 191 is the number of students in the 10 o'clock class. In **D**, the answer was reached by subtracting 53 from 329.

## SOLVE WORD PROBLEMS INVOLVING PERCENTS

52. *C* is the correct response. To arrive at this solution, first change 35% to 0.35. Then multiply 0.35 by 20 ounces to determine that the amount of sugar consumed was 7 ounces. To find the nonsugar amount, subtract 7 ounces from 20 ounces. In *A*, the 35% was computed, but not subtracted. In *B*, 20 ounces was divided by 2. In *D*, the 35% was incorrectly changed to 3.5 and then multiplied by 20 ounces.
53. *B* is the correct response. To arrive at this solution, first change 78% to 0.78. Then multiply  $0.78 \times 150$  to obtain 117. Subtract 117 from 150 to obtain 33. Or, subtract 78% from 100% to obtain 22%. Then multiply  $0.22 \times 150$  to obtain 33. In *A*, the 0.22 was multiplied by 100 instead of 150. In *C*, necessary steps were excluded. In *D*, 117 was not subtracted from 150.
54. *C* is the correct response. To arrive at this solution, first change 7% to 0.07. Multiply 0.07 by 300 to obtain 21. Change 20% to 0.2 and multiply 0.2 by 250 to obtain 50. Add 21 and 50 to obtain 71. In *A*, only the 7% of 300 was calculated. In *B*, only the 20% of 250 was calculated. In *D*, 20% and 7% were added to get 27%, and then 27% of 550 was calculated.
55. *D* is the correct response. To arrive at this solution, divide 76 by 190 to obtain 0.4. Then change 0.4 to 40%. In *A*, 76 was divided by 265. In *B*, 25 was divided by 75. In *C*, 101 was divided by 265.

## SOLVE PROBLEMS INVOLVING THE STRUCTURE AND LOGIC OF ARITHMETIC

56. *B* is the correct response. Only one number satisfies all the conditions. The numbers that leave a remainder of 3 when divided into 31 are 4, 7, 14, and 28. The numbers that leave a remainder of 4 when divided into 25 are 3, 7, and 21. So, 7 is the only number that appears in both groups. Therefore, there is only 1 number that satisfies all conditions.

57. *C* is the correct response. Answer *A* may be eliminated quickly since 2 is not a multiple of 4. The 8, 20, and 32 are multiples of 4. Which of those remaining leave a remainder of 2 when divided by 3?

$$\begin{array}{r} 2\ r2 \\ 3\overline{)8} \end{array} \quad \begin{array}{r} 6\ r2 \\ 3\overline{)20} \end{array} \quad \begin{array}{r} 10\ r2 \\ 3\overline{)32} \end{array}$$

Since all do, none can be eliminated.

Which number leaves a remainder of 0 when divided by 5?

$$\begin{array}{r} 1\ r3 \\ 5\overline{)8} \end{array} \quad \begin{array}{r} 4 \\ 5\overline{)20} \end{array} \quad \begin{array}{r} 6\ r2 \\ 5\overline{)32} \end{array}$$

Thus, 20 is the correct answer. Answers *B* and *D* do not satisfy the test.

### ESTIMATE SUMS, AVERAGES, AND PRODUCTS

58. *B* is the correct response. The correct response must fall between 58 and 82. Options *A*, *C*, and *D* do not fall between these numbers.
59. *B* is the correct response. In *A*, the number is greater than the highest possible score. In *C*, the lowest value in the set was used. In *D*, 18 and 8 were simply added.
60. *B* is the correct response. The correct response must fall between \$5.00 and \$20.00. Options *A*, *C*, and *D* do not fall between these values.
61. *C* is the correct response. In *A*, the numbers were multiplied. In *B*, the estimate is too high. In *D*, the estimate is too low.
62. *B* is the correct response. In *A*, the estimate is too high. In *C*, the estimate is too low. In *D*, the estimate is too low.

## FIND MISSING NUMBERS GIVEN A PATTERN

63. **C** is the correct response. The difference between successive terms is  $+4$ . The missing term is  $21 + 4$  or  $25$ . In **A**, the wrong relationship was used, as  $4$  was subtracted instead of added. In **B**,  $3$  was added instead of  $4$ . In **D**, the incorrect operation was used, as  $4$  was multiplied instead of added.
64. **A** is the correct response. The ratio between successive terms is  $-\frac{10}{5} = -2$ . So the missing term is  $80$  times  $-2$  or  $-160$ . In **B**, the wrong relationship was used. In **C**, the wrong relationship was used. In **D**,  $80$  was incorrectly multiplied by  $+2$  instead of  $-2$ .
65. **B** is the correct response. The numerator should be  $1$ . The denominators form an arithmetic progression with a difference of  $4$ . The denominator of the missing term will be  $18 + 4$  or  $22$ . The missing term of the harmonic progression is  $\frac{1}{22}$ . In **A**, the wrong relationship was used, as the denominator was incorrectly multiplied by  $3$ . In **C**, the wrong relationship was used, as  $\frac{1}{18}$  was multiplied by  $3$ . In **D**, the wrong relationship was used as the numerator was omitted from the fraction.
66. **B** is the correct response. The relationship between the first and second numbers in each set is that the second number is formed by subtracting  $4$  from the first number. When  $4$  is subtracted from  $3$ , the missing term is  $-1$ . In **A**,  $3$  was incorrectly multiplied by  $-4$ , instead of added. In **C**, the additive inverse of the correct answer is given. In **D**,  $4$  was incorrectly added to  $3$ .
67. **D** is the correct response. The relationship between the first and second number in each set is that the second number is the first number multiplied by  $3$ . The missing number is determined by multiplying  $5 \times 3$ , or  $15$ . In **A**,  $5$  was incorrectly multiplied by  $-3$  instead of  $3$ . In **B**,  $5$  was incorrectly multiplied by  $\frac{1}{3}$  instead of  $3$ . In **C**, the wrong relationship was used, as  $3$  was incorrectly subtracted from  $5$  instead of multiplied.